Note on the correct derivation of the measure transformation for anomaly from path integral

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Follow Weinberg 22.2 (p. 362)

Consider <u>local</u> transformation

$$\Psi(x) \to U(x)\Psi(x)$$
$$\bar{\Psi}(x) \to \bar{\Psi}(x)\bar{U}(x)$$
$$\bar{U} = \gamma^0 U^{+}\gamma^0$$

Fermionic measure transforms as

$$D\bar{\Psi}D\Psi \to \left[\operatorname{Det}\mathcal{U}\,\operatorname{Det}\bar{\mathcal{U}}\right]^{-1} D\bar{\Psi}D\Psi$$

with $\mathcal{U}(x,y) = U(x)\,\delta^4(x-y)$

$$\bar{\mathcal{U}} \quad = \bar{U}(x)\,\delta^4(x-y)$$

If $U(x) = \exp(i\alpha(x)) \Rightarrow \mathcal{U}\bar{\mathcal{U}} = \mathbb{1}$

and $\operatorname{Det}\mathcal{U}\,\operatorname{Det}\bar{\mathcal{U}} = \operatorname{Det}\mathcal{U}\bar{\mathcal{U}} = \mathbb{1}$

If $\mathcal{U}(x) = \exp(i\beta(x)\gamma_5)$

$$\mathcal{U}(x) = \bar{\mathcal{U}}(x), \quad \mathcal{U}\bar{\mathcal{U}} \neq \mathbb{1} \quad !$$

Identity: $\underline{\operatorname{Det}\mathcal{U} = \exp \operatorname{tr}\ln\mathcal{U}}$

Keep $\beta(x)$ infinitesimal

$D\bar{\Psi} D\Psi \longrightarrow$

$$\exp\left[-2i\,\mathrm{tr}\int d^4x\, \beta(x)\, \gamma_5\, \delta^4(x-y)\right]_{y\to x} D\bar{\Psi} D\Psi$$

$\mathrm{tr}\,\gamma_5 = 0$ but $\delta^4(x-y)_{y\to x} = \infty$

Regulator $\exp\left[-\frac{\cancel{D}_x^2}{\Lambda^2}\right]$ $\Lambda \to \infty$ at the end

$$\delta^4(x-y) = \int \frac{d^4k}{(2\pi)^4}\, e^{-ik(x-y)} \quad , \quad x=y \text{ at the end}$$

$$\hookrightarrow -2i\,\mathrm{tr}\left[\beta(x)\,\gamma_5 \int \frac{d^4k}{(2\pi)^4}\, e^{-ik(x-y)} \exp\left[-\frac{\cancel{D}_x^2}{\Lambda^2}\right]\right]$$

Then it goes as in the notes:

Pick $\frac{1}{2}\frac{\cancel{D}_x^4}{\Lambda^4}$ from the Taylor exp. of the exp; $\cancel{D}_x^4 \to \left(\frac{e}{2}\,\sigma_{\mu\nu}F^{\mu\nu}\right)^2$

Simple derivative acting on $e^{-ik(x-y)}$

$$\longrightarrow e^{-\frac{\partial_x^2}{\Lambda^2}}\, e^{-ik(x-y)}\Big|_{x=y} = e^{k^2/\Lambda^2}$$

$$\hookrightarrow \beta(x)\, 4i\,\varepsilon^{\mu\nu\alpha\beta} F_{\mu\nu}F_{\alpha\beta}\, \frac{e^2}{4} \underbrace{\int \frac{d^4k}{(2\pi)^4}\, \frac{e^{k^2/\Lambda^2}}{\Lambda^4}}_{= i\pi^2/16\pi^4}$$