EPR PARADOX & BELL'S INEQUALITIES

⇒ QUANTUM "INDETERMINISM"

CLASSICAL PHYSICS : POSITION & MOMENTUM OF PARTICLE ARE AT EACH TIME EXACTLY PREDICTED, ONCE WE KNOW INITIAL CONDITIONS

$\updownarrow$

QUANTUM PHYSICS : WE DO NOT KNOW FOR SURE IN WHICH STATE SYSTEM IS UNTIL WE MEASURE

ONLY PROBABILITIES CAN BE PREDICTED

e.g. $|\Psi\rangle$ SPIN STATE OF e^- (SPIN 1/2)

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle + |\downarrow\rangle \right)$$

$P_{\Uparrow}$ PROBABILITY THAT e^- IS FOUND IN SPIN UP STATE

$P_{\Downarrow}$ " " " " " " SPIN DOWN

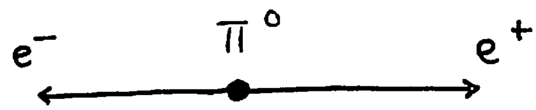
$$P_{\Uparrow} = \frac{1}{2} \qquad P_{\Downarrow} = \frac{1}{2}$$

⇒ EPR PARADOX (1935)

- EINSTEIN, PODOLSKY, ROSEN CHALLENGED QUANTUM INDETERMINISM IN A THOUGHT EXPERIMENT

  ↓

  VERSION OF D. BOHM (1952)

- 

π^0 SPIN 0

$e^- e^+$ PAIR ARE PRODUCED IN SPIN SINGLET STATE (ANGULAR MOMENTUM CONSERVATION)

$$|e^- e^+\rangle = \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

SPIN STATE OF e^- SPIN STATE OF e^+

THIS IS CALLED AN ENTANGLED STATE

- ENTANGLED STATE

$$|\Psi_{12}\rangle \neq |\Psi_1\rangle |\Psi_2\rangle$$

INCLUDES CORRELATIONS

PRODUCT OF 2 SINGLE PARTICLE STATES

e.g. $|\Psi_{12}\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$

$|\Psi_1\rangle = a_1|\uparrow\rangle + a_2|\downarrow\rangle \Rightarrow$ SPIN STATE OF PARTICLE 1

$|\Psi_2\rangle = b_1|\uparrow\rangle + b_2|\downarrow\rangle \Rightarrow$ PARTICLE 2

$$|\Psi_1\rangle|\Psi_2\rangle = a_1 b_1 |\uparrow\uparrow\rangle + a_1 b_2 |\uparrow\downarrow\rangle + a_2 b_1 |\downarrow\uparrow\rangle + a_2 b_2 |\downarrow\downarrow\rangle$$

if $|\Psi_{12}\rangle = |\Psi_1\rangle|\Psi_2\rangle$

$\Downarrow$

$$a_1 b_1 = a_2 b_2 = 0$$
$$a_1 b_2 = -a_2 b_1 = \frac{1}{\sqrt{2}}$$

$a_1 b_1 = 0 \nearrow a_1 = 0 \longrightarrow$ NOT POSSIBLE AS $a_1 b_2 = \frac{1}{\sqrt{2}}$

OR

$\searrow b_1 = 0 \longrightarrow$ NOT POSSIBLE AS $a_2 b_1 = -\frac{1}{\sqrt{2}}$

$\therefore \quad |\Psi_{12}\rangle \neq |\Psi_1\rangle|\Psi_2\rangle$

- EPR ARGUMENT

$|e^- e^+\rangle$ RESULTING FROM π^0 DECAY IS IN AN ENTANGLED STATE

ALICE $e^- \longleftarrow \bullet \longrightarrow e^+$ BOB

⇝ ALICE MEASURES e^- SPIN $\quad P_{e^-\Uparrow} = \frac{1}{2}$

$$P_{e^-\Downarrow} = \frac{1}{2}$$

⇝ BOB MEASURES e^+ SPIN $\quad P_{e^+\Uparrow} = \frac{1}{2}$

$$P_{e^+\Downarrow} = \frac{1}{2}$$

⇝ BUT BECAUSE $|\psi_{12}\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$

IF ALICE MEASURES $e^-\uparrow$ $\Rightarrow$ BOB WILL WITH 100% CERTAINTY SEE $e^+\downarrow$

IF ALICE MEASURES $e^-\downarrow$ $\Rightarrow$ BOB WILL WITH 100% CERTAINTY SEE $e^+\uparrow$

& VICE VERSA

⇝ ALICE & BOB CAN BE FAR REMOVED SUCH THAT BETWEEN TIME ALICE MADE MEASUREMENT & BOB MADE MEASUREMENT NO SIGNAL WHICH MOVES WITH SPEED OF LIGHT CAN REACH THE OTHER

→ EINSTEIN:

LOCALITY : NO SIGNAL CAN TRAVEL FASTER THAN SPEED OF LIGHT

QM : SEEMS TO EXHIBIT "SPOOKY ACTIONS AT A DISTANCE"

$\Downarrow$ EINSTEIN'S CONCLUSION

QM IS NOT WRONG
BUT IS INCOMPLETE

$|\psi\rangle$ DOES NOT REPRESENT ALL THERE IS TO KNOW ABOUT THE SYSTEM

OUR EXAMPLE :

$|\psi\rangle$ DEPENDS ONLY ON SPIN PROJ. OF PARTICLES 1 & 2

EINSTEIN : COMPLETE STATE MUST DEPEND ON SOME ADDITIONAL UNKNOWN PARAMETER λ (HIDDEN VARIABLE)

$\downarrow$

IF WE WOULD KNOW λ, WE COULD PREDICT OUTCOME OF INDIVIDUAL MEASUREMENT WITH CERTAINTY

$\therefore$ EINSTEIN : QM IS INCOMPLETE
↳ COMPLETE THEORY IS LOCAL, HIDDEN VARIABLE THEORY

⇒ BELL'S INEQUALITIES   J. BELL (1964)

- **EPR EXPERIMENT**

EXPECTATION VALUE OF PRODUCT OF SPIN ORIENTATIONS OF e^- & e^+ IN SINGLET STATE

$$\left\langle \tfrac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow) \right| \sigma_z^{(1)} \, \sigma_z^{(2)} \left| \tfrac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow) \right\rangle$$

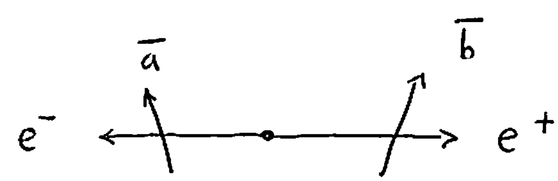
$$= \frac{1}{2} \langle \uparrow\downarrow | \sigma_z^{(1)} \, \sigma_z^{(2)} | \uparrow\downarrow \rangle$$

$$+ \frac{1}{2} \langle \downarrow\uparrow | \sigma_z^{(1)} \, \sigma_z^{(2)} | \downarrow\uparrow \rangle$$

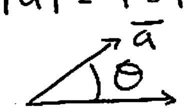
$$= \frac{1}{2}(1)(-1) + \frac{1}{2}(-1)(1) = -1$$

⇑ PREDICTION OF QM

- **GENERALIZATION OF EPR EXP.**



$|\bar{a}| = |\bar{b}| = 1$



$$\boxed{P(\bar{a}, \bar{b}) \equiv \left\langle \tfrac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow) \right| \bar{\sigma}^{(1)} \cdot \bar{a} \;\; \bar{\sigma}^{(2)} \cdot \bar{b} \left| \tfrac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow) \right\rangle}$$

⇑
EXPECTATION VALUE OF PRODUCT OF SPIN ORIENTATIONS OF e^- ALONG DIRECTION $\bar{a}$
e^+ ALONG DIRECTION $\bar{b}$

$$P(\hat{e}_z, \hat{e}_z) = -1$$

$$\sigma_x = \frac{1}{2}(\sigma_+ + \sigma_-)$$

$$\sigma_y = \frac{1}{2i}(\sigma_+ - \sigma_-)$$

$$\begin{aligned}
P(\bar a, \bar b) = {} & \frac{1}{2} \langle \uparrow\downarrow | \bar\sigma^{(1)}\cdot\bar a \;\; \bar\sigma^{(2)}\cdot\bar b | \uparrow\downarrow \rangle \\
& + \frac{1}{2} \langle \downarrow\uparrow | \bar\sigma^{(1)}\cdot\bar a \;\; \bar\sigma^{(2)}\cdot\bar b | \downarrow\uparrow \rangle \\
& - \frac{1}{2} \langle \uparrow\downarrow | \bar\sigma^{(1)}\cdot\bar a \;\; \bar\sigma^{(2)}\cdot\bar b | \downarrow\uparrow \rangle \\
& - \frac{1}{2} \langle \downarrow\uparrow | \bar\sigma^{(1)}\cdot\bar a \;\; \bar\sigma^{(2)}\cdot\bar b | \uparrow\downarrow \rangle
\end{aligned}$$

$$\begin{aligned}
= {} & \frac{1}{2} a_z b_z \langle \uparrow\downarrow | \sigma_z^{(1)} \sigma_z^{(2)} | \uparrow\downarrow \rangle \\
& + \frac{1}{2} a_z b_z \langle \downarrow\uparrow | \sigma_z^{(1)} \sigma_z^{(2)} | \downarrow\uparrow \rangle \\
& - \frac{1}{2}\frac{1}{4} (a_x - i a_y)(b_x + i b_y) \langle \uparrow\downarrow | \sigma_+^{(1)} \sigma_-^{(2)} | \downarrow\uparrow \rangle \\
& - \frac{1}{2}\frac{1}{4} (a_x + i a_y)(b_x - i b_y) \langle \downarrow\uparrow | \sigma_-^{(1)} \sigma_+^{(2)} | \uparrow\downarrow \rangle
\end{aligned}$$

$$\begin{aligned}
= {} & \frac{1}{2} a_z b_z (1)(-) + \frac{1}{2} a_z b_z (-1)(1) \\
& - \frac{1}{8}(a_x - i a_y)(b_x + i b_y)(2)(2) \\
& - \frac{1}{8}(a_x + i a_y)(b_x - i b_y)(2)(2)
\end{aligned}$$

$$= -a_z b_z - a_x b_x - a_y b_y = -\bar a \cdot \bar b$$

$$\boxed{P(\bar a, \bar b) = -\bar a \cdot \bar b}$$

PREDICTION OF QM

$|\bar a| = |\bar b| = 1$

$\bar a \cdot \bar b = \cos\theta$

- LOCAL HIDDEN VARIABLE THEORY

↳ COMPLETE KNOWLEDGE OF SYSTEM
IS GIVEN BY HIDDEN VARIABLE λ
(IF WE WOULD KNOW λ, WE WOULD KNOW e.g.
FOR EACH π^0 DECAY WHAT SPIN OF e^- IS)

