

Lecture 7

Previous lecture :

discussed quantization of
the non-abelian spin-1
gauge field

Quantization : find "normal modes"
current degrees of freedom

massless spin-1 : 2 physical
polarizations

in QFT we look for unitary (QM)
representations of Poincaré group
(special relativity)

Natural objects \rightarrow 4-vector fields
that have 4 degrees of freedom

To quantise = remove 2 unphysical
polarizations

For QED \rightarrow polarization vectors

$$\varepsilon_{\lambda=\pm 1}^{\mu}(q)$$

We could completely remove the
remaining 2 polarizations because

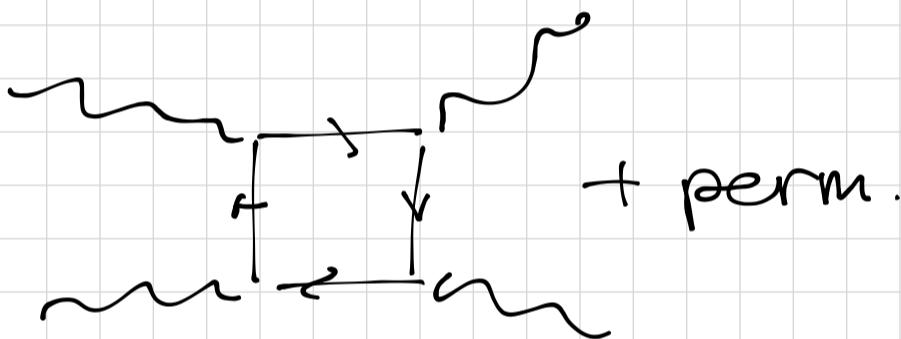
they would always decouple
from any physical observable

Why? $\rightarrow A^\mu$ only couples to
conserved current!

$$J^\mu = \bar{\psi} \gamma^\mu \psi \text{ or similar}$$

The only way to look for treeable
 \rightarrow self interaction

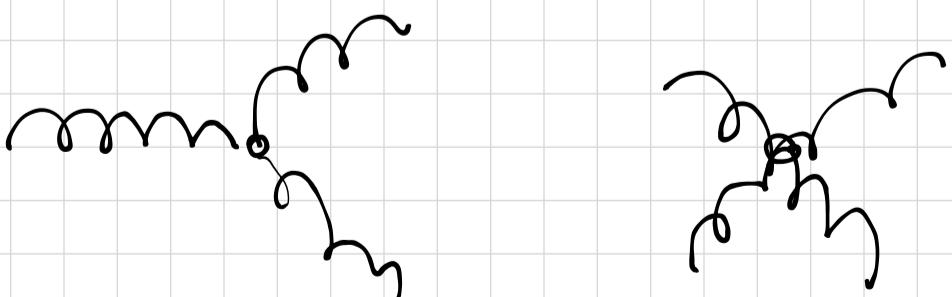
in QED light-by-light scatt.
only proceeds via fermion loops



Each vertex \rightarrow conserved current

Only explicitly gauge-invariant
terms $\sim (F_{\mu\nu} F^{\mu\nu})^2$ and $(\tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu})^2$

Now, in non-abelian theory this is
already different: 3- and 4-gluon
interaction arises at tree-level



What's the problem with that?

Unphysical polarizations couple

to the physical ones and affect their dynamics!

Ways out → introduce fictitious ghost fields (wrong spin-stat. massless fields) that cancel the unphysical polarization

- ↑ Manifest Lorentz-invariant
- ↓ Extra diagrams

→ give up Lorentz inv.
work in axial gauges

- ↑ Only work with physical polarizations

- ↓ Possible problems with

$$\text{Singularities} \\ \text{at } \overline{(r q)} \\ ; \nabla^{\mu\nu} = -g^{\mu\nu} + \frac{r^\mu q^\nu + r^\nu q^\mu}{\overline{(r q)}} \delta^{ab} \\ q^2 + i\varepsilon$$

Axial gauges are of limited use: only when the choice of kinematics is natural for problem at hand

OK, this is all great, but
 why all we derived for a non-abelian
 gauge-field theory is of any use?
 Until now it is just math: things can
 get more complicated than QED.

