SCHRÖDINGER'S CAT & KITTENS

1) QUASI-CLASSICAL STATES OF HARMONIC OSCILLATOR (H.O.)

2) SCHRÖDINGER - CAT STATE

3) FRAGILITY OF QUANTUM SUPERPOSITION

1) QUASI-CLASSICAL STATES OF H.O.

↳ H.O. OPERATOR METHOD

- $$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2} m \omega^2 \hat{x}^2$$

$$\downarrow \quad \left\| \begin{array}{l} \hat{X} \equiv \sqrt{\frac{m\omega}{\hbar}}\, \hat{x} \\ \hat{P} \equiv \frac{1}{\sqrt{m\hbar\omega}}\, \hat{p} \end{array} \right.$$

$$\hat{H} = \frac{\hbar\omega}{2} \left(\hat{P}^2 + \hat{X}^2 \right)$$

$$= \frac{\hbar\omega}{2} \left(\hat{X}^2 - (i\hat{P})^2 \right)$$

$$\begin{cases} \hat{a} \equiv \frac{1}{\sqrt{2}} \left(\hat{X} + i\hat{P} \right) \\ \hat{a}^+ \equiv \frac{1}{\sqrt{2}} \left(\hat{X} - i\hat{P} \right) \end{cases}$$

IN CHAPTER 2 WE DENOTED

$$\hat{a}_- = \hat{a} \qquad\qquad [\hat{a}, \hat{a}^+] = 1.$$

$$\hat{a}_+ = \hat{a}_-^+ = \hat{a}^+.$$

- SOLUTIONS

$$\hat{H} | n \rangle = E_n | n \rangle$$

$$\psi_n(x) \equiv \langle x | n \rangle$$

$$\begin{cases} \hat{a} | n \rangle = \sqrt{n} \, | n-1 \rangle \\ \hat{a}^+ | n \rangle = \sqrt{n+1} \, | n+1 \rangle \end{cases}$$

$$\hat{a}^+ \hat{a} \, | n \rangle = n \, | n \rangle \qquad n = 0, 1, 2 \ldots$$

$$\hat{H} = \hbar \omega \left(\hat{a}^+ \hat{a} + \frac{1}{2} \right)$$

$$E_n = \hbar \omega \left(n + \frac{1}{2} \right) \qquad n = 0, 1, 2, \ldots$$

- GROUND STATE $n = 0$

$$\hat{a} | 0 \rangle = 0$$

$$(\hat{X} + i \hat{P}) | 0 \rangle = 0$$

IN COORDINATE BASIS

$$\langle x | \hat{X} + i \hat{P} | 0 \rangle = 0$$

$$\left(\sqrt{\frac{m\omega}{\hbar}} \, x + \sqrt{\frac{\hbar}{m\omega}} \frac{d}{dx} \right) \psi_0(x) = 0$$

$$\psi_0(x) = C \, e^{-\frac{m\omega}{2\hbar} x^2} \qquad \left(C = \left(\frac{m\omega}{\pi \hbar} \right)^{1/4} \right)$$

QUASI - CLASSICAL STATE

- EIGENSTATE OF $\hat{a}$ WITH EIGENVALUE α (COMPLEX)

$$\underline{\underline{\hat{a}|\alpha\rangle = \alpha|\alpha\rangle}} \qquad \langle\alpha|\hat{a}^{+} = \langle\alpha|\alpha^{*}$$

$\rightsquigarrow$ $|n\rangle$ FORM A COMPLETE SET

$$|\alpha\rangle = \sum_{n=0}^{\infty} \alpha_n |n\rangle$$

$$\alpha_n = \langle n|\alpha\rangle$$

$$\rightsquigarrow \quad \hat{a}|\alpha\rangle = \sum_{n=1}^{\infty} \alpha_n \sqrt{n}\, |n-1\rangle$$

$$= \sum_{n=0}^{\infty} \alpha_{n+1} \sqrt{n+1}\, |n\rangle$$

$$\alpha|\alpha\rangle = \sum_{n=0}^{\infty} \alpha\,\alpha_n\, |n\rangle$$

$$\Downarrow$$

$$\alpha_{n+1} = \frac{1}{\sqrt{n+1}}\,\alpha\,\alpha_n$$

$\rightsquigarrow$ SUPPOSE WE CHOOSE $\alpha_0 = C$ (CONSTANT)

RECURSIVELY $\alpha_1 = \frac{\alpha}{\sqrt{1}} \cdot C$

$\alpha_2 = \frac{\alpha^2}{\sqrt{2!}}\, C$

$$\alpha_n = \frac{\alpha^n}{\sqrt{n!}} \, C$$

$$| \alpha \rangle = C \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} \, | n \rangle$$

$\rightsquigarrow$ NORMALIZATION $\quad \langle \alpha | \alpha \rangle = \underline{1}$

$$1 = |C|^2 \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \frac{\alpha^{*m} \alpha^n}{\sqrt{m! \, n!}} \underbrace{\langle m | n \rangle}_{\delta_{nm}}$$

$$1 = |C|^2 \sum_{n=0}^{\infty} \frac{|\alpha|^{2n}}{n!} = |C|^2 e^{|\alpha|^2}$$

$$\downarrow$$

$$|C| = e^{-|\alpha|^2/2}$$

$$\boxed{| \alpha \rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} \, | n \rangle}$$

(ALSO CALLED A COHERENT STATE)

$\langle \hat{H} \rangle$, $\langle \hat{x} \rangle$, $\langle \hat{p} \rangle$, σ_x^2, σ_p^2 IN $|\alpha\rangle$

- $$\langle \alpha | \hat{H} | \alpha \rangle = \hbar\omega \langle \alpha | \hat{a}^+ \hat{a} + \frac{1}{2} | \alpha \rangle$$
$$= \hbar\omega \left(|\alpha|^2 + \frac{1}{2} \right)$$