↳ MEASURED VALUE OF e^- SPIN SEEN BY ALICE
(IN UNITS $\frac{\hbar}{2}$)

$$A(\bar{a}, \lambda) = \pm 1$$

↳ MEASURED VALUE OF e^+ SPIN SEEN BY BOB

$$B(\bar{b}, \lambda) = \pm 1$$

(IN UNITS $\frac{\hbar}{2}$)

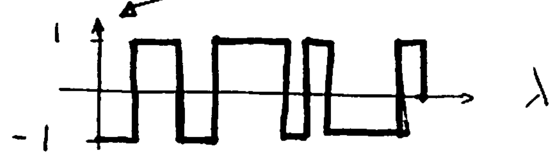
$$\left.\begin{array}{l} A \text{ DOES NOT DEPEND ON } \bar{b} \\ B \text{ DOES NOT DEPEND ON } \bar{a} \end{array}\right\}$$

LOCALITY ASSUMPTION
(ALICE & BOB MAY BE FAR APART)

↳ $P(\lambda) > 0 \quad \forall \lambda$ PROBABILITY DISTR OF HIDDEN VARIABLE

$$\int d\lambda \; P(\lambda) = 1$$

ALICE $\langle S=0 | \bar{\sigma}^{(1)} \cdot \bar{a} \, | S=0 \rangle \equiv \int d\lambda \; P(\lambda) \; A(\bar{a}, \lambda)$



$= 0$
(IN SINGLET STATE $P_\Uparrow = P_\Downarrow = 1/2$)

BOB $\langle S=0 | \bar{\sigma}^{(2)} \cdot \bar{b} | S=0 \rangle \equiv \int d\lambda \; P(\lambda) \, B(\bar{b}, \lambda)$

$= 0$

↳ FOR ALIGNED DETECTORS $\bar{b} = \bar{a}$

$$A(\bar{a}, \lambda) = - B(\bar{a}, \lambda)$$

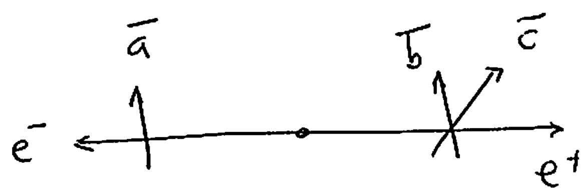
↑ MEASURED SPINS OPPOSITE

$$\hookrightarrow \quad P(\bar{a}, \bar{b}) = \int d\lambda \; P(\lambda) \; A(\bar{a}, \lambda) \; \underbrace{B(\bar{b}, \lambda)}_{-A(\bar{b}, \lambda)}$$

$$P(\bar{a}, \bar{c}) = - \int d\lambda \; P(\lambda) \; A(\bar{a}, \lambda) \; A(\bar{c}, \lambda)$$

$$P(\bar{a}, \bar{b}) - P(\bar{a}, \bar{c}) = - \int d\lambda \; P(\lambda) \; A(\bar{a}, \lambda) \left[A(\bar{b}, \lambda) - A(\bar{c}, \lambda) \right]$$

INSERT



↓ $A^2(\bar{b}, \lambda) = 1$

SINCE $A(\bar{b}, \lambda) = \pm 1$

$$P(\bar{a}, \bar{b}) - P(\bar{a}, \bar{c})$$

$$= - \int d\lambda \; P(\lambda) \; A(\bar{a}, \lambda) \, A(\bar{b}, \lambda) \left[1 - A(\bar{b}, \lambda) \, A(\bar{c}, \lambda) \right]$$

$$\downarrow$$

$$-1 \leq A(\bar{a},\lambda)\, A(\bar{b},\lambda) \leq +1$$

$$P(\lambda)\left[1 - A(\bar{b},\lambda)\, A(\bar{c},\lambda)\right] \geq 0$$

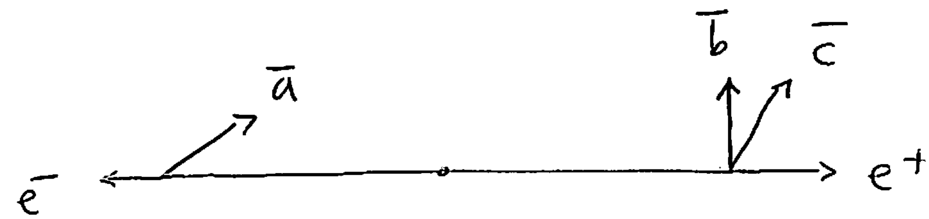
$$\left| P(\bar{a},\bar{b}) - P(\bar{a},\bar{c}) \right| \leq \int d\lambda\, P(\lambda)\left[1 - A(\bar{b},\lambda)\, A(\bar{c},\lambda)\right]$$

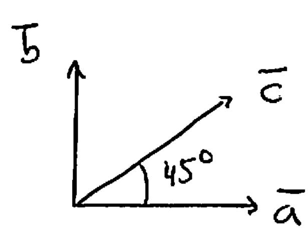
$$= 1 + P(\bar{b},\bar{c})$$

$$\boxed{\left| P(\bar{a},\bar{b}) - P(\bar{a},\bar{c}) \right| \leq 1 + P(\bar{b},\bar{c})}$$

BELL INEQUALITY

↳ EXAMPLE





- PREDICTION OF QM:

$$P(\bar{a},\bar{b}) = -\cos 90^\circ = 0$$

$$P(\bar{a},\bar{c}) = -\cos 45^\circ = -\frac{1}{\sqrt{2}}$$

$$P(\bar{b},\bar{c}) = -\cos 45^\circ = -\frac{1}{\sqrt{2}}$$

↓

INSERT IN BELL'S INEQUALITY

$$\underbrace{\frac{1}{\sqrt{2}}}_{0.707} \leq \underbrace{1 - \frac{1}{\sqrt{2}}}_{0.293}$$

∴ QM VIOLATES BELL'S INEQUALITY.

LOCAL HIDDEN VARIABLE THEORY (→ BELL'S INEQ.) IS INCOMPATIBLE WITH QUANTUM MECHANICS

↓

IT CAN BE DETERMINED BY EXPERIMENT WHICH OF TWO IS CORRECT (IF ANY)!

- **EXPERIMENT**

PIONEERING GROUP : ALAIN ASPECT et al., (ORSAY, FRANCE).

PHYS. REV. LETT. 49, 91 (1982)

" " " 49, 1804 (1982)

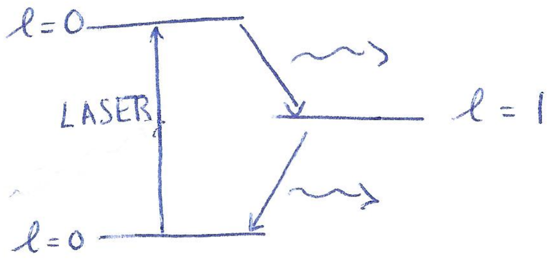
NATURE 526, 682 (2015)
PHYS. REV. LETT. 115, 250401 (2015)
PHYS. REV. LETT 115, 250402 (2015)

NOBEL PRIZE 2022 → ASPECT, CLAUSER, ZEILINGER

EXP. IS DONE WITH PHOTON ENTANGLED STATES INSTEAD OF SPIN 1/2 PARTICLES

PHOTON POLARIZATION (2 POSSIBLE STATES)
↳ TRANSVERSE ON DIRECTION OF MOTION

e^- IN CALCIUM ATOM EXCITED BY



$$|2\gamma\rangle = \frac{1}{\sqrt{2}} \left| \uparrow\uparrow + \rightarrow\rightarrow \right\rangle$$ ENTANGLED STATE

↳ LEADS TO SAME CORRELATIONS AS SPIN SINGLET STATE

$\therefore$ $\boxed{\text{QM WITHSTANDS TEST BY EXPERIMENT! LOCAL HIDDEN VARIABLE THEORY IS DISPROVED BY EXPERIMENT}}$

The Royal Swedish Academy of Sciences has decided to award the Nobel Prize in Physics 2022 jointly to

Alain Aspect, **John F. Clauser** and **Anton Zeilinger**

"for experiments with entangled photons, establishing the violation of Bell inequalities and pioneering quantum information science"

Entanglement and the Einstein, Podolsky and Rosen paradox

In his 1935 article 'Discussion of probability relations between separated systems' [1], Erwin Schrödinger stated:

> When two systems, of which we know the states by their respective representatives, enter into temporary physical interaction due to known forces between them, and when after a time of mutual influence the systems separate again, then they can no longer be described in the same way as before, viz. by endowing each of them with a representative of its own. I would not call that *one* but rather *the* characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought. By the interaction the two representatives (or ψ-functions) have become *entangled*.

