HIGGS MECHANISM

- PROBLEM

$SU(2) \times U(1)_Y$ GAUGE SYMMETRY OF STANDARD ELECTROWEAK THEORY

$\downarrow$

4 GAUGE BOSONS: γ, W^+, W^-, Z^0

MANIFEST GAUGE SYMM. REQUIRES GAUGE BOSONS TO BE MASSLESS

BUT EXPERIMENTALLY $M_W \simeq 80.4$ GeV

$M_Z \simeq 91.2$ GeV

$\Rightarrow$ ADDING MASS TERMS IN $\mathcal{L}$ (EXPLICIT SYMM. BREAKING) YIELDS INCONSISTENT THEORY

$\rightsquigarrow$ UNRENORMALIZABLE DIVERGENCES APPEAR.

$\Rightarrow$ SOLUTION: ONE HAS TO ADD MASS WITHOUT BREAKING GAUGE INVARIANCE

$\Downarrow$

SPONTANEOUS SYMMETRY BREAKING

• SPONTANEOUS SYMMETRY BREAKING OF DISCRETE SYMMETRY

CONSIDER EXAMPLE OF SCALAR FIELD THEORY

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2}\mu^2\phi^2 - \frac{\lambda}{4}\phi^4$$

$$= T - V$$

INTERACTION TERM ($\lambda > 0$)

$$V = \frac{1}{2}\mu^2\phi^2 + \frac{\lambda}{4}\phi^4$$

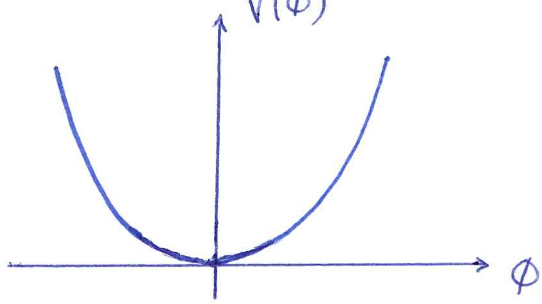
$\mathcal{L}$ HAS SYMMETRY w.r.t $\phi \to -\phi$ (Z_2 SYMMETRY)

↳ **CASE $\mu^2 > 0$**

⇓

MINIMUM OF V

$\phi = 0$



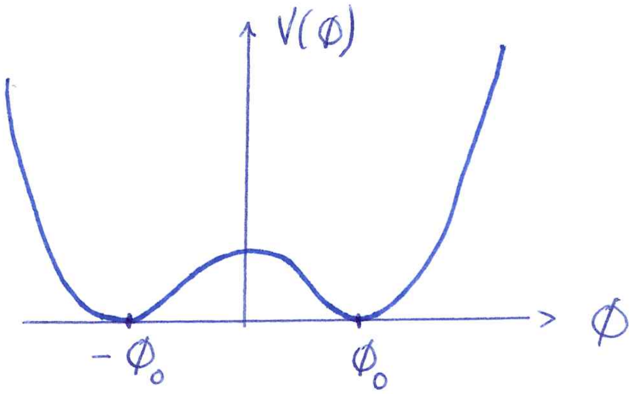
⇒ GROUND STATE ($\phi = 0$) REFLECTS SYMMETRY $\phi \to -\phi$ OF $\mathcal{L}$

∴ MANIFEST SYMMETRY : SYMMETRY OF DYNAMICS ($\mathcal{L}$)
= SYMMETRY OF GROUND STATE

↳ **CASE $\mu^2 < 0$**

⇓

MINIMUM OF V



$$\frac{dV}{d\phi} = \mu^2\phi + \lambda\phi^3$$

$$= \phi(\mu^2 + \lambda\phi^2) = 0$$

$\phi = 0$: MAXIMUM

$\phi = \pm\sqrt{-\frac{\mu^2}{\lambda}}$: MINIMUM

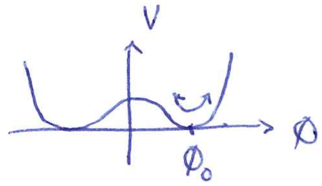
$\equiv \pm \phi_0$

2 DEGENERATE MINIMA / GROUND STATES $\phi_0, -\phi_0$

$\therefore$ <u>HIDDEN</u> SYMMETRY (SPONTANEOUSLY BROKEN SYMM.)