$|\alpha|^2$ CAN BE INTERPRETED AS AVERAGE VALUE OF EXCITATION LEVEL

- $$\langle \hat{x} \rangle = \sqrt{\frac{\hbar}{m\omega}} \langle \alpha | \hat{X} | \alpha \rangle$$
$$= \sqrt{\frac{\hbar}{m\omega}} \frac{1}{\sqrt{2}} \langle \alpha | \hat{a} + \hat{a}^+ | \alpha \rangle$$
$$= \sqrt{\frac{\hbar}{m\omega}} \frac{1}{\sqrt{2}} \left\{ \alpha + \alpha^* \right\} = \sqrt{\frac{\hbar}{m\omega}} \sqrt{2} \, \mathrm{Re}\,\alpha$$

- $$\langle \hat{p} \rangle = \sqrt{m\hbar\omega} \langle \alpha | \hat{P} | \alpha \rangle$$
$$= \sqrt{m\hbar\omega} \frac{1}{i} \frac{1}{\sqrt{2}} \langle \alpha | \hat{a} - \hat{a}^+ | \alpha \rangle$$
$$= \sqrt{m\hbar\omega} \frac{1}{\sqrt{2}} \frac{1}{i} \left\{ \alpha - \alpha^* \right\}$$
$$= \sqrt{m\hbar\omega} \sqrt{2} \, \mathrm{Im}\,\alpha$$

- FOR SCALED VARIABLES $\langle \hat{X} \rangle = \sqrt{2} \, \mathrm{Re}\,\alpha$

$$\langle \hat{P} \rangle = \sqrt{2} \, \mathrm{Im}\,\alpha$$

- $$\sigma_x^2 = \langle \alpha | (\hat{x} - \langle \hat{x} \rangle)^2 | \alpha \rangle$$

$$= \langle \alpha | \hat{x}^2 | \alpha \rangle - \langle \hat{x} \rangle^2$$

$$= \frac{\hbar}{m\omega} \frac{1}{2} \langle \alpha | (\hat{a} + \hat{a}^+)(\hat{a} + \hat{a}^+) | \alpha \rangle - \langle \hat{x} \rangle^2$$

$$= \frac{\hbar}{m\omega} \frac{1}{2} \langle \alpha | \hat{a}^2 + \hat{a}^{+2} + \underbrace{\hat{a}\hat{a}^+}_{1 + \hat{a}^+\hat{a}} + \hat{a}^+ a | \alpha \rangle$$

$$- \langle \hat{x} \rangle^2$$

$$= \frac{\hbar}{m\omega} \frac{1}{2} \left(\alpha^2 + \alpha^{*2} + 1 + 2\alpha\alpha^* \right)$$

$$- \langle \hat{x} \rangle^2$$

$$= \frac{\hbar}{m\omega} \frac{1}{2} \left((\alpha + \alpha^*)^2 + 1 \right) - \frac{\hbar}{m\omega} \frac{1}{2} (\alpha + \alpha^*)^2$$

$$\boxed{\sigma_x^2 = \frac{\hbar}{2m\omega}}$$ INDEPENDENT OF α $\left(\sigma_x = \frac{1}{\sqrt{2}} \right)$

- $$\sigma_p^2 = \langle \alpha | (\hat{p} - \langle \hat{p} \rangle)^2 | \alpha \rangle$$

$$= \hbar m\omega \frac{1}{2} \left(\left(\frac{\alpha - \alpha^*}{i} \right)^2 + 1 \right) - \hbar m\omega \frac{1}{2} \left(\frac{\alpha - \alpha^*}{i} \right)^2$$

$$\boxed{\sigma_p^2 = \frac{\hbar m\omega}{2}}$$ $\left(\sigma_p = \frac{1}{\sqrt{2}} \right)$

- $\sigma_x \sigma_p = \frac{\hbar}{2}$ MINIMUM UNCERTAINTY STATE !

↳ WAVEFUNCTION IN COORDINATE SPACE

$$\psi_\alpha(X) \equiv \langle X | \alpha \rangle$$

$$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$$

$$\langle X | \hat{a} | \alpha \rangle = \alpha \langle X | \alpha \rangle$$

$$\hat{a} = \frac{1}{\sqrt{2}}\left(\hat{X} + i\hat{P}\right) \qquad \hat{P} = -i\frac{d}{dX}$$

$$\frac{1}{\sqrt{2}}\left(X + \frac{d}{dX}\right)\psi_\alpha(X) = \alpha\,\psi_\alpha(X)$$

$$\frac{d\psi_\alpha}{dX} = -\left(-\sqrt{2}\alpha + X\right)\psi_\alpha(X)$$

$$\downarrow$$

$$\psi_\alpha(X) = C\, e^{-\frac{1}{2}\left(X - \sqrt{2}\alpha\right)^2}$$

NOTE $\langle \hat{X} \rangle = \sqrt{2}\,\mathrm{Re}\,\alpha$

C is NORMALIZATION CONSTANT

↳ WAVE FUNCTION IN MOMENTUM SPACE

$$\varphi_\alpha(P) \equiv \langle P | \alpha \rangle$$

$$\langle P | \hat{a} | \alpha \rangle = \alpha \langle P | \alpha \rangle$$

$$\langle P | \frac{1}{\sqrt{2}} (\hat{X} + i \hat{P}) | \alpha \rangle = \alpha \langle P | \alpha \rangle$$

$$\downarrow \quad \hat{X} = i \frac{d}{dP}$$

$$\frac{i}{\sqrt{2}} \left(\frac{d\varphi_\alpha}{dP} + P \varphi_\alpha(P) \right) = \alpha \varphi_\alpha(P)$$

$$\frac{d\varphi_\alpha}{dP} = -(P + i\sqrt{2}\alpha) \varphi_\alpha(P)$$

$$\varphi_\alpha(P) = C' e^{-\frac{1}{2}(P + i\sqrt{2}\alpha)^2}$$

$$\langle \hat{P} \rangle = \sqrt{2} \, \mathrm{Im}\, \alpha$$

↳ TIME EVOLUTION OF QUASI-CLASSICAL STATE

- SUPPOSE AT $t=0$, OSCILLATOR IS IN QUASI-CLASSICAL STATE

$$|\psi(t=0)\rangle = |\alpha_0\rangle \quad \text{WITH} \quad \alpha_0 = \rho e^{i\phi}$$

$$\rho, \phi \in \mathbb{R}$$

$$\rho > 0$$

- TIME EVOLUTION

$$|\psi(t)\rangle = e^{-\frac{|\alpha_0|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha_0^n}{\sqrt{n!}} e^{-\frac{i}{\hbar}E_n t} |n\rangle$$