That a pure quantum state is entangled means that it is not separable; for the simplest case of two distinct spinless particles moving on a line, being separable means that the wave function can be written as

$$\psi(x,y) = \psi_1(x)\psi_2(y)\,, \tag{1}$$

while the general form of the wave function is

$$\psi(x,y) = \sum_i c_i \psi_i(x)\psi_i(y)\,, \tag{2}$$

where the c_i are complex numbers. This basic definition can be generalized not only to states with many particles, and many quantum numbers (such as spin or charge), but also to what are called mixed states that describe classical statistical mixtures of pure states, typically thermal states.

From the beginnings of quantum mechanics, the electrons in an atom were recognized to be entangled because of their mutual Coulomb interaction. The simplest case is the helium atom, which has two electrons. To determine the spectrum of helium the effects of entanglement must be included, and the first successful calculation was made by Hylleraas in 1928 [2]. This first calculation was not, however, what prompted Schrödinger to call entanglement *the* characteristic trait of quantum mechanics. Rather, he was prompted by a paper published in 1935 by Albert Einstein, Boris Podolsky and Nathan Rosen (EPR) [3].

This seminal paper described the seemingly paradoxical consequences of entanglement between particles that are so distant that any interaction between them can be completely ignored. In another paper from 1935, Schrödinger described an additional apparent paradox related to entanglement between a microscopic and a macroscopic system — the latter symbolized by Schrödinger's unfortunate cat.

The basic notion in the EPR thought experiment is that distant members of an entangled pair are measured using operators that do not commute (see Figure 1). In the original formulation, the position, x, and momentum, p, of moving particles satisfied $[x,p] = i\hbar$. The experiment was reformulated in 1951 by David Bohm [4], who instead considered a pair of entangled spin one-half particles that can be measured by the operators S_i, satisfying $[S_i, S_j] = i\hbar\epsilon^{ijk} S_k$, where ϵ^{ijk} is the fully anti-symmetric Levi-Civita symbol.

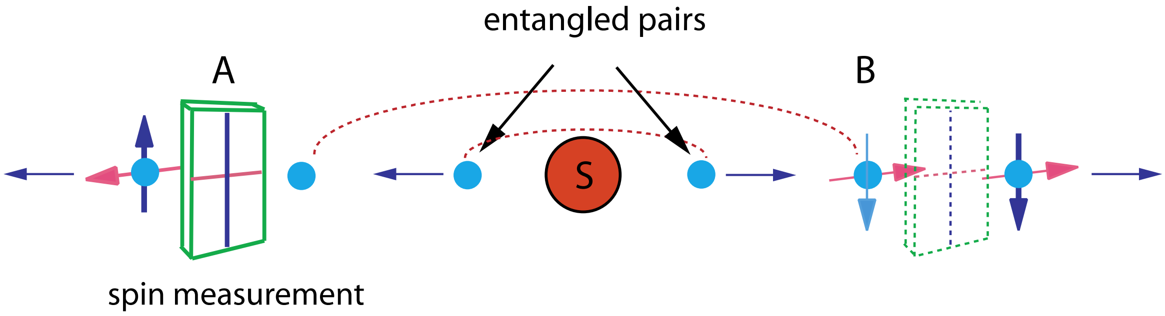


***Figure 1**. A schematic of an EPR experiment. Pairs of entangled particles are prepared in a singlet spin state and sent in opposite directions from a source S. (In the figure the entangled pairs are shown to be connected with dashed red curves.) The spin direction of each particle is, in this initial state, totally undetermined. At some distance from the source, one particle from each pair passes a measuring device operated by A, or Alice (shown by the green window), that measures the spin component in the z-direction (blue arrows). After passing the measuring apparatus, the particle appears with quantized spin in the z-direction, with either spin up, as shown in the figure, or with spin down. Due to the strict anti-correlation between the spin orientation of the particles in the pairs, at the same time as Alice's particle appears with spin up in the z-direction, then B or Bob's particle will appear with spin down in the z-direction. In this way, the measurement performed by Alice effectively acts as a measurement of the other particle (indicated by the dashed green window) even if no measuring device acts on this particle. The EPR paradox appears if Bob instead choses to measure in the x-direction (red arrows). It then looks like there are sharp values for the spin in perpendicular directions, in clear contradiction to quantum mechanics.*

In Bohm's version of the EPR experiment, a pair of spin one-half particles is created in an entangled singlet state $|\psi_-\rangle$, where the spin wave function is of the so-called Bell type,

$$|\psi_\pm\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle) \qquad (3)$$

In an experimental configuration such as in Figure 1, A and B, or Alice and Bob[1], can perform measurements of the spin components along the directions $\vec{a}$. and $\vec{b}$ respectively, where $|\vec{a}| = |\vec{b}| = 1$. Alice and Bob observe perfect anti-correlation if they measure in the same direction, $\vec{a} = \vec{b}$. For arbitrary orientations, the probability of measuring opposite spin values is $\frac{1}{2}(1+\vec{a}\cdot\vec{b})$. Note that the singlet state $|\psi_-\rangle$ is independent of which quantization axis is chosen for the spin.

[1] A(lice) and B(ob) are the conventional names of the parties in various quantum information protocols.

The conceptual puzzle presented by EPR is as follows: The components of the spin operators measured by Alice, or by Bob, do not commute. For example, for $\vec{a} = \hat{z}$, and $\vec{a} = \hat{x}$, we have $[S_z, S_x] = i\hbar S_y$. According to quantum mechanics, one cannot simultaneously assign a sharp quantum number to non-commuting observables, but in this case, this seemingly leads to a contradiction, which is seen as follows: Assume that Alice measures $S_z$and gets the result 1/2. Then from Eq. (3) it follows that *if Bob were also to measure* S_z, he would get $-1/2$ with 100% probability; this is true event by event. But Bob can choose instead to measure S_x, and get a definite answer +1/2 or -1/2, so it seems that both spin components have sharp values.

It thus *seems* that we can assign sharp values to both spin components. Einstein, Podolsky and Rosen concluded: 'From this follows that either *(1) the quantum-mechanical description of reality given by the wave function is not complete or (2) when the operators corresponding to two physical quantities do not commute the two quantities cannot have simultaneous reality'* [2] [3].

Clearly, Einstein and his younger colleagues believed in the first alternative, although they acknowledged the logical possibility of the second: 'Indeed, one would not arrive at our conclusion if one insisted that two or more physical quantities can be regarded as simultaneous elements of reality *only when they can be simultaneously measured or predicted*' [3]. This was indeed the position of Niels Bohr, and the debates between these luminaries of modern physics would form the understanding of quantum mechanics for generations of physicists that followed.

Bell inequalities

Most working physicists, if they were interested in the issue, sided with Bohr, and especially so after John von Neumann presented a proof showing that it is impossible to complement quantum mechanics with 'hidden variables' that would determine the outcome of any experiment. Still, some people kept pondering problems related to the foundations and interpretation of quantum mechanics.

For example, in 1957, Hugh Everett proposed the 'many-worlds interpretation' of quantum mechanics [5] as an alternative to the then-dominant 'Copenhagen interpretation'. The latter comes in different variations, but the basic claim, following Bohr, is that there is a sharp distinction between the microscopic phenomena described by quantum mechanics and the macroscopic detectors used to study them, which are assumed to obey the laws of classical physics.

The many-worlds interpretation makes no such division, but instead purports that whenever a measurement takes place, a different world is created and that there is no connection between the different worlds. In this interpretation, Schrödinger's cat would be alive in one world and dead in another.

Yet another alternative to the Copenhagen interpretation was the 'Bohmian' or pilot-wave version of quantum mechanics [6]. This is a fully deterministic theory that reproduces the results of non-relativistic quantum mechanics, but at the price of non-locality.

Because the theories of Bohm and Everett did not make any experimentally testable predictions that differed from standard quantum mechanics, most physicists regarded these kinds of proposals as rather esoteric and preferably only discussed during coffee breaks or in philosophy and history of science departments. This was the state of affairs in 1964 when John Bell published the paper "On the Einstein Podolsky Rosen paradox" [7].

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[2] Earlier in the paper, EPR defined 'element of physical reality' as follows: 'If, without in any way disturbing a system, we can predict with certainty (i.e. with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.'

Bell pointed out that von Neuman's proof was not correct (he gave the proof of this statement in a later publication [8]), and he also formulated the first Bell inequality, which was a spectacular theoretical discovery. Using a special version of the Bohmian-EPR thought experiment, he showed mathematically that *no theory based on local hidden variables would be able to reproduce all the results of quantum mechanics*.

With this mathematical illustration, Bell provided a proof of the assertions made by Bohr and Schrödinger, and thus showed that *all attempts to construct a local realist model of quantum phenomena are doomed to fail*. Bell used the words local and realist here in a technical sense: the former indicates the impossibility of instantaneous signalling, limited by the finite speed of light, and the latter means that the outcome of any experiment is fully determined by properties of the system, often referred to as hidden, that exist independently of any actual or potential measurement.

Bell first derived an inequality for a certain correlation function that has to be obeyed by any local realist theory, and he then showed that for some experimental conditions the predictions of quantum mechanics violate this inequality. The thought experiment considered by Bell is not suitable for experimental tests, simply because it makes assumptions about the detectors that cannot be justified for real equipment.