How do we know that $SU(3)$ gauge
 theory describes anything real?

1. How do we know that QED describes
 anything real?

Maxwell equations follow from

QED Lagrangian

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}(i\gamma^\mu + m)\psi - e A^\mu \bar{\psi} \gamma_\mu \psi$$

$\underbrace{\qquad\qquad\qquad}_{\text{Free EM field}}$ $\underbrace{\qquad\qquad\qquad}_{j^\mu - \text{source}}$

Because j^μ reduces to a classical current, everything about classical electrodynamics is automatically included

Also correctly describes low-energy quantum effects: hydrogen atom; $g-2$ of quantum fermions; Compton scattering; ...

This reflects the general path

from classical to quantum field theory: Start from familiar classical ingredients, then guess its quantum analog (unitary representation of the Poincaré group; conserved currents from Noether theorem → second quantization...)

Obviously, this is not how we may proceed in the case of QCD!

Previously shown that although there is a conserved current, it has no classical realization!

So, the question is: why we even try to describe strong interaction in terms of exchanges of massless spin-1 bosons?

To answer this → what is strong interaction?

First observed phenomenon: the existence of atomic nuclei
atoms are electrically neutral, can be ionized → contain bound electrons;

Atomic nucleus consists of equal number

of electron and proton, but is heavier than those put together.

Neutrons have to be there to make the periodic table work.

Also, atomic nucleus is $\sim 10^5$ times more compact than the atom \rightarrow why does the Coulomb repulsion not push the protons apart if they are pressed together so closely?

An heuristic idea \rightarrow there is a force that holds them together that is much stronger than Coulomb. Then why we have never experienced this force in the classical world? — Like electricity and chemistry

OK heuristic explanation: it has very short range, so that only at distances of order of the nuclear size $\sim 1 \text{ fm} \simeq (200 \text{ MeV})^{-1}$ this interaction manifests itself.

OK, this should not be too strange: at last, we didn't know that electrons exist before we found practical ways to knock them loose from the atom.

So, what should be the theory of strong interaction like?

From what we learned about interaction range \leftrightarrow mediator mass,

We should expect $m \sim \frac{1}{1 \text{ fm}} \sim 200 \text{ MeV}$

and strong coupling constant g , so nucleons feel the potential

$$V_s \sim -\frac{g^2}{4\pi} \cdot \frac{1}{r} e^{-mr}$$

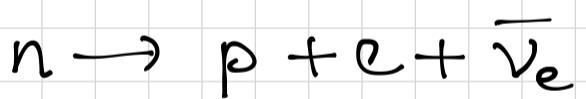
Actually, this guess is right : the long-range behavior of the nuclear force is conveyed by pion exchange with $m_\pi = 135 \text{ MeV}$. Additionally, there are short-range (δ -function) contributions. That, as mentioned in the context of Schwinger-Dyson eq., are the manifestation of the quantum nature of interaction.

This theory is not wrong at all! It is still the relevant picture of what is going on in nuclei!

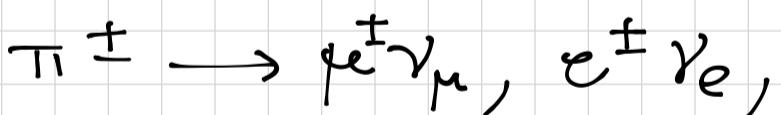
Modern nuclear theory operates with just these ingredients (well, "a bit" more involved, but at first approximation that's it): look for solutions of Schrödinger eq. for many-body problem with the nucleon-nucleon potential \sim pion exchange + contact terms. NR eq. is adequate because the nucleons are non-rel. in the bound states: few MeV kinetic energy per nucleon.

