Theoretical Physics 6a (QFT): SS 2025
Exercise sheet 9

16.06.2025

(0)(0 points) How much time did it take you to solve this exercise sheet?

Exercise 1. (100+25 points): Scalar QED

(a)(70 points) Calculate the square of the amplitude for scalar QED Compton Scattering:

$$p + k = p' + k'$$

And write the differential cross section **in the lab frame**.

(b)(30 points) In the previous question you calculated the cross-section for the Compton scattering in scalar QED, $p + k = p' + k'$.
It appears that this result can be also used to get the cross-section of pair production from two photons, $k + k' = p + p'$. To perform this, start from the matrix element squared, $|M|^2$, and replace $p \to -p$, $k' \to -k'$. Then write the cross-section using the general formula **in the center-of-mass frame**.
Hint: these crossing rules are very useful in QFT - using the symmetry properties of Feynman diagrams you are able to greatly reduce the computation time by expressing the matrix elements of different processes in terms of each other.

(c)(25 bonus points) Consider decay $1 \to 3$:

$$p_1 = p_2 + p_3 + p_4$$

It is possible if $m_1 > m_2 + m_3 + m_4$. Prove the formula for differential decay rate:

$$d\Gamma = \frac{dtdu}{32\,(2\pi)^3\, m_1^3}\,|M_{fi}|^2$$

Where denoted:

$$t = (p_1 - p_3)^2 = (p_2 + p_4)^2$$
$$u = (p_1 - p_4)^2 = (p_2 + p_3)^2$$