This obstacle was removed in 1969, by John Clauser, Michael Horne, Abner Shimony and Richard Holt (CHSH): they proposed a variation of the Bell inequality that was indeed possible to check by an experiment on entangled photons using existing technology [9]. Below, we first explain the CHSH inequality; then we describe the subsequent experiment performed by Stuart Freedman and Clauser [11].

The CHSH scenario shown in Figure 2 differs from the EPR thought experiment in that Alice can perform two different experiments that we denote by a_1 and a_2 (typically the measurement of the spin in two different directions, $\mathbf{a}_1$ and $\mathbf{a}_2$); similarly, Bob can measure b_1 or b_2. Assuming local realism, the measurement outcomes one *would* obtain for each individual quantum system are well defined even if a measurement is not made. Quantum mechanics does not predict the results of the measurements, but they nevertheless should be considered as elements of reality in the EPR sense.

Denoting the results of a potential measurement with A_1, B_2, etc., with the discrete outcomes $A_1 = \pm 1$, etc., we have

$$A_1(B_1 + B_2) + A_2(B_1 - B_2) = \pm 2, \tag{4}$$

since either $B_1 + B_2 = \pm 2$, in which case $B_1 - B_2 = 0$, or vice versa, and both A_1 and A_2 take the values ± 1.

The experiment can now be repeated many times, and in each instance, Alice chooses to measure *either* a_1 or a_2., and Bob chooses to measure *either* b_1 or b_2.. In a realist model, because the relation (4) holds for every individual measurement, we can obtain the correlations between the measurement outcomes simply by taking an ensemble average over the measurements; that is, $E(a_1, b_1) = \langle A_1 . B_1 \rangle$, etc. We then arrive at the inequality,

$$S = |E(a_1, b_1) + E(a_1, b_2) + E(a_2, b_1) - E(a_2, b_2)| < 2 \tag{5}$$

which holds for any realist theory.

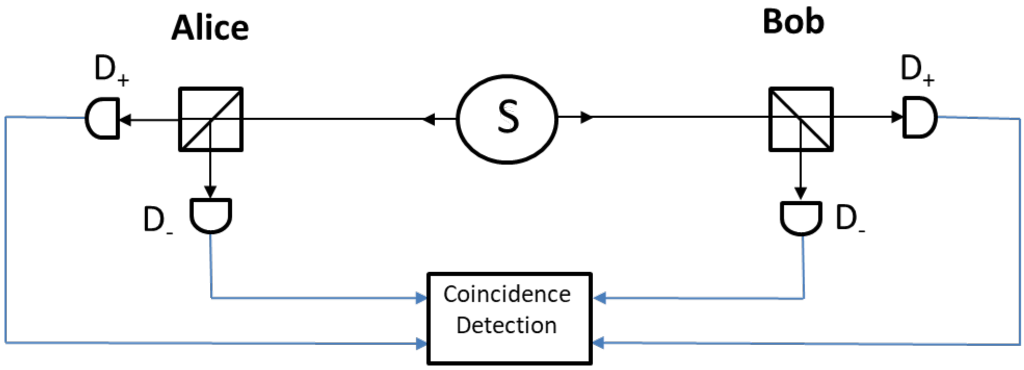

Figure 2. *The source S produces pairs of entangled photons, sent in opposite directions. Each photon encounters a two-channel polarizer whose orientation can be set by the Alice and Bob . Emerging signals from each channel are detected by single photon detector D_+ and D_- and coincidences counted by the coincidence unit. The correlation* $E(a,b) = (N_{++} - N_{+-} - N_{-+} + N_{--})/(N_{++} + N_{+-} + N_{-+} + N_{--})$ *where N_{++}, N_{+-}, N_{-+}, and N_{--} are the number of coincidence events recorded corresponding to the simultaneous detection at Alice's and Bob's detectors D_+ and D_+ , D_+ and D_-, D_- and D_+ , and D_- and D_- , respectively.*

We can now compare this result with what is predicted by quantum theory. If the pair is in the state $|\psi_-\rangle$ given in Eq. (3), it is straightforward to show that $E(a_1, b_1) = -\boldsymbol{a}_1.\boldsymbol{b}_1$, and the same holds true for the other combinations. It is now possible to see that one can choose the directions so that $\boldsymbol{a}_1.\boldsymbol{b}_1 = \boldsymbol{a}_1.\boldsymbol{b}_2 = \boldsymbol{a}_2.\boldsymbol{b}_1 = -\boldsymbol{a}_2.\boldsymbol{b}_2 = 1/\sqrt{2}$ so $S = 2\sqrt{2}$, which is in clear violation of the CHSH version of the Bell inequality (5).

The Freedman-Clauser experiment

The story might have stopped here. Some people said, 'Well, this is really weird', but dismissed that thought because the status quo already held that quantum mechanics is strange, Schrödinger's cat is bizarre, and so on. And despite the bizarreness, it all seemed to work, so the inclination of the research community at the time was to just carry on using quantum mechanics to study new and exciting phenomena.

Indeed, initially very few people took notice of Bell's work. However, those few who did, worried. Could it be that quantum mechanics does not always work? What about performing an experiment that tests quantum mechanics in one of those situations where it contradicts local realism? These were clearly the questions behind the CHSH work, and one of the authors, Clauser, set out to perform the experiment, together with the now-deceased Freedman.

Clauser had a background in molecular astrophysics from his Ph.D. thesis, working with Pat Thaddeus as his advisor at Columbia University in the City of New York. As a Ph.D. student, he had acquired an interest in the foundations of quantum mechanics. Thus, when he arrived at the University of California, Berkeley (UC Berkeley), to work as a postdoctoral researcher with Charles Townes in 1970, Clauser was prepared: he knew that Carl Kocher had built experimental equipment as part of his Ph.D. thesis at UC Berkeley in 1967 to study the time correlation between pairs of photons originating from a common source [10].

Clauser thought this experimental equipment could be used and improved to experimentally test the Bell–CHSH inequality. The problem was that Townes had hired Clauser to work on radio astronomy and search for molecules in the interstellar medium, a research field that Townes had pioneered.

Figure 3 shows the energy level diagram of calcium, which is the atom used by Kocher and his adviser Eugene Commins [10] to create pairs of photons, one at wavelength 5513 Å and one at 4227 Å.

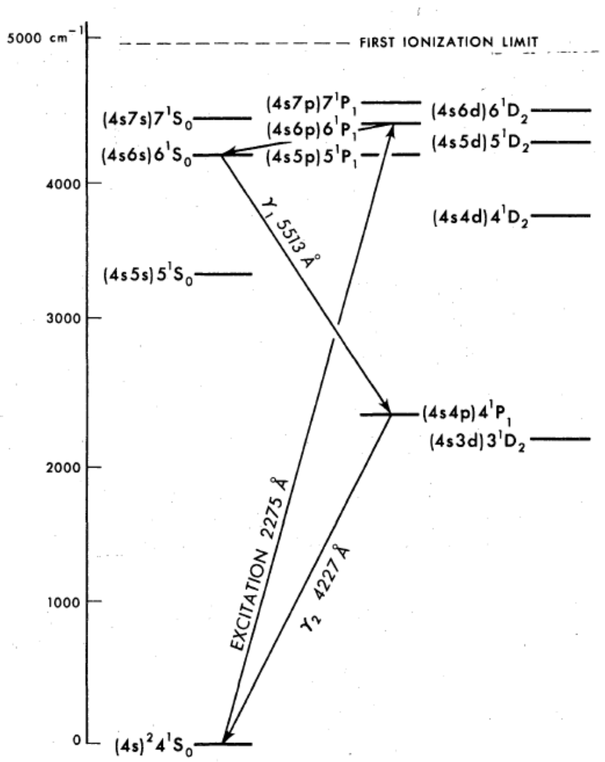

Figure 3. *Energy level diagram for calcium (Kocher and Commins [10]). The ground state is excited to the 6* $^1P_1$ *state by a photon from a hydrogen arc lamp with a wavelength of 2275 Å. The initial state in the cascade is then populated by a transition 6* $^1P_1 \rightarrow 6\ ^1S_0$*. The parities of the initial and final atomic states are even, and therefore the photon state must have even parity. This means a + sign is necessary in Eq. (3). Of course, the equation must be rewritten to take into account that it is now photons with helicities equal to ±1 and not spin half particles. For a photon pair emitted in cascade in calcium, the two polarization states, with zero total angular momentum, are* $|++\rangle$ *(both photon helicities positive) and* $|--\rangle$ *(both photon helicities negative). The photon state then becomes* $|\psi_+\rangle = \frac{1}{\sqrt{2}}(|++\rangle + |--\rangle)$*. Here + and – refer to the helicity quantum number. Positive helicity means that the spin angular momentum is in the direction of the linear momentum of the particle, while for negative helicity it is the opposite. (From Kocher's PhD thesis, Polarization correlation of photons emitted in an atomic cascade, Lawrence Berkeley National Laboratory, 1967.)*

Because the two photons have a common origin, they can be shown to be entangled. The coincidence between Alice's and Bob's detection rates for antiparallel photons is R = ½cos²φ, where φ is the angle between the axes of the polarizers. In the original experiment, the polarizers were oriented at 0° and 90° with respect to each other, which means that a time correlation could be measured at 0°, whereas no correlation was observed at an angle of 90°.

Kocher's PhD thesis clearly showed that he was aware of Bell's inequality, but with the angles he chose, he was not able to test the inequality. Kocher had already left UC Berkeley when Clauser arrived, but his experiment was still in the lab.

Clauser was by now well aware that no previous experiment had tested the Bell–CHSH inequality. He got Townes interested, and Townes made an agreement with both Clauser and Commins: Clauser could work half time on the Bell tests, and Commins' Ph.D. student at the time, Freedman, was allowed to work together with Clauser.