SYMMETRY OF DYNAMICS ($\mathcal{L}$)

IS <u>NOT</u> SYMMETRY OF GROUND STATE



PERTURBATIVE EXPANSION AROUND $\phi_0 = +\sqrt{-\frac{\mu^2}{\lambda}}$

$$\phi(x) = \phi_0 + \eta(x)$$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \frac{\lambda}{4}\left(\phi^2 + \frac{\mu^2}{\lambda}\right)^2 + \frac{\mu^4}{4\lambda}$$

$$= \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \frac{\lambda}{4}\left(\cancel{\phi_0^2} + 2\phi_0\eta + \eta^2 + \cancel{\frac{\mu^2}{\lambda}}\right)^2 + \frac{\mu^4}{4\lambda}$$

$$= \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \underbrace{\lambda\phi_0^2}_{-\mu^2}\eta^2 - \lambda\phi_0\eta^3 - \frac{\lambda}{4}\eta^4 + \text{CONSTANT}$$

$$\boxed{\mathcal{L} = \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \frac{1}{2}\underbrace{(-2\mu^2)}_{\equiv\, m_\eta^2}\eta^2 - \underbrace{\lambda\phi_0\eta^3 - \frac{\lambda}{4}\eta^4}_{\text{INTERACTION TERMS}}}$$

MASS TERM

DUE TO SPONTANEOUS SYMM. BREAKING

$\Rightarrow$ MASS TERM (m_η^2) IS CREATED IN MASSLESS THEORY

$\rightsquigarrow$ PHYSICAL EXAMPLES :

FERROMAGNETISM : SPINS CAN ALIGN ALONG ONE DIRECTION

2 DEGENERATE GROUND STATES

$\uparrow\uparrow\uparrow\uparrow \qquad \downarrow\downarrow\downarrow\downarrow$

• SPONTANEOUS SYMMETRY BREAKING OF CONTINUOUS GLOBAL SYMMETRY

↳ COMPLEX SCALAR FIELD ϕ , ϕ^*

$$\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$$ ϕ_1, ϕ_2 : REAL FIELDS.

$$\mathcal{L} = (\partial_\mu \phi)^* (\partial^\mu \phi) - \mu^2 \phi^* \phi - \lambda (\phi^* \phi)^2$$

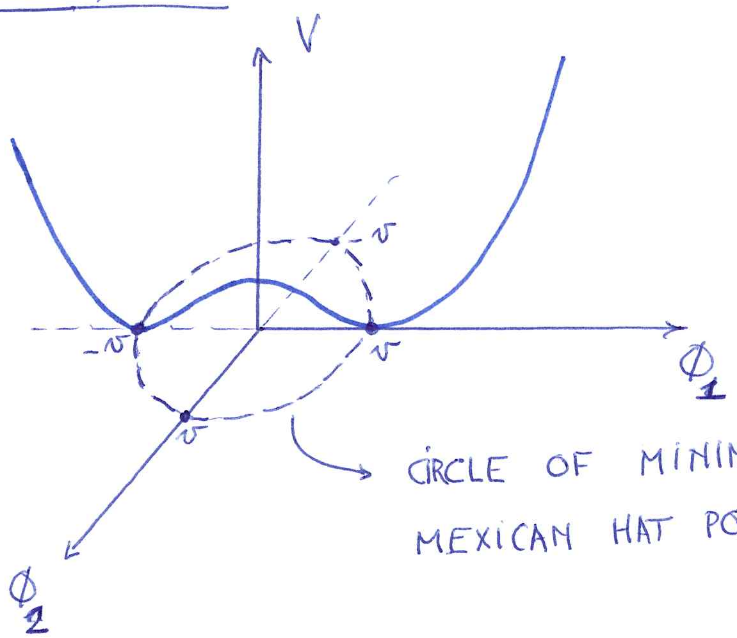
$\mathcal{L}$ HAS CONTINUOUS GLOBAL SYMMETRY

$$\phi(x) \xrightarrow{U(1)} e^{i\alpha} \phi(x)$$: α CONSTANT : GLOBAL PHASE TF.

IN TERMS OF ϕ_1, ϕ_2

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi_1)(\partial^\mu \phi_1) + \frac{1}{2}(\partial_\mu \phi_2)(\partial^\mu \phi_2) - \underbrace{\left[\frac{1}{2}\mu^2(\phi_1^2 + \phi_2^2) + \frac{\lambda}{4}(\phi_1^2 + \phi_2^2)^2 \right]}_{V(\phi_1, \phi_2)}$$

FOR $\mu^2 < 0, \lambda > 0$: MINIMA OF V : $\phi_1^2 + \phi_2^2 = v^2 = -\frac{\mu^2}{\lambda}$



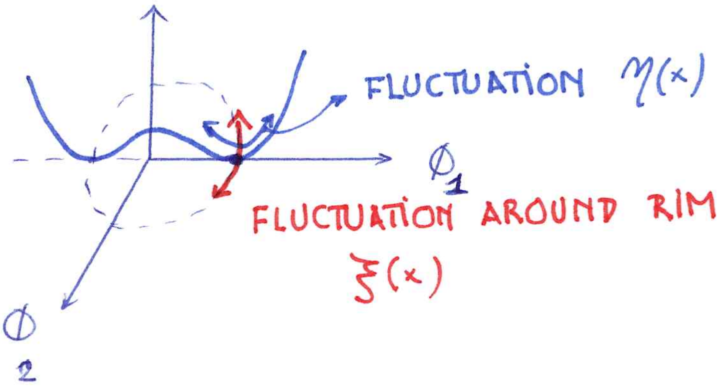
CIRCLE OF MINIMA

MEXICAN HAT POTENTIAL

↳ PERTURBATIVE EXPANSION AROUND A MINIMUM

$$\phi(x) = \frac{1}{\sqrt{2}}\left(v + \eta(x) + i\,\xi(x)\right)$$

$$v = \sqrt{-\frac{\mu^2}{\lambda}}$$



$$\mathcal{L} = \frac{1}{2}(\partial_\mu \xi)(\partial^\mu \xi) + \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta)$$

