Exercise sheet 11
Theoretical Physics 3: QM WS2022/2023
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Exercise 1. Spin operator (25 points)

a) *(15 p.)* Reduce an arbitrary function of the argument $a+\mathbf{b}\boldsymbol{\sigma}$ to a linear function:

$$f\left(a+\mathbf{b}\boldsymbol{\sigma}\right)=A+\mathbf{B}\boldsymbol{\sigma}$$

By writing the coefficients A and $\mathbf{B}$ explicitly ($\boldsymbol{\sigma}$ stands for Pauli matrices).

Hint: use the rotational invariance and act on the eigenstates of σ_3.

b) *(10 p.)* One of the most important properties of Pauli matrices is the expansion of the exponential form:

$$e^{i\alpha(\mathbf{n}\boldsymbol{\sigma})}=I_2\cos\alpha+i\left(\mathbf{n}\boldsymbol{\sigma}\right)\sin\alpha$$

Where I_2 denotes a unit 2×2 matrix and $\mathbf{n}$ is a unit vector in an arbitrary direction. Prove this formula using the result of the part a).

Exercise 2. Entangled states (25 points)

Let $|0\rangle$ and $|1\rangle$ denote the orthonormal basis of 1-particle states.

a) *(10 p.)* What are the conditions for a state:

$$|\psi\rangle=A\,|0\rangle\otimes|0\rangle+B\,|0\rangle\otimes|1\rangle+C\,|1\rangle\otimes|0\rangle+D\,|1\rangle\otimes|1\rangle$$

To be entangled?

b) *(15 p.)* Check if the following states are entangled or not:

$$|\psi_1\rangle=\frac{1}{\sqrt{2}}\,|0\rangle\otimes|0\rangle+\frac{1}{\sqrt{8}}\,|0\rangle\otimes|1\rangle+\frac{1}{\sqrt{8}}\,|1\rangle\otimes|0\rangle+\frac{1}{\sqrt{4}}\,|1\rangle\otimes|1\rangle$$
$$|\psi_2\rangle=\frac{1}{2}\left(e^{i\varphi_1}|0\rangle\otimes|0\rangle+e^{i\varphi_2}|0\rangle\otimes|1\rangle+e^{i\varphi_3}|1\rangle\otimes|0\rangle+e^{i\varphi_4}|1\rangle\otimes|1\rangle\right)$$

Where φ_i are some real constants.

Exercise 3. Hidden variables and Bell inequalities (50 points)

Consider the three-particle state:

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(\begin{pmatrix}0\\1\end{pmatrix}\otimes\begin{pmatrix}0\\1\end{pmatrix}\otimes\begin{pmatrix}0\\1\end{pmatrix}-\begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix}\right)$$

a) *(25 p.)* Calculate all possible expectation values:

$$\langle\psi\,|\sigma_i\otimes\sigma_j\otimes\sigma_k|\,\psi\rangle$$

With $i, j, k = 1, 2$ (8 in total).

b) *(25 p.)* Alice, Bob and Charlie were able to create the state $|\psi\rangle$ from the previous part and now keep one particle each. Additionally, they have sets of devices which allow them to measure either σ_1 or σ_2 on their particles. After solving the part a), how can they test the hidden variables theory?