Clauser and Freedman had identified the polarizers as the weak point in Kocher's experiment. The polarizers' inefficiency would make it prohibitively time consuming to carry out the experiment at a number of different angles between the polarizers. The two researchers chose instead 'pile-of-plates' polarizers, which have much better efficiency. It took two years for Freedman and Clauser to rebuild Kocher's experiment so that it could be used to test the Bell–CHSH inequality, and about 200 hours to record the data.

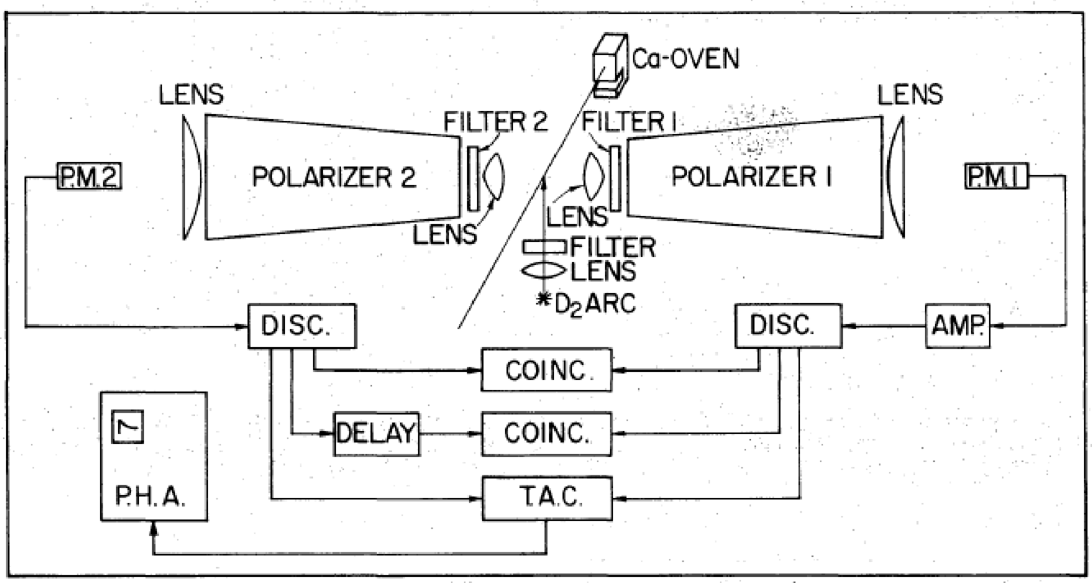

Figure 4. *Schematic diagram of the apparatus and associated electronics used by Freedman and Clauser [11]. The distance between the detectors was 5 metres (m).*

Figure 4 shows the modified experiment and Figure 5 the experimental data. Freedman and Clauser [11] rewrote the inequality as

$$\delta = \left|\frac{R(22½^0)}{R_0} - \frac{R(67½^0)}{R_0}\right| - \frac{1}{4} \leq 0, \qquad (6),$$

where $R(22\frac{1}{2}°)$ and $R(67\frac{1}{2}°)$ are the coincidence rates at the angles between the polarizer axes of 22½° and 67½°, respectively, and R_0 is the coincidence rate with the polarizers removed [11]. The measurement gave $\delta = 0.050 \pm 0.008$, in clear violation of the inequality in Eq. (6). The measured values for $R(\phi)/R_0$ at different angles ϕ are shown in Figure 5. The curve is that calculated from quantum mechanics and not a fit to the data points.

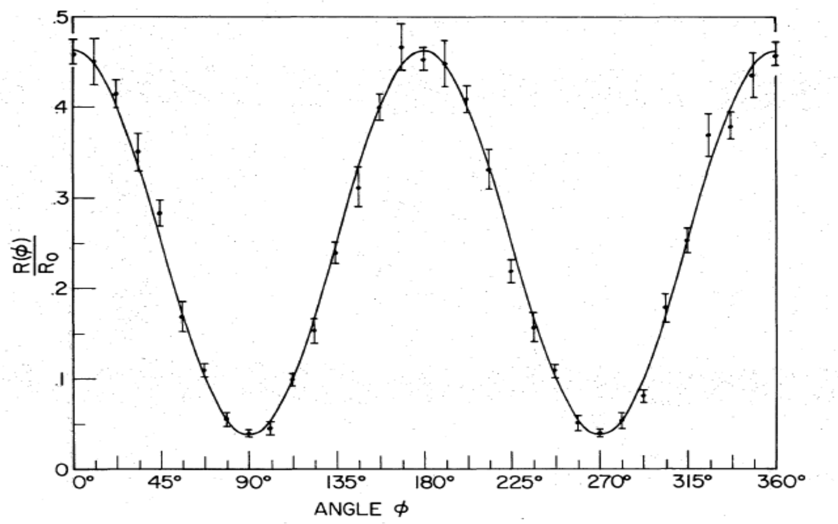

***Figure 5**. The experimentally measured ratio $R(\phi)/R_0$ as a function of the angle ϕ between the axes of the polarizers. The solid line is not a fit to the data points but the polarization correlation predicted by quantum mechanics. (From Freedman's PhD thesis, Experimental Test of Local Hidden-Variable Theories, Lawrence Berkeley National Laboratory, 1972.)*

The Aspect experiments

Like all theoretical results, Bell inequalities are derived under certain assumptions. One of these was of particular concern to Bell himself: the assumption that the two observers, Alice and Bob, make random choices of what to measure independent of each other.

For this to be true, one must make sure that Alice cannot send a message to Bob about whether A_1 or A_2 is measured, which Bob receives before he decides to measure B_1 or B_2. In other words, Alice will not influence Bob's choices. Assuming that special relativity is correct, this locality condition amounts to making sure that such a message would have to travel with a speed greater than that of light. There are also some other assumptions that we shall briefly discuss in a later section.

Alain Aspect was the first to design an experiment that avoided the locality 'loophole'. In 1981 and 1982, together with collaborators Phillipe Grangier, Gérard Roger and Jean Dalibard, Aspect performed a series of experiments using improved techniques and novel instruments [12–14]. He established a violation of a Bell inequality with very high precision, tens of standard deviations [13], as compared with the six standard deviations of the Freedman–Clauser experiment [11]. More importantly, Aspect ensured the independence of Alice and Bob by using polarization settings that changed randomly during the time of flight of the photons between the detectors [14].

In the first experiment [12], two laser systems were used to directly excite the 6 $^1S_0$ state by means of two-photon absorption. This was much more effective than using a filtered hydrogen (deuterium) arc lamp and populating the 6 $^1S_0$ state via the 6 $^1P_1 \rightarrow$ 6 $^1S_0$ transition (see Fig. 3).

In the next experiment, Aspect and collaborators used two-channel polarizers in dichromatic measurements. This allowed them to obtain excellent statistics and the largest violation of Bell inequalities at the time [13].

However, it was the third experiment [14] that garnered the most attention. Rotating the orientation of the polarizers was known to be impossible on a timescale comparable to the photon flight times. With a distance of 6 m from the calcium photon source to each polarizer, this leaves no more than 20 nanoseconds (ns) to rotate the polarizers. In 1976, Aspect proposed an experiment [15] in which acousto-optical devices could be used to switch the photons into two different branches of the apparatus on timescales shorter than the available 20 ns (see Figure 6).

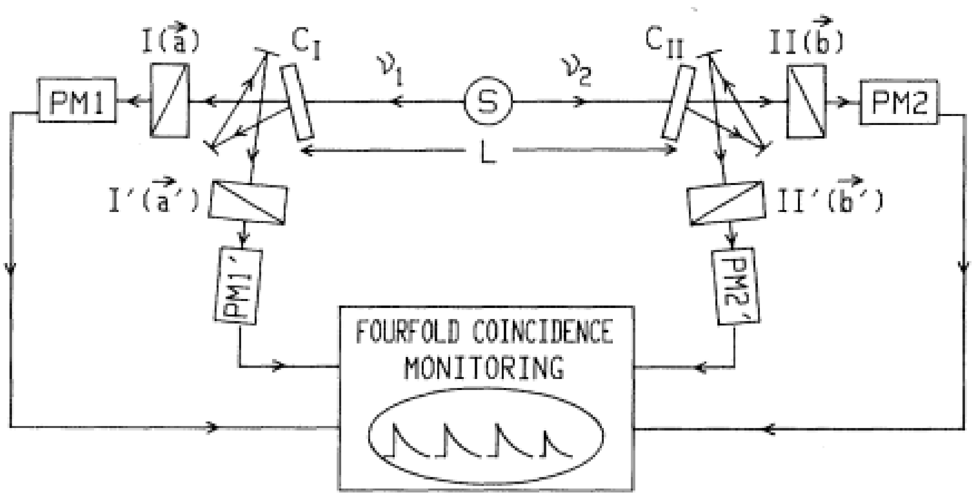

***Figure 6**. Schematic of the experiment proposed by Aspect in 1976 [15] and performed with collaborators in 1982 [14]. The photons emitted by the calcium cascade source first meet the optical switches C_I and C_{II}, where they can either be transmitted to polarizers and detectors PM1 and PM2, or be reflected to another set of polarizers and detectors PM1' and PM2'. Switching between the two channels occurs approximately every 10 ns. The distance between the polarizers was 12 m. The optical switches are ultrasonic standing waves resulting from interference between counter-propagating acoustic waves produced by two electro-acoustical transducers.*

The optics in this experiment were much more complicated than in the earlier experiments, and only single-channel polarizers were used. The inequality used required $-1 \leq S \leq 0$, whereas the experiment gave $S = 0.101 \pm 0.020$, in clear violation of the inequality (five standard deviations), but in good agreement with the quantum mechanical value 0.112.