So why is this not the whole story ??

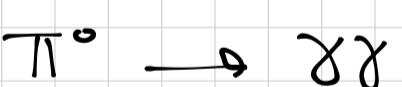
1. This theory does not explain why neutrons and pions decay



$$T_n = 877.75(28) \text{ s}$$



$$T_{\pi^\pm} \approx 26 \text{ ns}$$



$$T_{\pi^0} \approx 8.5 \cdot 10^{-17} \text{ s}$$

Compare to muon lifetime

$$\mu \rightarrow e \bar{\nu}_e \nu_\mu, T_\mu \approx 2.2 \text{ ps}$$

All numbers are very different;

But weak n, π^\pm decays are due to the same weak interaction as the muon decay (models phase space)

2. Such a theory is non-renormalizable: if you attempt to compute quantum corrections, they will diverge. Back in the 1950-60's it was a no-go, one looked for fundamental renormalizable theories like QED that proved its validity and unlimited validity (well, that's also not true: pure QED is incomplete, coupling constant becomes infinitely large at asymptotic energies).

If nucleons and pions are not fundamental degrees of freedom, then what is?

To find out \rightarrow probe the nucleon structure with higher energies, like we did to split atoms into nuclei (or ions, in general) and e^- ; produce other chemical elements in nuclear reactions etc.

Boom of accelerator physics in 1950's
 \rightarrow a huge number of new particles

produced: look for patterns.

2 general kinds of particles —

→ baryons (half-integer spin $\frac{1}{2}, \frac{3}{2} \dots$)

observed conserved number of
baryons $b \rightarrow +1, \bar{b} \rightarrow -1$

→ mesons (integer spins)

meson number not conserved

e.g. $\gamma + p \rightarrow \pi^+ N$
 $\pi^+ \pi^- N$
 $3\pi^+ N, \dots$

All these new particles decay (rates —
depending on e.-m., weak or strong
decay channel) — so also they are not
fundamental.

Then, the idea was: all hadrons
(= baryons and mesons) are bound
states of fundamental fermions
(quarks — coined by Gell-Mann).

The quarks have spin $\frac{1}{2}$

and would give a baryon if odd number
or a meson as $n[q + \text{anti-}q]$ (to build a
Lorentz scalar, vector, etc.).

This was just a guess: hadrons are

quantum objects, so they cannot be reduced to fixed number of constituents, only infinite Fock space

$$\text{e.g. } (q\bar{q} \rightarrow (q\bar{q})(q\bar{q}) + \dots)$$

$$(q\bar{q}q + q\bar{q}q(q\bar{q}) + \dots)$$

One observed symmetries in the hadron spectrum: pions (π^+, π^-, π^0) had very close mass ($\pi^\pm - \pi^0$ mass difference can be related to e-m quantum effect \rightarrow em self energy)

→ can produce a vector from 2 spinor fields

SU(2) symmetry $\rightarrow \begin{pmatrix} u \\ d \end{pmatrix}$ quarks

transform as one

SU(2) group generators T^a
Pauli matrices

Rotations $\begin{pmatrix} u \\ d \end{pmatrix} \rightarrow T^a \begin{pmatrix} u \\ d \end{pmatrix}$

$$U = e^{i\theta^a T^a} = \mathbb{1} + i\theta^a T^a + \dots$$

Symmetry \rightarrow means 2 particles,
1 mass, 1 coupling

$SU(2)$ symmetry in flavor
Space called isospin symmetry
(isotopic spin)

Spin algebra in the isospin space

If adding strange quark

$\rightarrow \begin{pmatrix} u \\ d \\ s \end{pmatrix}$ transform as one

Generators \rightarrow Gell-Mann's τ^a

$SU(3)$ algebra

How you can put the blocks together?

Mesons

Quarks are in fundamental repr. 3

Antiquarks \rightarrow antifundamental repr.