$$- \frac{\lambda}{4}\left((v+\eta)^2 + \xi^2 + \frac{\mu^2}{\lambda}\right)^2 + \text{CONSTANT}$$

$$\downarrow \quad v^2 = -\frac{\mu^2}{\lambda}$$

$$= \frac{1}{2}(\partial_\mu \xi)(\partial^\mu \xi) + \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta)$$

$$- \frac{\lambda}{4}\left(4v^2\eta^2 + 4v\eta(\eta^2+\xi^2) + (\eta^2+\xi^2)^2\right)$$

$$= \frac{1}{2}(\partial_\mu \xi)(\partial^\mu \xi) + \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \frac{1}{2}\underbrace{(-2\mu^2)}_{\equiv\, m_\eta^2}\eta^2$$

$$\underbrace{- \lambda v \eta(\eta^2+\xi^2) - \frac{\lambda}{4}(\eta^2+\xi^2)^2}_{\text{INTERACTION TERMS}}$$

η FIELD HAS ACQUIRED A MASS $m_\eta = \sqrt{-2\mu^2}$

ξ FIELD IS MASSLESS !

$\Downarrow$

FLUCTUATIONS ALONG RIM (FROM ONE MINIMUM TO OTHER MINIMUM)

COST NO ENERGY $\Rightarrow$ GOLDSTONE BOSONS

(MASSLESS EXCITATIONS)

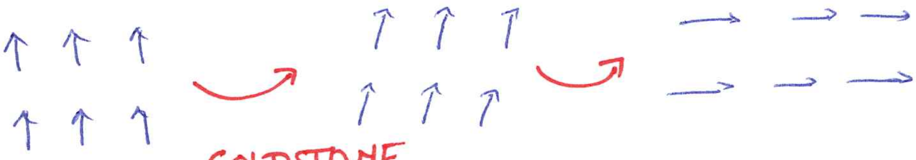
↳ PHYSICAL EXAMPLE : FERROMAGNETISM WITH SPINS IN 3 DIM

$$H = -J\sum_{i,j} \bar{S}_i \cdot \bar{S}_j$$

H : HAS ROTATIONAL SYMM.

IN FERROMAGNETIC PHASE

ROTATIONAL SYMM IS SPONTANEOUSLY BROKEN



GOLDSTONE BOSON (SPIN WAVES)

↳ EXAMPLE IN QCD : $SU(2)_L \times SU(2)_R$ CHIRAL SYMMETRY

$$\mathcal{L}_{QCD} = \bar{u}\left(i\gamma^\mu D_\mu\right)u + \bar{d}\left(i\gamma^\mu D_\mu\right)d - m_u \bar{u}u - m_d \bar{d}d$$
$$- \frac{1}{4} F_{a\mu\nu} F_a^{\mu\nu}$$

$m_u \simeq 2$ MeV , $m_d \simeq 5$ MeV SMALL COMPARED TO $\Lambda_{QCD} \simeq 200$ MeV

IN LIMIT $m_u = m_d = 0$ (CHIRAL LIMIT)

LEFT & RIGHT HANDED QUARKS DECOUPLE

$$q^L \equiv \frac{1}{2}(1-\gamma_5)\, q \qquad q^R \equiv \frac{1}{2}(1+\gamma_5)\, q \quad \Rightarrow \quad q = q^L + q^R$$

$$\mathcal{L}_{QCD}^{\text{CHIRAL LIMIT}} = \bar{u}^L\left(i\gamma^\mu D_\mu\right)u^L + \bar{u}^R\left(i\gamma^\mu D_\mu\right)u^R$$
$$+ \bar{d}^L\left(i\gamma^\mu D_\mu\right)d^L + \bar{d}^R\left(i\gamma^\mu D_\mu\right)d^R$$
$$- \frac{1}{4} F_{a\mu\nu} F_a^{\mu\nu}$$

$\mathcal{L}_{QCD}^{\text{CHIRAL LIMIT}}$ HAS GLOBAL $SU(2)_L \times SU(2)_R$ SYMMETRY

$$\begin{pmatrix} u^L \\ d^L \end{pmatrix} \xrightarrow{SU(2)_L} \exp\left\{ i\,\theta_i^L \frac{\tau_i}{2} \right\} \begin{pmatrix} u^L \\ d^L \end{pmatrix}$$