The experiment was not ideal, since the distance between the polarizers was too small to allow for a truly random settings between them.[3] It would take more than 15 years before Anton Zeilinger's

[3] See discussion by A. Aspect, *Nature* **398**, 189 (1990).

group could test the inequality under strict local conditions [16], with the observers separated by 400 m and with a number of other technical improvements.

Entanglement as a resource

The experiments of Clauser and Aspect opened the eyes of the physics community to the profound importance of entanglement, and they provided the tools to use distant, but still entangled, photons, such as Bell pairs, as a *quantum resource*. This resource has become central to the rapidly developing field of quantum information science.

We now discuss some of the highlights of the development that has taken place in the 40 years that have passed since Aspect's first experiment. The examples refer both to fundamental discoveries in quantum physics and to results that are of direct relevance for practical applications.

The no-cloning theorem [17] says that no unitary transformation can produce a copy of an arbitrary quantum state while maintaining the original. To show this, assume that there exists a unitary 'cloning operator' $\hat{U}$ that acts on two arbitrary states $|\psi_1\rangle$ and $|\psi_2\rangle$ as

$$\hat{U}\,|\psi_1\rangle \otimes |\phi\rangle = |\psi_1\rangle \otimes |\psi_1\rangle \quad \text{and} \quad \hat{U}\,|\psi_2\rangle \otimes |\phi\rangle = |\psi_2\rangle \otimes |\psi_2\rangle\,.$$

By using the linearity of the unitary operator, we find for the unknown quantum state $|\psi\rangle = \alpha|\psi_1\rangle + \beta|\psi_2\rangle$ that

$$\hat{U}\,|\psi\rangle \otimes |\phi\rangle = \alpha|\psi_1\rangle \otimes |\psi_1\rangle \; + \beta|\psi_2\rangle \otimes |\psi_2\rangle$$

but also

$$|\psi\rangle \otimes |\psi\rangle = (\alpha|\psi_1\rangle + \beta|\psi_2\rangle) \otimes (\alpha|\psi_1\rangle + \beta|\psi_2\rangle)\,. \tag{7}$$

Since the results are different, we conclude that such a cloning operator does not exist. Clearly, the problem is that the right-hand sides of Eq. (7) are nonlinear in $|\psi_1\rangle$ and $|\psi_2\rangle$.

Thus, quantum states cannot be copied. However, the possibility to 'teleport' an arbitrary quantum state from one position to another remains, so long as the original copy is destroyed. The first proposal for how to do this was given in 1993 by Bennett *et al.* [18], and the first experiments were performed in 1997 by the groups of Anton Zeilinger [19] and Francesco De Martini [20].

We outline the general idea of teleportation protocols using a simple example. For a two-level system, there are four Bell states. In addition to $|\psi_\pm\rangle$, defined in Eq. (3), we also have $|\phi_\pm\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle \pm |\downarrow\downarrow\rangle)$, and it is easy to show that these states are orthogonal and span the full Hilbert space for two two-level systems. To teleport an unknown state $|\chi\rangle_1$ from Alice to Bob, one first arranges for them to share one Bell pair, say $|\phi_-\rangle$, so the total state of A and B is

$$|\Psi\rangle_{123} = |\chi\rangle_1 \otimes |\phi_-\rangle_{23},$$

where Alice possesses the states 1 and 2, Bob possesses state 3, and 2 and 3 form the shared Bell state. With a bit of algebra, one can rewrite this combined state in terms of the four Bell states,

$$|\Psi\rangle_{123} = \frac{1}{2}[|\phi_+\rangle_{12} \otimes V_3|\chi\rangle_3 + |\phi_-\rangle_{12} \otimes V_4|\chi\rangle_3 + |\psi_+\rangle_{12} \otimes V_2|\chi\rangle_3 + |\psi_-\rangle_{12} \otimes V_1|\chi\rangle_3],$$

where the operators V_i all have the property that $V_i^2 = 1$.

Next, Alice performs a joint measurement on the states 1 and 2, with an operator that projects them into one of the Bell states, say $|\phi_-\rangle_{12}$, so the total state becomes

$$|\Psi\rangle_{123} \rightarrow |\phi_-\rangle_{12} \otimes V_4|\chi\rangle_3.$$

Finally, Alice sends a classical two-bit message to Bob that he should apply V_4 on his state, and in doing so, he will recover $|\chi\rangle_3$, since $V_4^2 = 1$.

A closely related phenomenon is that of entanglement swapping, which, as we shall see below, is of practical importance for quantum communication. In a swapping event, one source emits the photons 1 and 2 in a Bell state, and another source emits the photons 3 and 4 in another Bell state. Photons 1 and 4 are received by Alice and Bob, respectively, while 2 and 3 are arranged to arrive simultaneously at C, to Cecilia, as illustrated in Figure 7. Thus, using the same notation as above, the original state can be written as

$$|\Psi\rangle_{1234} = |\phi_+\rangle_{12} \otimes |\phi_+\rangle_{34},$$

and by manipulations similar to the case of the teleportation example, we can rewrite it as

$$\begin{aligned}|\Psi\rangle_{1234} = \frac{1}{2}(\, &|\psi_+\rangle_{14} \otimes |\psi_+\rangle_{23} + |\psi_-\rangle_{14} \otimes |\psi_-\rangle_{23} \\ + \, &|\phi_+\rangle_{14} \otimes |\phi_+\rangle_{23} + |\phi_-\rangle_{14} \otimes |\phi_-\rangle_{23}\,).\end{aligned}$$

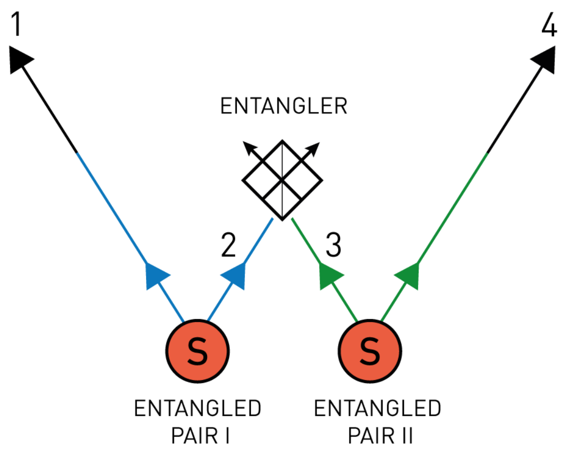

***Figure** 7. Schematic setup for an entanglement swapping experiment [22]. The two sources of entangled pairs of photons were in this case by parametric down-conversion in nonlinear crystal, which is a much more efficient source of entangled photons than can be obtained from an atomic cascade. The entangler is in the text called C for Cecilia.*

In the next step, Cecilia makes a joint measurement projecting on one of the four Bell states of 2 and 3, and as in the teleportation case, this measurement can now be chosen to be projected on a particular Bell state, say $|\psi_-\rangle_{23}$, as

$$|\Psi\rangle_{1234} \rightarrow -|\psi_-\rangle_{14} \otimes |\psi_-\rangle_{23}.$$

Thus, although they were never close to each other, photons 1 and 4 are now entangled, and Alice and Bob share a Bell pair. The first discussion of this possibility was by Bennett *et al.* [18], and later in the same year Marek Zukowski, Zeilinger, Horne, and Artur Ekert coined the term entanglement swapping, and showed how, via an initiating event, it could be used to decide when an entangled pair has been produced [21]. The first experiment to demonstrate this was published in 1998 by Jian-Wei Pan, Dik Bouwmeester, Harald Weinfurter and Zeilinger [22].

The problem facing Zeilinger's group was how to implement the simple scenario shown in Figure 7 in a real experiment. Figure 8 shows the experimental setup and how a Bell measurement on photons 2 and 3 leads to entanglement of photons 1 and 4. The details of the experiment are explained in the caption of Figure 8, and the data shown in Figure 9 clearly demonstrate the polarization correlation, and hence entanglement, between photons 1 and 4.

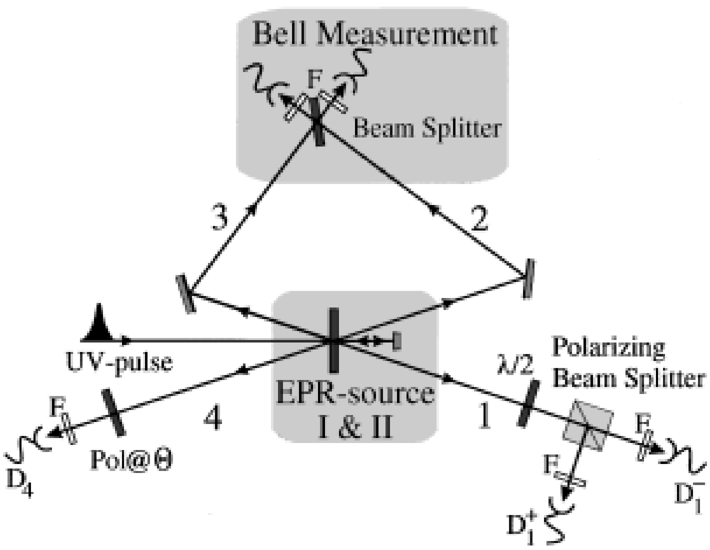

Figure 8*. A UV pulse passing through a BBO crystal (β-BaB_2O_4) creates down-converted pairs of entangled photons 1 and 2. After a reflection, during its second passage through the crystal, a second pair, 3 and 4, of entangled photons is created. Photons 2 and 3 are directed to the beam splitter in the upper grey box. When a coincidence between photons 2 and 3 is registered, they are projected into the $|\psi_-\rangle_{23}$ state. This projection results in photons 1 and 4 becoming entangled. To demonstrate entanglement between photons 1 and 4, coincidences between detectors D_4 and D_1^+ and D_4 and D_1^- are measured. From ref. [22].*