SATISFIES $\hat{H}\,|\psi(t)\rangle = i\hbar \frac{\partial}{\partial t}|\psi(t)\rangle$

$E_n = \hbar\omega(n+\frac{1}{2})$ ↓

$$|\psi(t)\rangle = e^{-\frac{|\alpha_0|^2}{2}} e^{-i\omega t/2} \sum_{n=0}^{\infty} \frac{\left(\alpha_0 e^{-i\omega t}\right)^n}{\sqrt{n!}} |n\rangle$$

$$\underline{\underline{\alpha(t) \equiv \alpha_0 e^{-i\omega t} = \rho e^{-i(\omega t - \phi)}}}$$

NOTE $|\alpha(t)| = |\alpha_0| = \rho$

$$\underline{\underline{|\psi(t)\rangle = e^{-i\omega t/2} |\alpha(t)\rangle}}$$

AT LATER TIME t, OSCILLATOR IS STILL IN A QUASI-CLASSICAL STATE $\alpha(t)$

- $$\langle x \rangle_t = \langle \psi(t) | \hat{x} | \psi(t) \rangle$$
$$= \langle \alpha(t) | \hat{x} | \alpha(t) \rangle$$
$$= \sqrt{\frac{2\hbar}{m\omega}} \; \mathrm{Re}\, \alpha(t)$$
$$= \sqrt{\frac{2\hbar}{m\omega}} \; \rho \cos(\omega t - \phi) = x_0 \cos(\omega t - \phi)$$

- $$\langle p \rangle_t = \langle \psi(t) | \hat{p} | \psi(t) \rangle$$
$$= \langle \alpha(t) | \hat{p} | \alpha(t) \rangle$$
$$= \sqrt{2m\hbar\omega} \; \mathrm{Im}\, \alpha(t)$$
$$= -\sqrt{2m\hbar\omega} \; \rho \sin(\omega t - \phi)$$
$$= -p_0 \sin(\omega t - \phi)$$

NOTE $$\langle p \rangle_t = m \frac{d}{dt} \langle x \rangle_t$$

$\langle x \rangle_t$, $\langle p \rangle_t$ SOLUTIONS OF CLASSICAL OPERATOR !

NOTE $$\frac{\sigma_x}{|\langle x \rangle_t|} \simeq \frac{1}{2\rho} \ll 1$$

FOR $|\alpha| \gg 1$

$$\frac{\sigma_p}{|\langle p \rangle_t|} \simeq \frac{1}{2\rho} \ll 1$$

BOTH POSITION & MOMENTUM QUITE ACCURATELY DEFINED

↳ EXAMPLE PENDULUM $\ell = 1\,m$

$m = 1\,g$

DESCRIBED BY QUASI-CLASSICAL STATE

$t = 0$: $x_0 = 10^{-6}\,m$

- $\omega = \sqrt{\frac{g}{\ell}} = 3.13\ s^{-1}$

- $\alpha(t=0) = \alpha_0 = \rho = x_0 \sqrt{\frac{m\omega}{2\hbar}}$

$$\simeq 10^{-6} \left(\frac{10^{-3} \cdot 3.13}{(2)\, 1.054\, 10^{-34}} \right)^{1/2}$$

$$\alpha_0 \simeq (3.9) . 10^{9}$$

- $\frac{\sigma_x}{|\langle x \rangle_t|} \simeq 10^{-10}$

$\frac{\sigma_p}{|\langle p \rangle_t|} \simeq 10^{-10}$

- AFTER $\frac{T}{4}$ $\omega . \frac{T}{4} = \frac{\pi}{2}$

$$\alpha\left(\frac{T}{4}\right) = -i\,\alpha_0$$

⇒ 2) SCHRÖDINGER - CAT STATE

- AT $t=0$: STATE IS IN QUASI-CLASSICAL STATE

$$|\psi(t=0)\rangle = |\alpha\rangle$$

- BETWEEN $[0,T]$ ONE ADDS A COUPLING

$$\hat{H}_1 = \hbar\omega_a(\hat{a}^+\hat{a})^2 \quad \text{ANHARMONIC OSCILLATOR}$$

WITH $\omega_a \gg \omega$ AND $\omega T \ll 1$

$\Downarrow$

HAMILTONIAN DURING $[0,T]$ IS TO GOOD APPROXIMATION GIVEN BY $\hat{H}_1$

- EIGENSTATES OF $\hat{H}_1$ $\rightarrow |n\rangle$
  EIGENVALUES $\rightarrow \hbar\omega_a n^2$
  $\Big\}\ \hat{H}_1|n\rangle = \hbar\omega_a n^2|n\rangle$

$$|\psi(t)\rangle = e^{-|\alpha|^2/2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}\,e^{-i\omega_a n^2 t}\,|n\rangle$$

- FOR SPECIAL CASE $\underset{\sim\sim\sim}{T = \dfrac{\pi}{2\omega_a}}$

$$e^{-i\omega_a n^2 T} = e^{-i\frac{\pi}{2}n^2} = \begin{cases} -i, & n \text{ ODD} \\ 1, & n \text{ EVEN} \end{cases}$$

$$= \frac{1}{\sqrt{2}}\left\{e^{-i\frac{\pi}{4}} + e^{i\frac{\pi}{4}}(-1)^n\right\}$$

$$\boxed{|\psi(T)\rangle = \frac{1}{\sqrt{2}}\left\{ e^{-i\frac{\pi}{4}} |\alpha\rangle + e^{i\frac{\pi}{4}} |-\alpha\rangle \right\}}$$