θ_i^L, θ_i^R CONSTANTS

$$\begin{pmatrix} u^R \\ d^R \end{pmatrix} \xrightarrow{SU(2)_R} \exp\left\{ i\,\theta_i^R \frac{\tau_i}{2} \right\} \begin{pmatrix} u^R \\ d^R \end{pmatrix}$$

OR EQUIVALENTLY

$$\begin{pmatrix} u \\ d \end{pmatrix} \longrightarrow \exp\left\{ i\,\theta_i \frac{\tau_i}{2} + i\,\eta_i \frac{\tau_i}{2}\gamma_5 \right\} \begin{pmatrix} u \\ d \end{pmatrix}$$

SUBGROUP OF TRANSFORMATIONS θ_i WITH $\eta_i = 0$
↳ ISOSPIN

NOETHER THEOREM : SYMMETRIES ⟷ CONSERVATION LAWS.

SYMM. CURRENTS

$$J_i^\mu = \bar{q}\,\gamma^\mu \frac{\tau_i}{2}\, q \qquad \text{VECTOR CURRENT}$$

$$J_{5i}^\mu = \bar{q}\,\gamma^\mu \gamma_5 \frac{\tau_i}{2}\, q. \qquad \text{AXIAL VECTOR CURRENT}$$

SPECTRUM OF QCD

SYMMETRY UNDER $u \leftrightarrow d$

$p : uud \Rightarrow 938.3$ MeV

$n : ddu \Rightarrow 939.6$ MeV

- ISOSPIN SYMM IS MANIFEST
- AXIAL SYMM IS <u>NOT</u> MANIFEST $\Rightarrow$ SPONTANEOUSLY BROKEN

$SU(2)_L \times SU(2)_R \longrightarrow SU(2)_{\text{ISO-SPIN}}$

↳ 3 GOLDSTONE BOSONS π^+, π^-, π^0 :

FOR $SU(3)_L \times SU(3)_R \longrightarrow SU(3)$
8 GOLDSTONE BOSONS π, K, η

GAP ↕

$N(1535)$ $\frac{1}{2}^-$

P 983.3 MeV $\frac{1}{2}^+$

• SPONTANEOUS SYMMETRY BREAKING OF LOCAL GAUGE SYMMETRY U(1) : ABELIAN HIGGS MODEL

COMPLEX SCALAR FIELD

$$\mathcal{L}_0 = (\partial_\mu \phi)^* (\partial^\mu \phi) - \mu^2 \phi^* \phi - \lambda (\phi^* \phi)^2$$

REQUIRE $\mathcal{L}$ TO BE INVARIANT UNDER
<u>LOCAL</u> U(1) GAUGE SYMMETRY

$$\phi(x) \xrightarrow{U(1)} e^{i\alpha(x)} \phi(x)$$

WE NEED TO INTRODUCE GAUGE FIELDS

$$\partial^\mu \Rightarrow D^\mu = \partial^\mu + ieA^\mu$$

$$A^\mu(x) \xrightarrow{U(1)} A^\mu - \frac{1}{e} \partial^\mu \alpha$$

$$\mathcal{L} = \left[(\partial_\mu - ieA_\mu)\phi^*\right]\left[(\partial^\mu + ieA^\mu)\phi\right] - \mu^2 \phi^* \phi - \lambda (\phi^* \phi)^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$= (\partial_\mu \phi^*)(\partial^\mu \phi) - \mu^2 \phi^* \phi - \lambda (\phi^* \phi)^2$$
$$- ieA_\mu \left[\phi^* (\partial^\mu \phi) - (\partial^\mu \phi^*) \phi\right] + e^2 A_\mu A^\mu \phi^* \phi$$
$$- \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

↳ FOR <u>$\mu^2 > 0$</u> : QED LAGRANGIAN FOR CHARGED SPIN 0 PARTICLE WITH MASS μ + INTERACTION TERM $\lambda(\phi^*\phi)^2$

↳ FOR <u>$\mu^2 < 0$</u> : SPONTANEOUS SYMMETRY BREAKING

CASE $\mu^2 < 0$: EXPANSION AROUND A MINIMUM

$$\phi(x) = \frac{1}{\sqrt{2}} \left\{ v + \eta(x) + i\,\xi(x) \right\} \quad \text{WITH } v = \sqrt{-\frac{\mu^2}{\lambda}}$$

$$\begin{aligned}
\mathcal{L} = {} & \frac{1}{2}(\partial_\mu \xi)(\partial^\mu \xi) + \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) \\
& - \frac{\lambda}{4}\left((v+\eta)^2 + \xi^2 + \frac{\mu^2}{\lambda} \right)^2 + \text{CONSTANT} \\
& - ie A_\mu \frac{1}{2} \Big[(v + \eta - i\xi)(\partial^\mu \eta + i \partial^\mu \xi) \\
& \qquad - (v + \eta + i\xi)(\partial^\mu \eta - i \partial^\mu \xi) \Big] \\
& + e^2 A_\mu A^\mu \frac{1}{2}\left((v+\eta)^2 + \xi^2 \right) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\
= {} & \frac{1}{2}(\partial_\mu \xi)(\partial^\mu \xi) + \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \frac{1}{2}(+2v^2\lambda)\eta^2 \\
& + ev A_\mu(\partial^\mu \xi) + \underbrace{\frac{1}{2} e^2 v^2 A_\mu A^\mu} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\
& + \text{INTERACTION TERMS}
\end{aligned}$$