$\bar{3}$

This is similar to simple spinors

$$x = \begin{pmatrix} f \\ g \end{pmatrix} \quad x^+ = (f^+, g^+)$$

Out of these you can construct
a matrix xx^+ or scalar x^+x

$$3 \otimes \bar{3} = 8 \oplus 1$$

↑
trivial repr. (singlet)
adjoint repr. (octet)

Quantum Nrs.

Charge

$$e_u = \frac{2}{3}$$

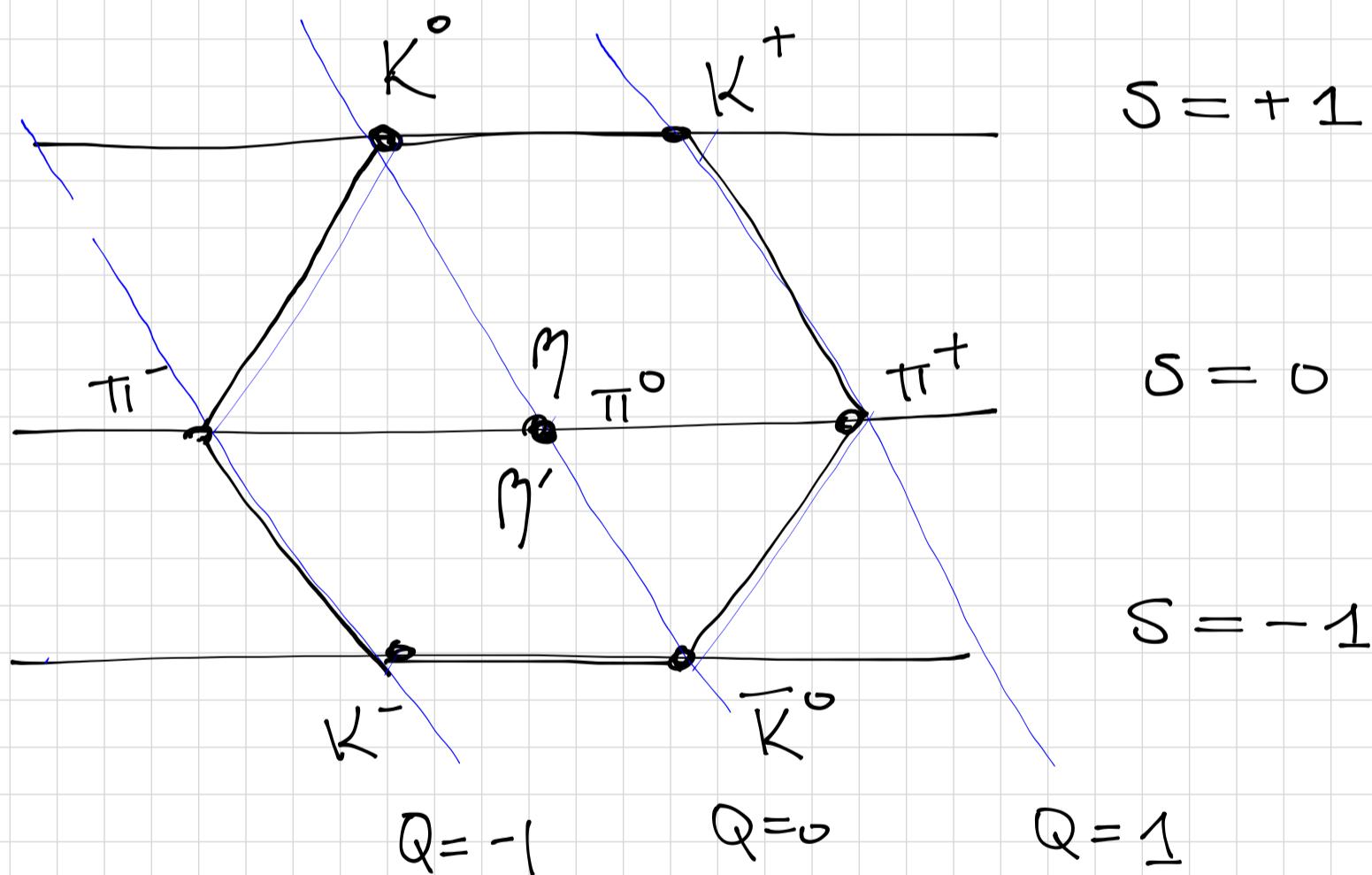
$$e_d = e_s = -\frac{1}{3}$$

Strangeness $S=0$ (u, d)