SCHRÖDINGER CAT STATE

- FOR $\alpha = i\rho$ $\quad \rho \in \mathbb{R}$

$$\langle \hat{x} \rangle = 0$$

$$\langle \hat{p} \rangle = \pm \underbrace{\sqrt{2m\hbar\omega}\, \rho}_{p_0} \quad \text{FOR } |\pm\alpha\rangle$$

FOR $|\alpha| \gg 1$ $\quad |\alpha\rangle$ AND $|-\alpha\rangle$

ARE MACROSCOPICALLY DIFFERENT

(e.g. 'DEAD' OR 'ALIVE' STATES OF SCHRÖDINGER'S CAT)

INITIAL COHERENT STATE $|\alpha\rangle$ HAS EVOLVED AFTER TIME T TO A COHERENT SUPERPOSITION OF COHERENT STATES $|\alpha\rangle$ AND $|-\alpha\rangle$

- PROBABILITY DISTRIBUTION FOR POSITION

$$P(X) = |\langle X | \psi(T)\rangle|^2$$

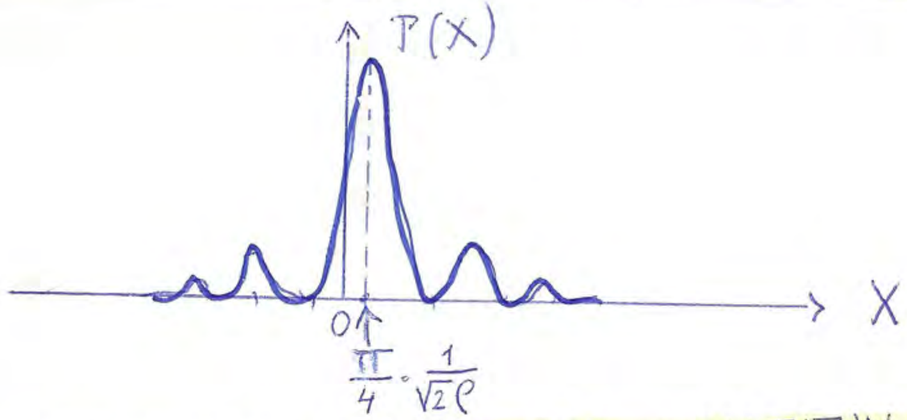
$$= \frac{1}{2} \left| e^{-i\frac{\pi}{4}} \psi_\alpha(X) + e^{i\frac{\pi}{4}} \psi_{-\alpha}(X) \right|^2$$

$$\Big\downarrow \quad \psi_\alpha(X) = C\, e^{-\frac{1}{2}(X - \sqrt{2}\alpha)^2} \qquad \left(C = \frac{1}{\pi^{1/4}} e^{-\rho^2} \right)$$

$$= |C|^2 \frac{1}{2} \left| e^{-i\frac{\pi}{4}} e^{-\frac{1}{2}(X - i\sqrt{2}\rho)^2} + e^{i\frac{\pi}{4}} e^{-\frac{1}{2}(X + i\sqrt{2}\rho)^2} \right|^2$$

$$= \frac{|C|^2}{2} e^{-X^2} e^{2\rho^2} \left| e^{-i\frac{\pi}{4} + i\sqrt{2}\rho X} + e^{i\frac{\pi}{4} - i\sqrt{2}\rho X} \right|^2$$

$$\boxed{P(X) \sim e^{-X^2} \cos^2\left(X\rho\sqrt{2} - \frac{\pi}{4}\right)}$$



- PROBABILITY DISTRIBUTION FOR MOMENTUM

$$P(P) = |\langle P | \psi(T) \rangle|^2$$

$$= \frac{1}{2} \left| e^{-i\frac{\pi}{4}} \varphi_\alpha(P) + e^{i\frac{\pi}{4}} \varphi_{-\alpha}(P) \right|^2$$

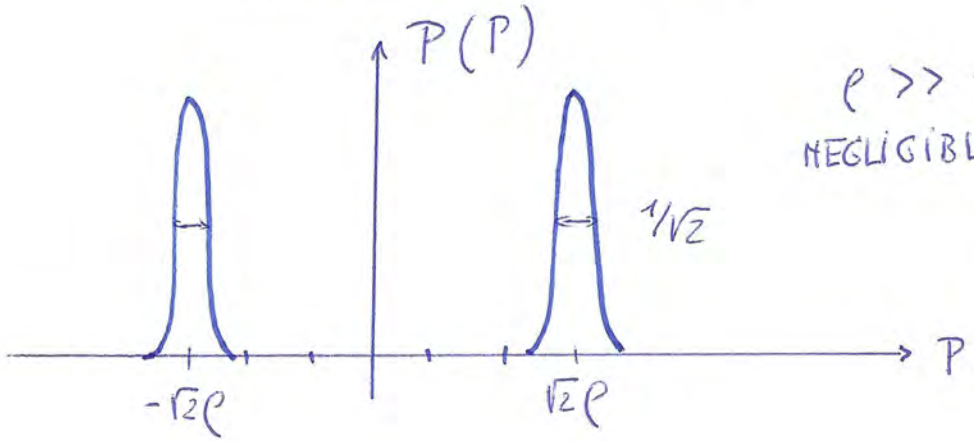
$$\downarrow \quad \varphi_\alpha(P) = C' e^{-\frac{1}{2}(P + i\sqrt{2}\alpha)^2}$$

$$= \frac{|C'|^2}{2} \cdot \left| e^{-i\frac{\pi}{4}} e^{-\frac{1}{2}(P - \sqrt{2}\rho)^2} + e^{i\frac{\pi}{4}} e^{-\frac{1}{2}(P + \sqrt{2}\rho)^2} \right|^2$$

$$\boxed{P(P) \approx e^{-(P - \sqrt{2}\rho)^2} + e^{-(P + \sqrt{2}\rho)^2}}$$

GAUSSIAN CENTERED AT

$\langle \hat{P} \rangle = \sqrt{2}\rho$ $\qquad$ $\langle \hat{P} \rangle = -\sqrt{2}\rho$