(CUBIC & QUARTIC TERMS IN η, ξ, A^μ FIELDS)

$\Downarrow$

GAUGE FIELD A^μ HAS ACQUIRED A MASS !

$$\underline{\underline{m_A = ev}}$$

BUT : MASSIVE SPIN 1 PARTICLE HAS.

3 POLARIZATIONS : 2 TRANSVERSE (± 1)
1 LONGITUDINAL (0)

$\updownarrow$

MASSLESS SPIN 1 (GAUGE) PARTICLE
ONLY HAS 2 TRANSVERSE POLARIZATIONS

$\therefore$ WE CAN MAKE GAUGE TF. TO ELIMINATE GOLDSTONE BOSON (ξ) & CONVERT IT TO LONGITUDINAL DEGREE OF FREEDOM

$$\phi(x) = \frac{1}{\sqrt{2}}\left(v + \eta(x)\right) e^{i\frac{\xi(x)}{v}}$$

TO LOWEST ORDER IN ξ THIS IS EQUIVALENT TO FORM ABOVE

$$\underset{\sim\sim\sim}{\text{GAUGE TF.}} \qquad \phi(x) \longrightarrow \frac{1}{\sqrt{2}}\left(v + \eta(x)\right) \underbrace{e^{i\frac{\xi(x)}{v}}}_{\text{PHASE}}$$

$$A^\mu \longrightarrow A^\mu - \frac{1}{e}\frac{1}{v}\left(\partial^\mu \xi\right)$$

$$\left(D_\mu \phi\right)^* \left(D^\mu \phi\right) = \frac{1}{2}\left(\partial_\mu \eta - ie\left(v+\eta\right)A_\mu\right) \cdot \left(\partial^\mu \eta + ie\left(v+\eta\right)A^\mu\right)$$

$$\boxed{\begin{aligned}\mathcal{L} = {} & \frac{1}{2}\left(\partial_\mu \eta\right)\left(\partial^\mu \eta\right) - \frac{\lambda}{4}\left(\left(v+\eta\right)^2 - v^2\right)^2 \\ & + \frac{1}{2}\left(e^2 v^2\right) A_\mu A^\mu + e^2 v \eta A_\mu A^\mu + \frac{1}{2} e^2 \eta^2 A_\mu A^\mu \\ & - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}\end{aligned}}$$

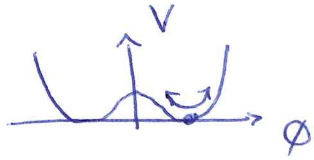
ABELIAN HIGGS MODEL

↳ GOLDSTONE BOSON (ξ) DISAPPEARED
HAS BEEN CONVERTED TO LONGITUDINAL POLARIZATION
OF A^μ FIELD

↳ GAUGE BOSON HAS ACQUIRED A MASS

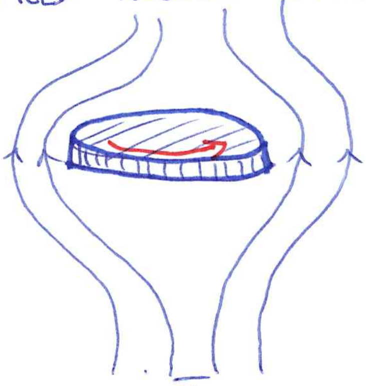
$$m_A = e v$$

↳ HIGGS FIELD (η) : EXCITATIONS



MASS $m_\eta^2 = 2v^2\lambda$ (MASSIVE SCALAR PARTICLE)

⇝ PHYSICAL EXAMPLE : TYPE II SUPERCONDUCTOR
$v \longleftrightarrow$ CONDENSATE OF (ELECTRICALLY CHARGED) COOPER PAIRS
SCREENING OF MAGNETIC FIELD INSIDE SUPERCONDUCTOR
MEISSNER EFFECT



$\bar{B}$ FIELD CAN ONLY PENETRATE SHORT DISTANCE R
INSIDE SUPERCONDUCTOR $\Rightarrow$ SCREENING
POTENTIAL $\dfrac{e^{-r/R}}{r} \Longleftrightarrow$ MASS $m_A \sim \dfrac{1}{R}$

WHEN $\bar{B} \uparrow$: MAGNETIC FIELD WILL FORM VORTICES
OF SIZE ξ INSIDE SUPERCONDUCTOR
WHERE SUPERCONDUCTIVITY IS LOST