$S=-1$ (s)

$S=+1$ (\bar{s})

(Spin $\frac{1}{2}$)



Octet states :

$$K^0 = q\bar{s}$$

$$K^+ = u\bar{s}$$

$$\bar{K}^0 = s\bar{d}$$

$$K^- = s\bar{u}$$

$$\bar{\pi}^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$$

$$\bar{\pi}^+ = u\bar{d} \quad \bar{\pi}^- = d\bar{u}$$

$$\begin{pmatrix} u \\ d \\ s \end{pmatrix} \otimes \begin{pmatrix} \bar{u} \\ \bar{d} \\ \bar{s} \end{pmatrix}$$

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = O_\eta \begin{bmatrix} \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s}) \\ \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s}) \end{bmatrix}$$

$$O_\eta = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$\theta \approx 11.5^\circ$$

These are pseudoscalar mesons
consistent with $(q\bar{q})$ composition

$$J^{PC} = 0^- \quad P = (-1)^{L+1}$$

usual selection rules

$$|L-S| \leq J \leq L+S$$

$$\underline{PS} \rightarrow \underline{S=L=0}$$

$$S=0, 1, \frac{1}{2}, \frac{1}{2}$$

L - angular mom.

If $SU(3)$ symmetry were exact
 \rightarrow all masses would be degenerate

In general : if related by a symmetry,
different particles are
just different projections
of one particle (8-dim.
or 1-dim. octet/singlet)

Similar for Spin-1 mesons

$$VM \rightarrow S=1, L=0 \quad J^{PC} = 1^{--}$$

$$P = (-1)^{L+1}$$

$$C = (-1)^{L+S}$$

In principle, L can take any value
 → infinite tower of states
 not all are observed → so-called
 "missing states"
 (artifact of naive quark model)

In a similar way → baryons
 Flavor

$$3 \otimes 3 \otimes 3 = 10_S \oplus 8_H \oplus 8_H + 1_A$$

Useful to combine spin and flavor

$$\rightarrow \text{SU}(6) \text{ symmetry}$$

$$\begin{pmatrix} u_{\uparrow\uparrow} \\ d_{\uparrow\downarrow} \\ s_{\uparrow\downarrow} \end{pmatrix}$$

$$6 \otimes 6 \otimes 6 = 56_S \oplus 70_H \oplus 70_H \oplus 20_A$$

$$56 = 10^{\frac{3}{2}} \oplus 8^{\frac{1}{2}}$$

\uparrow \leftarrow N-octet (spin- $\frac{1}{2}$)
 Δ -decuplet (spin- $\frac{3}{2}$)

One member of the Δ -decuplet, Δ^{++}

$$|\Delta^{++}\rangle = |u_{\uparrow} u_{\uparrow} u_{\uparrow}\rangle$$

Is observed experimentally but is prohibited by Fermi statistics!

This contradiction was formally solved

by postulating a new quantum number color under which the quarks would be charged

↪ can still build an antisymmetric combination

$$| f^{abc} u_\uparrow^a u_\uparrow^b u_\uparrow^c \rangle$$

This was the starting point of our building the $SU(3)$ non-abelian gauge theory.

| This color symmetry turns out to be exact

Another example of exact symmetry of nature $\rightarrow U(1)$ of QED charge conservation

To implement $SU(3)$ gauge symmetry introduce an octet of gluon fields that transform as adjoint representation