$\rho \gg 1$

NEGLIGIBLE OVERLAP

Schrödinger cat states

SCHRÖDINGER CAT STATES WITHOUT DISSIPATION

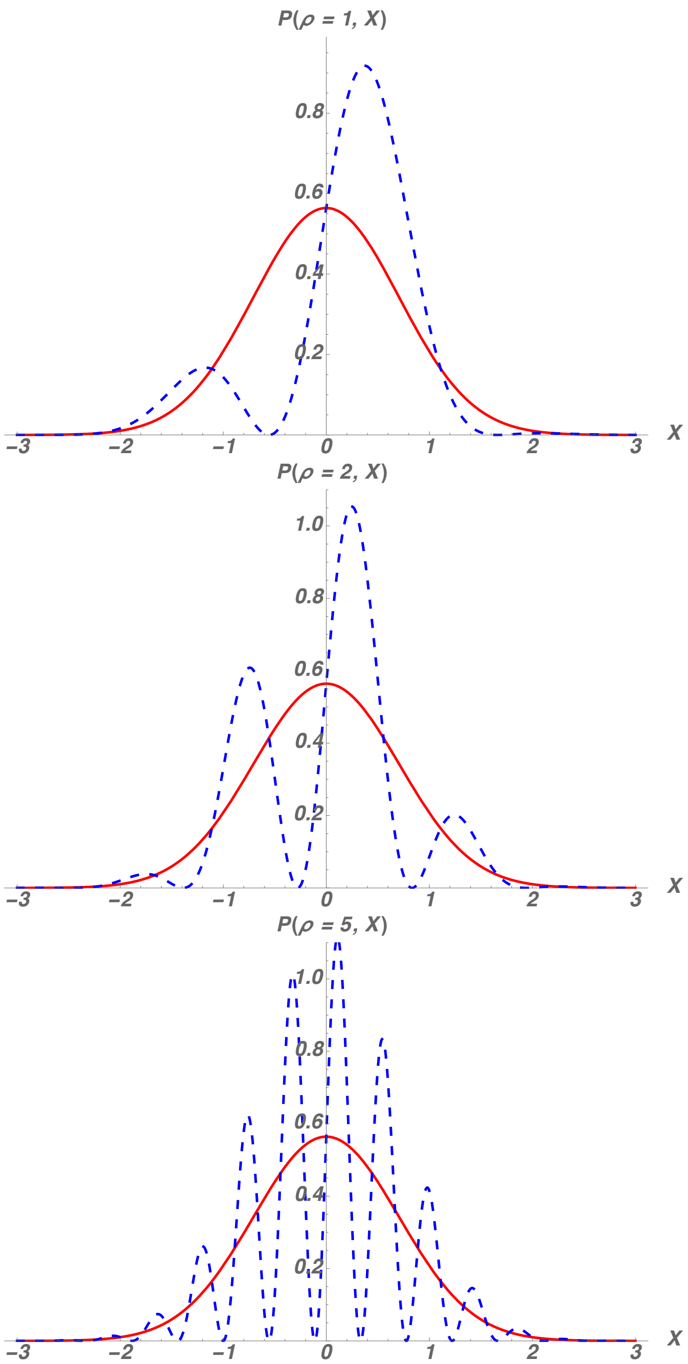


FIG. 1: Red curves: probability distribution for coherent state $\psi_\alpha(x)$, with $\alpha = i\rho$. Blue curves: probability distribution for Schrödinger cat state $\frac{1}{\sqrt{2}}\{e^{-i\frac{\pi}{4}}\psi_\alpha(x) + e^{i\frac{\pi}{4}}\psi_{-\alpha}(x)\}$, with $\alpha = i\rho$. The results are shown for $\rho = 1, 2, 5$.

- QM SUPERPOSITION

PREPARE N SYSTEMS IN
STATE $|\Psi(T)\rangle$: QM SUPERPOSITION OF 2 QUASI-CLASSICAL STATES (i.e. MACROSCOPICALLY DIFFERENT)

& PERFORM MEASUREMENTS OF P ON EACH SYSTEM

$$\underbrace{\frac{1}{\sqrt{2}}}_{\sigma_P} \ll \delta P \ll \underbrace{\sqrt{2}\,\rho}_{\langle P \rangle}$$

δP: RESOLUTION OF MEASURING APPARATUS

FOR $N \gg 1$: $\sim N/2$ TIMES ONE MEASURES $P = \sqrt{2}\,\rho$
$\sim N/2$ " " " $P = -\sqrt{2}\,\rho$

- STATISTICAL MIXTURE

INSTEAD OF N SYSTEMS IN STATE $|\Psi(T)\rangle$
CONSIDER STATISTICAL MIXTURE OF
$N/2$ SYSTEMS IN $|\alpha\rangle$
$N/2$ " " $|-\alpha\rangle$

& PERFORM MEASUREMENTS OF P ON EACH SYSTEM

$N/2$ SYSTEMS IN $|\alpha\rangle \longrightarrow P = \sqrt{2}\,\rho$
$N/2$ " " $|-\alpha\rangle \longrightarrow P = -\sqrt{2}\,\rho$

∴ SAME RESULT IN BOTH CASES

- CAN WE DISTINGUISH BETWEEN BOTH CASES ?

