Exercise sheet 7
Theoretical Physics 3: QM WS2022/2023
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Exercise 1. Energy-time uncertainty relation (60 points)

Consider a particle in the infinite square well:

$$V(x) = \begin{cases} 0, & 0 \leq x \leq l \\ +\infty, & \text{otherwise} \end{cases}$$

Which has as it's initial wave function a mixture of the first and second excited stationary states:

$$\Psi(x,0) = A\left(\Psi_2(x) + \Psi_3(x)\right)$$

a) *(45 p.)* Calculate σ_H, σ_x and $d\langle x\rangle/dt$.

b) *(15 p.)* Check that energy-time uncertainty relation holds.

Exercise 2. Coherent states (40 points)

a) *(5 p.)* Prove that eigenfunctions $|\alpha\rangle$ of annihilation operator $\hat{a}$ can be written in the following form:

$$|\alpha\rangle = e^{-|\alpha|^2/2} e^{\alpha \hat{a}^\dagger} |0\rangle$$

b) *(15 p.)* Show that these functions form a set of normalized but non-orthogonal states. *Hint*: use the Baker-Campbell-Hausdorff formula:

$$e^{\hat{A}} e^{\hat{B}} = e^{[\hat{A},\hat{B}]} e^{\hat{B}} e^{\hat{A}}$$

c) *(20 p.)* Harmonic oscillator Hamiltonian can be written in the form:

$$\hat{E} = \hbar\omega\left(\hat{n} + \frac{1}{2}\right); \quad \hat{n} = \hat{a}^\dagger \hat{a}$$

Calculate the expectation value of $\hat{n}$ in a state $|\alpha\rangle$, σ_n and additionally:

$$P(n) = |\langle\alpha|n\rangle|^2$$

Which can be interpreted as a probability to observe a coherent state $|\alpha\rangle$ with an energy E_n.

d) *(15 p.)* (*Bonus*) Prove that the creation operator $\hat{a}^\dagger$ does not form a set of eigenfunctions.