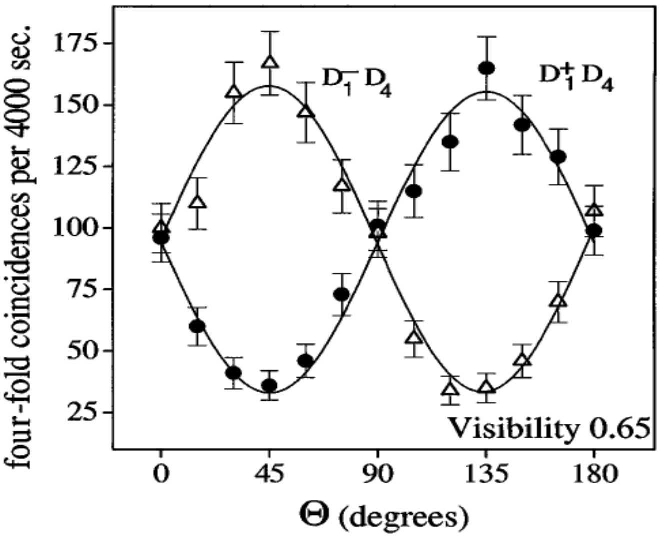

Figure 9. *Fourfold coincidences resulting from twofold coincidences $D_1^+D_4$ and $D_1^-D_4$ as a function of polarizer angle Θ at D_4. The two complementary sine curves show that photons 1 and 4 are entangled. From ref. [22].*

Quantum repeaters and quantum networks

An important goal of quantum technology is to be able to distribute entanglement over very large distances, in order to communicate quantum information. The simplest means is to use an optical fibre, but the problem is that light is attenuated, so that on average every second photon is lost in a 10-kilometre–long fibre. In classical communication networks, the problem is solved by placing amplifiers along the fibre links. Because of the no-cloning theorem, this is not possible in the quantum case, since classical amplifiers work by effectively making many copies of the original message. There are two ways to deal with this problem of loss.

The simplest solution is to avoid the loss by sending signals through space using satellites. Since the effective depth of the atmosphere is about 10 km, and the loss in empty space is very small, one can establish entanglement over very large distances. This approach was spearheaded by a team lead by Jian-Wei Pan, using the first quantum communication satellite [23, 24], *Micius,* launched by China in 2016.

Pan and colleagues demonstrated satellite-based distribution of entangled photon pairs between two locations separated by 1203 km on Earth, through two satellite-to-ground downlinks with a total length varying from 1600 to 2400 km. They observed survival of two-photon entanglement and a violation of the Bell inequality by 2.37 ± 0.09. Later, in collaboration with Zeilinger's group, they used the same satellite as a trusted relay to distribute a secure key between Beijing and Vienna. With a higher orbit satellite, which is under construction, it will be possible to directly distribute entangled photon pairs over 10,000 km.

The second approach to long distance quantum communication uses quantum repeaters, which are devices based on entanglement swapping. Repeaters can break a communication line into shorter parts and enable the use of swapping to transfer the entanglement from Alice to Bob through several nodes. In addition to an efficient swapping protocol, one also needs good quantum memory, since Alice must store her state until it is entangled with Bob's, which it will be after a number of swapping events. Combining these technologies could lead to the construction of a global quantum network, wherein distant nodes are connected by satellite and then to regional nodes by optical fibres and quantum repeaters.

Entanglement between many particles

Thus far, we have only discussed Bell pairs, or entangled states of two particles. However, entanglement is a basic property of any interacting quantum state. An important problem in condensed matter physics is to classify the possible ground states of systems with many electrons, and in particular those of relevance for the recently discovered topological materials. A classification in terms of entanglement could provide a sharp distinction between various phases of matter, and a quantity called entanglement entropy could be used to identify many of them.

Controlled entanglement is by no means restricted to polarization states of photons. Enormous effort has gone into establishing other good two-level systems that can be used in quantum technology, and in particular in quantum computing. Examples are spins in trapped atoms or ions; certain colour centres in crystals, such as diamond; and charge or flux states in superconducting electric circuits. In the last example, the two-level system arises as a collective effect in the motion of a macroscopically large number of electrons.

To better understand and classify different types of entanglement, and to find good measures of entanglement when more than two parties are involved, are important tasks for both theoretical physicists and mathematicians, and there are many open questions. We discuss here just one example where entanglement between three particles resulted in a spectacular new insight. In 1989, Daniel Greenberger, Horne and Zeilinger (GHZ) treated four-particle states [25], and a year later, together with Shimony, a three-photon state [26]:

$$|\Psi_{GHZ}\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\uparrow\rangle + |\downarrow\downarrow\downarrow\rangle)\ .$$

Here, as above, we use the spin notation for the polarization directions. By measuring S_x or S_y in different combinations for the three particles one can establish certain perfect correlations, meaning that knowing the result of the measurements of two particles, the result for the third can be inferred with certainty. This is akin to the absolute correlations in the original EPR experiment with both detectors having the same polarization. Such absolute EPR correlations are easily explained by local hidden variables, but the absolute correlations in the GHZ state, which follow from the quantum description, defy any such explanation. Thus, it is enough to experimentally establish these correlations to exclude local realism, and there is no need to evoke any inequality. A very clear and simple proof of this surprising result was given by Mermin [27]. In 1999, Zeilinger's group managed to produce the three-photon GHZ state [28], and a year later, they showed that it violated a Bell inequality, as predicted by the theory [29].

Quantum cryptography and recent developments based on Bell tests

Today, quantum technology refers to a very broad range of research and development. As an illustration we mention that the EU financed *Quantum Technology Flagship* [30] lists four main areas: quantum computing, quantum simulation, quantum communication and quantum metrology and sensing. In all of these areas quantum entanglement plays a fundamental role. This is an inappropriate venue to survey this vast landscape of innovative research. Rather, we briefly discuss quantum key distribution, since it is closely connected to the developments we have covered so far. The only known way to send messages that are guaranteed to be safe from eavesdropping is to have a shared and secret cryptographic key that is only used once. The problem is how to distribute a key shared by Alice and Bob in a secure way, and here is where quantum key distribution (QKD) comes in.

In 1991 Artur Ekert proposed an entanglement based QKD protocol [31].[4] In brief it works as follows: A and B share a Bell pair that is emitted from an independent source. Then A measures the spin in one of the three directions, $\mathbf{a}_i$, i =1, 2, 3, which is picked randomly, and B does the same in the directions $\mathbf{b}_i$. After the measurements Alice and Bob openly broadcast which spin component has been measured and divide the events in two groups. The first group contains events in which the same direction was chosen and the second in which they were different. The next step is to openly reveal the results of the measurements in the second group. This data can then be used to construct a variable S, as in Eq. (4), and perform a Bell test. If this is as expected from quantum theory, that is, if the correlations are violating a Bell inequality, then the events in the first group, which are totally anti-correlated, can be used to create a secret key. In 2006 the Zeilinger group used this scheme, and an optical free space link, to establish a secure key between the two Canary Islands, La Palma and Tenerife, that are separated by 144 km [32]. The experimenters used polarization entangled photon pairs and in a test of the CHSH inequality they obtained $S = 2.508 \pm 0.037$, demonstrating violation of the local realistic limit by more than 13 standard deviations.

The above example demonstrates that violating Bell inequalities are not “just” a matter of quantum mechanical ontology, but can be put to practical use. In this context we briefly return to the different “loopholes” in Clauser’s original experiment. We already discussed the locality loophole in relation to Aspect’s experiment. This was the loophole that was stressed by Bell, and of the other loopholes [33], the one considered to be the most important is the “detection loophole”. This arises because no detector has 100% efficiency — some photons are invariably lost, and if Nature is cruel, these photons might conspire to fake a violation of a Bell inequality. This can be avoided by having sufficiently good photon detectors, but the detection loophole was first closed in an experiment using trapped ions [34], and later in other systems. However, in these experiments one could not close the locality loophole, and it was only in the years 2015-17, that four groups, one of them led by Zeilinger, managed to simultaneously close both the locality and detection loopholes [35-38].

The main importance of these results is not to once again confirm that quantum mechanics is correct, but rather to enable even more secure QKD protocols. Since these depend on Bell tests the issue here is not whether Nature conspires to violate Bell inequalities, but whether the evil eavesdropper Eve does. In 2022 three groups used loophole free Bell tests to experimentally realize device independent QKD protocols [39-41]. This means that the key is secure, even if Eve has access to the quantum hardware that runs the distribution. For details we refer the reader to the original papers.

Conclusions

The founding fathers of quantum mechanics were well aware of its potentially revolutionary physical *and* philosophical implications, and held very different, and sometimes bluntly contradictory, views on the subject. By proving that quantum mechanics makes predictions that cannot be reproduced by any conceivable theory based on local hidden variables, John Bell transformed philosophy into empirical science, forever changing the field.

The transformation did not expunge controversy. Indeed, the overwhelming empirical evidence in the realms of atomic and optical physics was, to most practitioners, confirmation of the potent predictive power of quantum mechanics. Thus, to them, the experiments of Clauser and Aspect came as no surprise. Others saw them as fundamental discoveries about the nature of physical reality, providing an ultimate verification of quantum mechanics in a regime that is far removed from classical laws and reasoning.