OR
- N SYSTEMS IN QM SUPERPOSITION $|\psi(T)\rangle$
- STATISTICAL MIXTURE OF N/2 SYSTEMS IN $|\alpha\rangle$
  N/2 " " $|-\alpha\rangle$

MEASURE POSITION IN BOTH CASES
ASSUMING THAT RESOLUTION

$$\delta X \ll \frac{1}{\rho}$$

(i.e. ONE CAN FOLLOW OSCILLATIONS IN $\cos^2(X\rho\sqrt{2} - \pi/4)$)

→ FOR QM SUPERPOSITION
WE MEASURE $P(X) \sim e^{-X^2} \cos^2\left(X\rho\sqrt{2} - \frac{\pi}{4}\right)$

(OSCILLATING FUNCTION MODULATED BY GAUSSIAN)

→ FOR STATISTICAL MIXTURE
WE MEASURE (SAME FOR $|\alpha\rangle$ AND $|-\alpha\rangle$)

$$P(X) \sim |\psi_\alpha(x)|^2 \simeq e^{-X^2} \quad \text{(GAUSSIAN)}$$

∴ WE CAN DISTINGUISH BOTH SITUATIONS

IF $\delta X \ll \frac{1}{\rho}$

$$\delta x \ll \sqrt{\frac{\hbar}{m\omega}}\,\frac{1}{\rho}$$

FOR EXAMPLE OF PENDULUM

$$\delta x \ll \frac{x_0}{\rho^2} \sim \frac{10^{-6}}{10^{19}}\,m \sim 10^{-25}\,m$$

RESOLUTION NOT POSSIBLE IN PRACTICE

⟹ 3) FRAGILITY OF QUANTUM SUPERPOSITION

- IN REALISTIC SITUATION, OSCILLATOR IS COUPLED TO ENVIRONMENT

  ↳ THIS WILL DETERMINE HOW LONG ONE CAN DISCRIMINATE BETWEEN QM SUPERPOSITION AND A STATISTICAL MIXTURE (HALF IN ONE STATE, HALF IN OTHER STATE)

- DISSIPATION MODEL FOR QUASI-CLASSICAL STATE

$t=0$ OSCILLATOR IS IN QUASI-CLASSICAL STATE $|\alpha_0\rangle$

ENVIRONMENT $|\chi_e(0)\rangle$

STATE OF TOTAL SYSTEM

$$|\Psi(0)\rangle = |\alpha_0\rangle\,|\chi_e(0)\rangle$$

$t>0$
$$|\Psi(t)\rangle = |\alpha_1\rangle\,|\chi_e(t)\rangle$$

$$\alpha_1 = \alpha(t)\,e^{-\gamma t}$$

$$\alpha(t) = \alpha_0\,e^{-i\omega t}$$

$\gamma > 0$ ($\gamma \in \mathbb{R}$) DAMPING OF STATE DUE TO COUPLING

- ENERGY OF OSCILLATOR AT TIME t

$$\langle\alpha_1|\hat{H}|\alpha_1\rangle = \hbar\omega\left(|\alpha_1|^2 + \frac{1}{2}\right)$$
$$= \hbar\omega\left(|\alpha_0|^2 e^{-2\gamma t} + \frac{1}{2}\right)$$

FOR $t \gg \frac{1}{\gamma}$ → OSCILLATOR IS IN ITS GROUND STATE

FOR $t \ll \frac{1}{2\gamma}$ → ENERGY ACQUIRED BY ENVIRONMENT

$$E(0) - E(t) \simeq \hbar\omega|\alpha_0|^2\, 2\gamma t$$

DISSIPATION MODEL FOR SCHRÖDINGER CAT STATE

$t = 0$: OSCILLATOR IS IN SCHRÖDINGER CAT STATE

$$\frac{1}{\sqrt{2}}\left\{ e^{-i\pi/4} |\alpha_0\rangle + e^{i\pi/4} |-\alpha_0\rangle \right\}$$

TOTAL SYSTEM

$$|\Psi(0)\rangle = \frac{1}{\sqrt{2}}\left\{ e^{-i\pi/4} |\alpha_0\rangle + e^{i\pi/4} |-\alpha_0\rangle \right\} |\chi_e(0)\rangle$$

$t > 0$

$$|\Psi(t)\rangle = \frac{1}{\sqrt{2}}\left\{ e^{-i\pi/4} |\alpha_1\rangle |\chi_e^+(t)\rangle + e^{i\pi/4} |-\alpha_1\rangle |\chi_e^-(t)\rangle \right\}$$

$|\chi_e^+(t)\rangle$, $|\chi_e^-(t)\rangle$ 2 NORMALIZED STATES OF ENVIRONMENT THAT ARE A PRIORI DIFFERENT (NOT ORTHOGONAL)

↳ PROBABILITY DISTRIBUTION OF OSCILLATOR'S POSITION INDEPENDENT OF STATE OF ENVIRONMENT

$$P(x) = |\langle x | \Psi(t)\rangle|^2$$

$$= \frac{1}{2}\left\{ e^{i\pi/4} \psi_{\alpha_1}^*(x) \langle \chi_e^+(t)| + e^{-i\pi/4} \psi_{-\alpha_1}^*(x) \langle \chi_e^-(t)| \right\}$$

$$\cdot \left\{ e^{-i\pi/4} \psi_{\alpha_1}(x) |\chi_e^+(t)\rangle + e^{i\pi/4} \psi_{-\alpha_1}(x) |\chi_e^-(t)\rangle \right\}$$

$$P(X) = \frac{1}{2}\Big\{ |\Psi_{\alpha_1}(X)|^2 + |\Psi_{-\alpha_1}(X)|^2$$
$$+ i\,\Psi^*_{\alpha_1}(X)\,\Psi_{-\alpha_1}(X)\,\langle \chi^+_e(t)|\chi^-_e(t)\rangle$$
$$- i\,\Psi^*_{-\alpha_1}(X)\,\Psi_{\alpha_1}(X)\,\langle \chi^-_e(t)|\chi^+_e(t)\rangle \Big\}$$
$$= \frac{1}{2}\Big\{ |\Psi_{\alpha_1}(X)|^2 + |\Psi_{-\alpha_1}(X)|^2$$
$$+ 2\,\mathrm{Re}\Big[i\,\Psi^*_{\alpha_1}(X)\,\Psi_{-\alpha_1}(X)\,\underbrace{\langle \chi^+_e(t)|\chi^-_e(t)\rangle}_{\eta} \Big]\Big\}$$