CORRELATION LENGTH $\xi < R = \dfrac{1}{m_A}$: TYPE II SUPERCONDUCTOR $\Rightarrow$ ABRIKOSOV VORTICES
$\xi > R$: TYPE I SUPERCONDUCTOR : VORTICES DO NOT FORM SPONTANEOUSLY

- SPONTANEOUS SYMMETRY BREAKING OF LOCAL NON-ABELIAN GAUGE SYMMETRY $SU(2) \times U(1) \rightarrow U(1)_{EM}$

↳ COMPLEX DOUBLET FIELD UNDER $SU(2) \times U(1)_Y$

$$H = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

	t	t_3	$\frac{Y}{2}$	Q
ϕ^+	$\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$	$+1$
ϕ^0	$\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$	0

$$\mathcal{L} = (D_\mu H)^\dagger (D^\mu H) - \lambda \left(H^\dagger H - \frac{v^2}{2} \right)^2 - \frac{1}{4} W_{i\mu\nu} W^{i\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

$$D^\mu H = \left[\partial^\mu + i g W_i^\mu \frac{\tau_i}{2} + i g' B^\mu \frac{Y}{2} \right] H$$

UNDER $\boxed{SU(2) \times U(1)_Y \rightarrow U(1)_{EM}}$

POTENTIAL TERM INDUCES SPONTANEOUS SYMM. BREAKING

$$D^\mu H \rightarrow \exp\left\{ i \, \xi_i(x) \frac{\tau_i}{2} \right\} D^\mu H$$

WE CAN CHOOSE H IN UNITARY GAUGE (IN WHICH ONLY PHYSICAL FIELDS APPEAR) AS:

INSTEAD OF

$$\frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1(x) + i\phi_2(x) \\ \phi_3(x) + i\phi_4(x) \end{pmatrix} \Rightarrow H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} \qquad H_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

WITH $h(x)$ A REAL, SCALAR FIELD (HIGGS) & $v/\sqrt{2}$ VACUUM EXPECTATION VALUE OF H

$$\hookrightarrow \quad D^\mu H = \frac{1}{\sqrt{2}} \begin{bmatrix} i\frac{g}{2}(W_1^\mu - iW_2^\mu)(v+h) \\ \partial^\mu h + \frac{1}{2}(-gW_3^\mu + g'B^\mu)(v+h) \end{bmatrix}$$

USING $W^\mu = \frac{1}{\sqrt{2}}(W_1^\mu - iW_2^\mu)$

& Z^μ FIELDS

$$\begin{cases} W_3^\mu = \cos\theta_W Z^\mu + \sin\theta_W A^\mu \\ B^\mu = -\sin\theta_W Z^\mu + \cos\theta_W A^\mu \end{cases}$$

$$-gW_3^\mu + g'B^\mu = (-g\cos\theta_W - g'\sin\theta_W) Z^\mu + \underbrace{(-g\sin\theta_W + g'\cos\theta_W)}_{0} A^\mu$$

$$\underline{\underline{g'\cos\theta_W = g\sin\theta_W = e}}$$

$$\downarrow$$

$$-gW_3^\mu + g'B^\mu = -\frac{g}{\cos\theta_W} Z^\mu$$

$$\therefore \quad D^\mu H = \frac{1}{\sqrt{2}} \begin{bmatrix} i\frac{g}{\sqrt{2}} W^\mu (v+h) \\ \partial^\mu h - \frac{ig}{2\cos\theta_W} Z^\mu (v+h) \end{bmatrix}$$

$$\hookrightarrow (D^\mu H)^\dagger (D_\mu H)$$

$$= \frac{1}{2}(\partial^\mu h)(\partial_\mu h) + \frac{1}{4}(v+h)^2 g^2 W_\mu^+ W^\mu + \frac{1}{2}(v+h)^2\left(\frac{g}{2\cos\theta_W}\right)^2 Z_\mu Z^\mu$$

$$= \frac{1}{2}(\partial^\mu h)(\partial_\mu h) + \left(\frac{gv}{2}\right)^2 W_\mu^+ W^\mu + \frac{1}{2}\left(\frac{gv}{2\cos\theta_W}\right)^2 Z_\mu Z^\mu$$

$$+\left(2\frac{h}{v} + \frac{h^2}{v^2}\right)\left[\left(\frac{gv}{2}\right)^2 W_\mu^+ W^\mu + \frac{1}{2}\left(\frac{gv}{2\cos\theta_W}\right)^2 Z_\mu Z^\mu\right]$$

$W^\pm$, Z BOSONS ACQUIRE MASS !

$$\boxed{m_W = \frac{gv}{2} = \frac{v\,e}{2\sin\theta_W}}$$

$$m_Z = \frac{gv}{2\cos\theta_W} \Rightarrow \boxed{m_Z = \frac{m_W}{\cos\theta_W}}$$