[4] The first QKD protocol, proposed by Bennett and Brassard in 1984 (in *Advances in Cryptology: Proceedings of Crypto '84*, August 1984, Springer-Verlag, New York, pp. 475-480), did not utilize entangled pairs.

This year's Nobel prize is for experimental work. Apart from the disparities in philosophical interpretation, the early Bell experiments drove the development of what is often referred to as the "Second Quantum Revolution". Two of this year's laureates, John Clauser and Alain Aspect, are honoured for work that initiated a new era, opening the eyes of the physics community to the importance of entanglement, and providing techniques for creating, processing and measuring Bell pairs in ever more complex and mind-boggling scenarios. The experimental work of the third laureate, Anton Zeilinger, stands out for its innovative use of entanglement and Bell pairs, both in curiosity driven fundamental research and in applications such as quantum cryptography.

References

1. E. Schrödinger, *Mathematical Proceedings of the Cambridge Philosophical Society* **31**, 555 (1935).
2. E.A. Hylleraas, *Z. Physik* **48**, 469 (1928).
3. A. Einstein, B. Podolsky and N. Rosen, *Phys. Rev.* **47**, 777 (1935).
4. D. Bohm, *Quantum Theory*, Prentice-Hall, Englewood Cliffs, NJ (1951).
5. H. Everett, *Rev. Mod. Phys.* **29**, 454 (1957).
6. D. Bohm, *Phys. Rev.* **85**, 166 (1952), *ibid.* **85**, 180 (1952).
7. J.S. Bell, *Physics* **1**, 195 (1964).
8. J.S. Bell, *Rev. Mod. Phys.* **38**, 447 (1966).
9. J.F. Clauser, M.A. Horne, A. Shimony and R.A. Holt, *Phys. Rev. Lett.* **23**, 880 (1969).
10. C.A. Kocher and E.D. Commins, *Phys. Rev. Lett.* **18**, 575 (1967).
11. S.J. Freedman and J.F. Clauser, *Phys. Rev. Lett.* **28**, 938 (1972).
12. A. Aspect, P. Grangier and G. Roger, *Phys. Rev. Lett.* **47**, 460 (1981).
13. A. Aspect, P. Grangier and G. Roger, *Phys. Rev. Lett.* **49**, 91 (1982).
14. A. Aspect, J. Dalibard and G. Roger, *Phys. Rev. Lett.* **49**, 1804 (1982).
15. A. Aspect, *Phys. Rev. D* **14**, 1944 (1976).
16. G. Weihs, T. Jennewein, C. Simon, H. Weinfurter and A. Zeilinger, *Phys. Rev. Lett.* **81**, 5039 (1998).
17. W.K. Wootters, and W.H. Zurek, *Nature* **299**, 802 (1982); D. Dieks, *Phys. Lett. A* **92**, 271 (1982).
18. C.H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres and W.K. Wotters, *Phys. Rev. Lett.* **70**, 1895 (1993).
19. D. Bouwmeester, J.-W. Pan, K. Mattle, M. Eibl, H. Weinfurter and A. Zeilinger, *Nature* **390**, 575 (1997).
20. D. Boschi, S: Branca, F. De Martini, L. Hardy and S. Popescu, *Phys. Rev. Lett.* **80**, 1121 (1998).
21. M. Zukowski, A. Zeilinger, M.A. Horne and A.K. Ekert, *Phys. Rev. Lett.* **71**, 4287 (1993).
22. J.-W. Pan, D. Bouwmeester, H. Weinfurter and A. Zeilinger, *Phys. Rev. Lett.* **80**, 3891 (1998).

23. S.-K. Liao, W.-Q. Cai, W.-Y. Liu, L. Zhang, Y. Li, J.-G. Ren, J. Yin, Q. Shen, Y. Cao, Z.-P. Li, F.-Z. Li, X.-W. Chen, L.-H. Sun., J.-J. Jia, J.-C. Wu, X.-J. Jiang, J.-F. Wang, Y.-M. Huang, Q. Wang, Y.-L. Zhou, L. Deng, T.Xi, L. Ma, T. Hu, Q. Zhang, Y.-A. Chen, N.-L. Liu, X.-B. Wang, Z.-C. Zhu, C.-Y. Lu, R. Shu, C.-Z. Peng, J.-Y Wang and J.-W. Pan, *Nature* **549**, 43 (2017).
24. J.-G. Ren, P. Hu, H.-L. Yong, L. Zhang, S.-K. Liao, J. Yin, W.-Y. Liu, W.-Q. Cai, M. Yang, L. Li, K.-X. Yang, X. Han, Y.-Q. Yao, J. Li, H.-Y. Wu, S. Wan, L. Liu, D.-Q. Liu, Y.-W. Kuang, Z.-P. He, P. Shang, C. Guo, R.-H. Zheng, K. Tian, Z.-C. Zhu, N.-L. Liu, C.-Y. Lu, R. Shu, Y.-A. Chen, C.-Z Peng, J.-Y. Wang and J.-W. Pan, *Nature* **549**, 70 (2017).
25. D.M. Greenberger, M.A. Horne and A. Zeilinger, “Going beyond Bell’s theorem”, in *Bell’s Theorem, Quantum Theory and Conceptions of the Universe*, edited by M. Kafatos (Kluwer, Dordrecht, 1989), pp. 69–72.
26. D.M. Greenberger, M.A. Horne, A. Shimony and A. Zeilinger, *Am. J. Phys.* **58**, 1131 (1990).
27. D. Mermin, *Am. J. Phys.* **58**, 731 (1990).
28. D. Bouwmeester, J.-W. Pan, M. Daniell, H. Weinfurter and A. Zeilinger, *Phys. Rev. Lett.* **82**, 1345 (1999).
29. J.-W. Pan, D. Bouwmeester, M. Daniell, H. Weinfurter and A. Zeilinger, *Nature* **403**, 515 (2000).
30. European Commission. *Quantum Technologies Flagship*. https://digital-strategy.ec.europa.eu/en/policies/quantum-technologies-flagship [2022-09-27]
31. A.K. Ekert, *Phys. Rev. Lett.* **67**, 661 (1991).
32. R. Ursin, F. Tiefenbacher, T. Schmitt-Manderbach, H. Weier, T. Scheidl, M. Lindenthal, B. Blauensteiner, T. Jennewein, J. Perdigues, P. Trojek, B. Ömer, M. Fürst, M. Meyenburg, J. Rarity, Z. Sodnik, C. Barbieri, H. Weinfurter and A. Zeilinger, *Nature Physics* **3**, 481 (2007).
33. N. Brunner, D. Cavalcanti, S. Pironio, V. Scarani and S. Wehner, *Rev. Mod. Phys.* **86**, 419 (2014); *Erratum ibid.* **86**, 839 (2014).
34. M.A. Rowe, D. Kielpinski, V. Meyer, C.A. Sacket, W.M. Itano, C. Monroe and D.J. Wineland, *Nature* **409**, 791 (2001).
35. B. Hensen, H. Bernien, A.E. Dréau, A. Reiserer, N. Kalb, M.S Blok, J. Ruitenberg, R.F.L Vermeulen, R.N. Schouten, C. Abellán, W. Amaya, V. Pruneri, M.W. Mitchell, M. Markham D.J. Twitchen, D. Elkouss, S. Wehner, T.H. Taminiau and R. Hanson, *Nature* **526**, 682 (2015).
36. M. Giustina, M. A. M. Versteegh, S. Wengerowsky, J. Handsteiner, A. Hochrainer, K. Phelan, F. Steinlechner, J. Kofler, J.-Å. Larsson, C. Abellán, W. Amaya, V. Pruneri, M. W. Mitchell, J. Beyer, T. Gerrits, A. E. Lita, L.K. Shalm, S. W. Nam, T. Scheidl, R. Ursin, B. Wittmann and A. Zeilinger *Phys. Rev. Lett.* **115**, 250401 (2015).
37. L.K. Shalm, E. Meyer-Scott, B.G. Christensen, P. Bierhorst, M.A. Wayne, M.J. Stevens, T. Gerrits, S. Glancy, D.R. Hamel, M.S. Allman, K.J. Coakley, S.D. Dyer, C. Hodge, A.E. Lita, V.B. Verma, C. Lambrocco, E. Tortorici, A.L. Migdall, Y. Xhang, D.R. Kumor, W.H. Farr, F. Marsili, M.D. Shaw, J.A. Stern, C. Abellán, W. Amaya, V. Pruneri, T. Jennewein, M.W. Mitchell, PG. Kwiat, J.C. Bienfang, R.P. Mirin, E. Knill and S. W. Nam, *Phys. Rev. Lett.* **115**, 250402 (2015).
38. W. Rosenfeld, D. Burchardt, R. Garthoff, K. Redeker, N. Ortegel, M. Rau and H. Weinfurter, *Phys. Rev. Lett.* **119**, 010402 (2017).

39. D. P. Nadlinger, P. Drmota, B. C. Nichol, G. Araneda, D. Main, R. Srinivas, D. M. Lucas, C. J. Ballance, K. Ivanov, E. Y.-Z. Tan, P. Sekatski, R. L. Urbanke, R. Renner, N. Sangouard and J.-D. Bancal, *Nature* **607**, 682 (2022)

40. W. Zhang, T. van Leent, K. Redeker, R. Garthoff, R. Schwonnek, F. Fertig, S. Eppelt, W. Rosenfeld, V. Scarani, C. C.-W. Lim and Harald Weinfurter, *Nature* **607**, 687 (2022).

41. W.-Z. Liu, Y.-Z. Zhang, Y.-Z. Zhen, M.-H. Li, Y. Liu, J. Fan, F. Xu, Q. Zhang and J.-W. Pan, *Phys. Rev. Lett.* **129**, 050502 (2022).