SAME AS BEFORE BUT OSCILLATION DAMPED BY FACTOR η $\quad 0 \leq \eta \leq 1$

- PROBABILITY DISTRIBUTION FOR MOMENTUM

$$P(P) = \frac{1}{2}\Big\{ |\varphi_{\alpha_1}(P)|^2 + |\varphi_{-\alpha_1}(P)|^2$$
$$+ 2\,\mathrm{Re}\Big[i\,\varphi^*_{\alpha_1}(P)\,\varphi_{-\alpha_1}(P)\,\eta \Big]\Big\}$$

FOR $\rho \gg 1$ OVERLAP OF 2 GAUSSIANS IS NEGLIGIBLE

$$\simeq \frac{1}{2}\Big\{ |\varphi_{\alpha_1}(P)|^2 + |\varphi_{-\alpha_1}(P)|^2 \Big\}.$$

- POSITION MEASUREMENT CAN DISTINGUISH BETWEEN QM SUPERPOSITION & STATISTICAL MIXTURE PROVIDED η IS NOT TOO SMALL e.g. $\eta \gtrsim \frac{1}{10}$

- ENVIRONMENT MODEL: 2° OSCILLATOR (SAME m, ω)

$t = 0 \qquad | \chi_e^{\pm}(0) \rangle = | 0 \rangle$ GROUND STATE

$t > 0 \qquad | \chi_e^{\pm}(t) \rangle = | \pm \beta \rangle$ QUASI-CLASSICAL STATES

AND ASSUME FOR $\gamma t \ll 1 \Rightarrow |\beta|^2 = 2 \gamma t |\alpha_0|^2$

$$\hookrightarrow \eta = \langle \chi_e^{+}(t) | \chi_e^{-}(t) \rangle$$

$$= \langle \beta | -\beta \rangle$$

$$= e^{-|\beta|^2} \sum_{n=0}^{\infty} (-1)^n \frac{|\beta|^{2n}}{n!}$$

$$= e^{-2|\beta|^2}$$

$\hookrightarrow$ IN ORDER TO DISTINGUISH BETWEEN QM SUPERPOSITION & STATISTICAL MIXTURE

$$\eta = e^{-2|\beta|^2} \gtrsim \frac{1}{10} \quad \longleftrightarrow \quad |\beta| \lesssim 1$$

↳ ENERGY OF 1° OSCILLATOR FOR $t \ll \frac{1}{\gamma}$

$$E(t) \doteq E(0) - 2\gamma t\, |\alpha_0|^2 \hbar\omega$$

↳ ENERGY OF 2° OSCILLATOR

$$E'(t) = \frac{\hbar\omega}{2} + 2\gamma t\, |\alpha_0|^2 \hbar\omega$$

$$= \hbar\omega \left(|\beta|^2 + \frac{1}{2} \right)$$

$\therefore \quad |\beta| \lesssim 1 \quad \longleftrightarrow \quad 2\gamma t\, |\alpha_0|^2 \lesssim 1$

IF A SINGLE ENERGY QUANTUM IS EXCHANGED IT IS DIFFICULT TO DISTINGUISH BETWEEN QM SUPERPOSITION & STATISTICAL MIXTURE

↳ FOR PENDULUM EXAMPLE $\alpha_0 \simeq 10^9$

$$(2\gamma)^{-1} = 1 \text{ year} \simeq 3\ 10^7\ s$$

$$|\beta| \simeq 1 \quad \longleftrightarrow \quad t \simeq 10^{+7}\ 10^{-18} \simeq \underline{\underline{10^{-11}\ s}}\ !$$

LIFETIME OF QM SUPER-POSITION

EVEN WITH VERY WEAK DISSIPATION AFTER 10^{-11} s WE CANNOT DISTINGUISH ANY MORE BETWEEN QM SUPERPOSITION & STATISTICAL MIXTURE !