EXPERIMENT : $W^\pm$, Z BOSONS DISCOVERED AT CERN (1983)

$\Downarrow$

1984 NOBEL PRIZE (RUBBIA, VAN DER MEER)

$m_W = 80.4$ GeV

$m_Z = 91.2$ GeV

HIGGS VACUUM EXPECTATION VALUE $\underline{v = 251 \text{ GeV}}$

↳ HIGGS PART OF $\mathcal{L}$

$$\mathcal{L}_{HIGGS} = \frac{1}{2}(\partial^\mu h)(\partial_\mu h) - \frac{\lambda}{4}\left(h^2 + 2hv\right)^2$$
$$+ \left(\frac{2h}{v} + \frac{h^2}{v^2}\right)\left[m_W^2 W_\mu^+ W^\mu + \frac{1}{2} m_Z^2 Z_\mu Z^\mu\right]$$

↓ HIGGS IS MASSIVE

$$m_H^2 = 2\lambda v^2$$

m_H IS FREE PARAMETER

2012: HIGGS DISCOVERY@CERN

$m_H = 125.25$ GeV

2013: NOBEL PRIZE (ENGLERT, HIGGS)

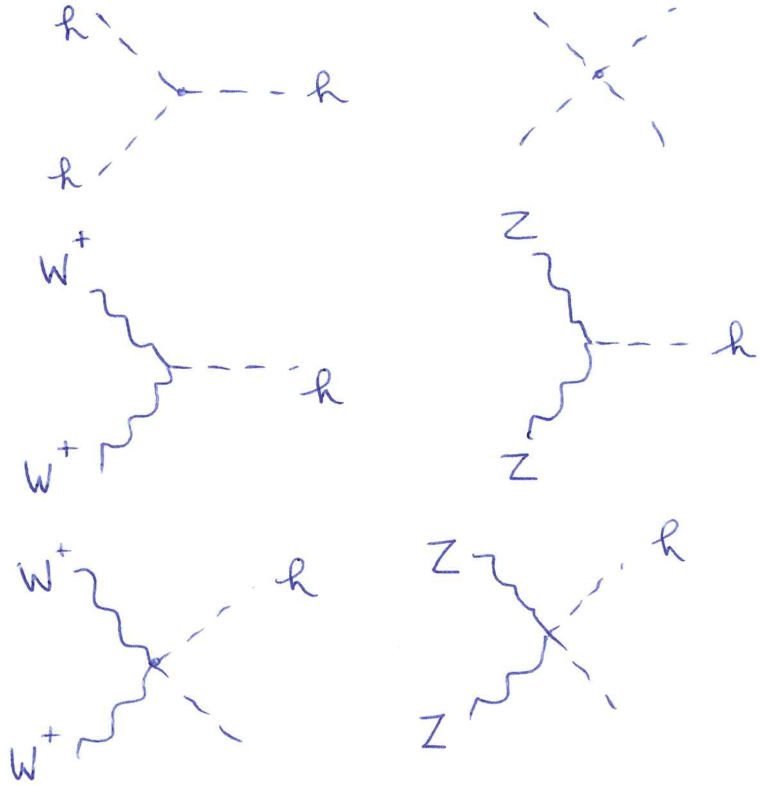
$$\boxed{\begin{aligned}\mathcal{L}_{HIGGS} &= \frac{1}{2}(\partial^\mu h)(\partial_\mu h) - \frac{1}{2} m_H^2 h^2 \left(1 + \frac{h}{2v}\right)^2 \\ &+ \left(\frac{2h}{v} + \frac{h^2}{v^2}\right)\left[m_W^2 W_\mu^+ W^\mu + \frac{1}{2} m_Z^2 Z_\mu Z^\mu\right]\end{aligned}}$$

WE CAN TRADE v FOR m_W

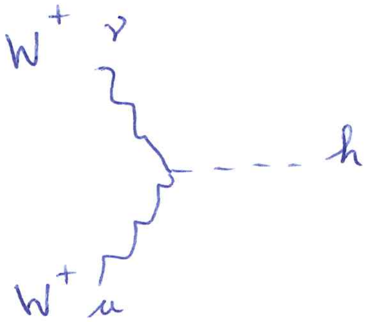
$$\frac{1}{v} = \frac{g}{2m_W} = \frac{e}{2\sin\theta_W}\frac{1}{m_W}$$

$$\mathcal{L}_{HIGGS} = \frac{1}{2}(\partial^\mu h)(\partial_\mu h) - \frac{1}{2} m_H^2 h^2$$
$$- g\frac{m_H^2}{4m_W} h^3 - \frac{g^2}{32}\frac{m_H^2}{m_W^2} h^4$$
$$+ \frac{gh}{m_W}\left(1 + \frac{g}{4m_W} h\right)\left[m_W^2 W_\mu^+ W^\mu + \frac{1}{2} m_Z^2 Z_\mu Z^\mu\right]$$

INTERACTION TERMS:

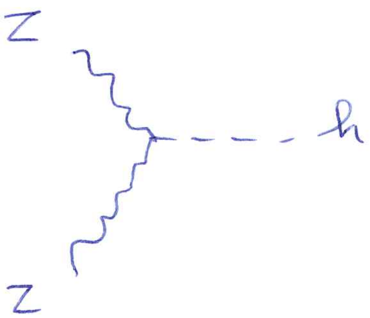


e.g. FEYNMAN RULES



$$\boxed{ig\, m_W\, g^{\mu\nu} = \frac{i\,e}{\sin\theta_W}\, m_W\, g^{\mu\nu}}$$

HIGGS COUPLING
PROPORTIONAL TO MASS



$$\boxed{ig\, \frac{m_Z^2}{m_W}\, g^{\mu\nu} = \frac{i\, e\, m_W}{\sin\theta_W \cos^2\theta_W}\, g^{\mu\nu}}$$

↳ FERMION MASSES : YUKAWA TERMS

e.g. e^-

$$\Psi_e^L = \begin{pmatrix} \psi_{\nu_e}^L \\ \psi_{e^-}^L \end{pmatrix}$$

$(\bar{\Psi}_e^L H) \cdot \psi_e^R + h.c.$ INVARIANT UNDER $SU(2)$

$$\mathcal{L}_e = - g_e \left[(\bar{\Psi}_e^L H) \cdot \psi_e^R + \bar{\psi}_e^R H^\dagger \Psi_e^L \right]$$

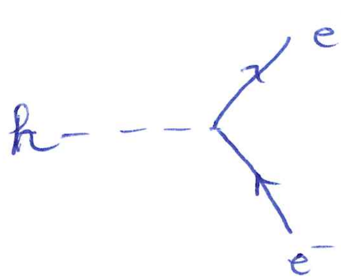
$$= - \frac{g_e v}{\sqrt{2}} \left(1 + \frac{h}{v}\right) \underbrace{\left[\bar{\psi}_e^L \psi_e^R + \bar{\psi}_e^R \psi_e^L \right]}_{\bar{\psi} \psi}$$

$$\mathcal{L}_e = - \frac{g_e v}{\sqrt{2}} \left(1 + \frac{h}{v}\right) \bar{\psi} \psi$$

TRADE g_e FOR LEPTON MASS $\qquad \frac{g_e v}{\sqrt{2}} = m_e$

$$\boxed{\mathcal{L}_e = - m_e \bar{\psi} \psi \left(1 + \frac{h}{v}\right)}$$

INTERACTION TERM



$$\boxed{- \frac{i m_e}{v}}$$

HIGGS COUPLING TO FERMIONS IS PROPORTIONAL TO MASS

PARAMETERS OF STANDARD MODEL

- COUPLINGS (3) : E.M. e
  STRONG α_s
  ELECTROWEAK UNIFICATION $\sin\Theta_W$

- SPONTANEOUS SYMM. BREAKING (1)
  HIGGS V.E.V $\rightarrow m_W$

- HIGGS MASS (1) $\rightarrow m_H$

- CHARGED LEPTON MASSES (3)
  m_e, m_μ, m_τ

- QUARK MASSES (6)
  $m_u, m_d, m_s, m_c, m_b, m_t$

- QUARK MIXING MATRIC (CHARGED WEAK INT)
  CKM : (4) 3 ANGLES + 1 PHASE

- STRONG CP VIOLATION
  (P & T VIOLATION TERM IN QCD ALLOWED)

$$\mathcal{L}_{CP} = \Theta \frac{\alpha_s}{8\pi} G_{a\mu\nu} \tilde{G}_a^{\mu\nu}$$

WITH DUAL FIELD TENSOR $\tilde{G}_a^{\mu\nu} = \frac{1}{2} \varepsilon^{\mu\nu\alpha\beta} G_{a\alpha\beta}$

Θ : PARAMETER

↓

SUCH TERM INDUCES ELECTRIC DIPOLE MOMENT $\bar{d}$

$$H_{eff} = - \, d \, \bar{E} . \bar{S} \qquad \bar{S} \text{ SPIN OF PARTICLE}$$

e.g. NEUTRON $|d_n| \sim 10^{-16} |\theta| \; e.cm$

EXP. LIMIT $|d_n| \lesssim 3 \; 10^{-26} \; e.cm$

$\Updownarrow$

$$|\theta| \lesssim 10^{-10}$$

$\therefore$ 18 PARAMETERS + θ

↳ NEUTRINOS ARE FOUND TO BE MASSIVE
NEUTRINO OSCILLATIONS $\Rightarrow$ BEYOND SM PHYSICS

3 MASSES $m_{\nu_1}, m_{\nu_2}, m_{\nu_3}$

4 PARAMETERS : ν MIXING MATRIX
PMNS MATRIX (PONTECORVO - MAKI - NAKAGAWA - SAKATA)

(3 ANGLES + 1 PHASE)