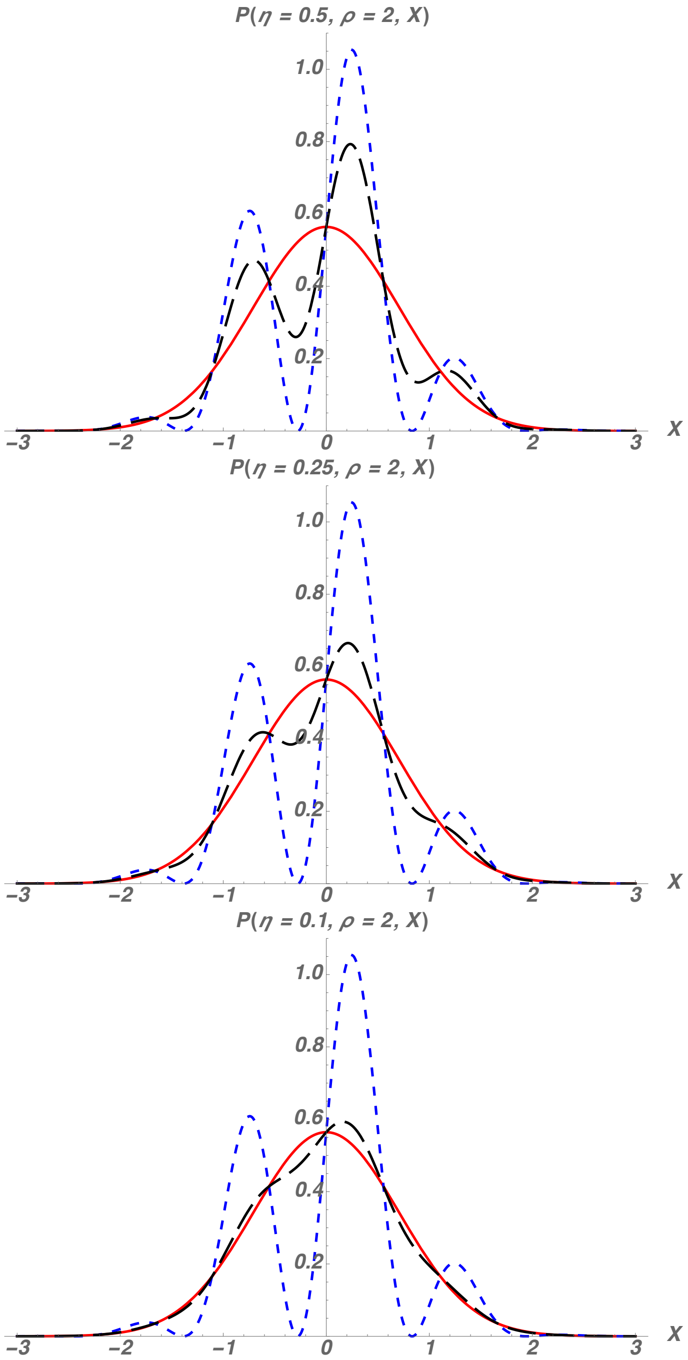


FIG. 2: Red curves: probability distribution for coherent state $\psi_\alpha(x)$, with $\alpha = i\rho$. Blue curves: probability distribution for Schrödinger cat state, with $\alpha = i\rho$. Black curves: probability distribution for corresponding Schrödinger cat state with dissipation: $P(x) = \frac{1}{2}\left\{|\psi_\alpha(x)|^2 + |\psi_{-\alpha}(x)|^2 + 2\eta Re[\psi^*_\alpha(x)\psi_{-\alpha}(x)]\right\}$. The results are shown for $\rho = 2$, for different value of the dissipation parameter $\eta = 0.5, 0.25, 0.1$.

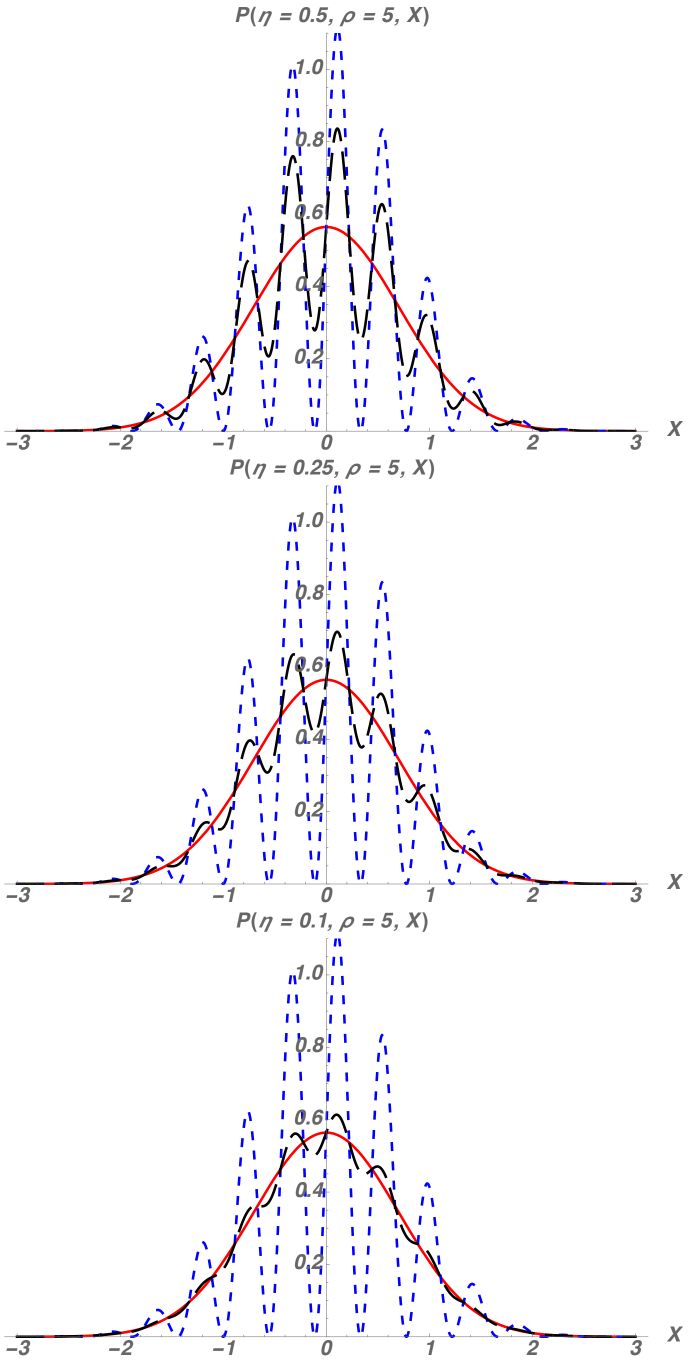


FIG. 3: Red curves: probability distribution for coherent state $\psi_\alpha(x)$, with $\alpha = i\rho$. Blue curves: probability distribution for Schrödinger cat state, with $\alpha = i\rho$. Black curves: probability distribution for corresponding Schrödinger cat state with dissipation: $P(x) = \frac{1}{2}\left\{|\psi_\alpha(x)|^2 + |\psi_{-\alpha}(x)|^2 + 2\eta Re[\psi^*_\alpha(x)\psi_{-\alpha}(x)]\right\}$. The results are shown for $\rho = 5$, for different value of the dissipation parameter $\eta = 0.5, 0.25, 0.1$.

- MEASURING & MANIPULATING INDIVIDUAL QUANTUM SYSTEMS (SCHRÖDINGER'S KITTENS)

NOBEL PRIZE 2012 :

SERGE HAROCHE & DAVID WINELAND

⇝ MESOSCOPIC SYSTEMS (SCHRÖDINGER'S KITTENS)

↳ SMALL # TRAPPED IONS OR PHOTONS IN CAVITY

$$|\alpha_0|^2 \sim 10$$

ALLOWS TO STUDY TIME EVOLUTION
OF SCHRÖDINGER CAT STATE
TOWARDS CLASSICAL STATE
BY DECOHERENCE

SEE e.g. NATURE 455, 510 (2008)

⇝ MACROSCOPIC SYSTEMS (SCHRÖDINGER'S CAT)

↳ WOULD REQUIRE CONSPIRACY OF $\sim 10^{27}$ PARTICLES

STATE IS DESTROYED AFTER VERY SHORT TIME

(cf. PENDULUM $\alpha_0 \sim 10^9$)