

Lecture 19

Precision tests of the Standard Model

We have constructed a $SU(3)_c \times SU(2)_w \times U(1)_Y$ gauge field theory, embedded spontaneous symmetry breaking, determined the set of parameters that specify the Standard Model completely.

We checked that anomalies do not destroy the symmetries

We checked that the inclusion of the dynamical Higgs boson ensures a correct HE behavior.

Now: the SM was built to explain observations in a mathematically sound way. The mathematical consistency is a constraint: can it be tested?

1. Prediction: existence of weak neutral current and Z^0 -boson.

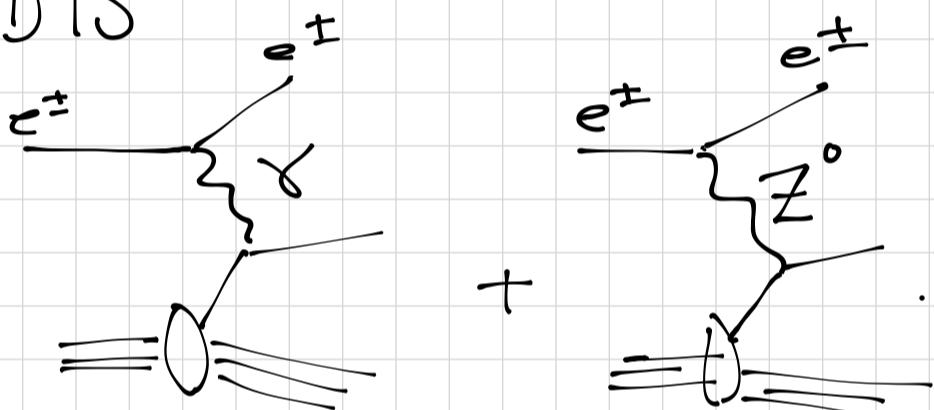
Weak decays were what one wanted to describe. They involve doublets

(quarks or baryons) \rightarrow hence $SU(2)$
 But then the mediator (weak boson)
 has to transform as adjoint, (W^\pm, W^3)

There is no way to do SSB such
 that W^\pm are massive and couple to
 L fermions, while W^3 massless that
 couples to L and R (γ). As a result,
 massive Z^0 that couples to L and R
 differently is a prediction.

Parity violation in DIS

$$e^\pm h \rightarrow e^\pm + X$$



Kinematics :

$$\begin{aligned} q^2 &= (k - k')^2 = -Q^2 \\ v &= E - E' \end{aligned} \quad \left| \begin{array}{l} x = \frac{Q^2}{2E} \\ y = \frac{v}{E} \end{array} \right.$$

Diff. cross section with

longitudinally polarized e^- ($\lambda = \pm 1$)

$$\frac{d\sigma}{dx dy} (\lambda) = \frac{2\pi y \alpha^2}{Q^4} \sum_{i=\gamma, \gamma Z, Z} n_i L_{\mu\nu}^i W_i^{\mu\nu}$$

$$L_{\mu\nu}^\gamma = 2 \left[k_\mu k_\nu' + k_\nu k_\mu' - \frac{Q^2}{2} g_{\mu\nu} - i \epsilon_{\mu\nu\alpha\beta} k^\alpha k'^\beta \right]$$

$$L_{\mu\nu}^{Z\gamma} = (g_V^e \mp \lambda g_A^e) L_{\mu\nu}^\gamma$$

$$L_{\mu\nu}^Z = (g_V^e \mp \lambda g_A^c)^2 L_{\mu\nu}^{\gamma}$$

Weak charges : "−" for e^- beam

$$g_V^e = -\frac{1}{2} + 2S_W^2, \quad g_A^c = -\frac{1}{2}$$

And factors

$$\eta_\gamma = 1$$

$$\eta_{\gamma Z} = \frac{G_F Q^2}{2\sqrt{2}\pi\alpha} \frac{M_Z^2}{Q^2 + M_Z^2}$$

$$\eta_Z = \eta_{\gamma Z}^2$$

$$W^{\mu\nu} = -\hat{g}^{\mu\nu} F_1(x, Q^2) + \frac{\hat{P}^\mu \hat{P}^\nu}{Pq} F_2(x, Q^2) + i \epsilon^{\mu\nu\alpha\beta} \frac{P_\alpha q_\beta}{2Pq} F_3(x, Q^2)$$

For $M^2 \ll Q^2 \ll M_Z^2$
use parton model for S.F.

$$F_2^{up} = \left(\frac{2}{3}\right)^2 \times (u + \bar{u}) + \left(-\frac{1}{3}\right)^2 \times (d + \bar{d}) \quad F_1 = \frac{1}{2x} F_2$$

$$F_2^{dn} = \left(-\frac{1}{3}\right)^2 \times (u + \bar{u}) + \left(\frac{2}{3}\right)^2 \times (d + \bar{d})$$

Used $u^p = d^n$, $d^p = u^n$ (charge symmetry)

$$F_2^{\gamma Z, p} = 2 \cdot \frac{2}{3} \left(\frac{1}{2} - \frac{4}{3} S_W^2 \right) \times (u + \bar{u})$$

$e^a \quad g_V^a$

$$+ 2 \left(-\frac{1}{3}\right) \left(-\frac{1}{2} + \frac{2}{3} S_W^2\right) \times (d + \bar{d})$$

$$F_3^{\gamma Z, p} = 2 \cdot \frac{2}{3} \cdot \frac{1}{2} (u - \bar{u}) + 2 \cdot (-\frac{1}{3})(-\frac{1}{2}) (d - \bar{d})$$

For isoscalar target (e.g. deuteron)

$$F_2^\gamma = \frac{5}{9} \times (u + \bar{u} + d + \bar{d})$$

$$F_2^{\gamma Z} = \underbrace{\left[\frac{2}{3}(1 - \frac{8}{3}S_w^2) + \frac{1}{3}(1 - \frac{4}{3}S_w^2) \right]}_{\sim 1 - \frac{20}{9}S_w^2} \times (u + \bar{u} + d + \bar{d})$$

$$F_3^{\gamma Z} = (u - \bar{u} + d - \bar{d})$$

$$A^{PV} = \frac{dG^R - dG^L}{dG^R + dG^L} \approx - \frac{G_F Q^2}{4\sqrt{2}\pi d} \left[a_1 + a_2 \frac{1 - (1-y)^2}{1 + (1-y)^2} \right]$$

$$a_1 = -2g_A^c \frac{F_2^{\gamma Z}}{F_2^\gamma} \approx \left(\frac{9}{5} - 4S_w^2 \right)$$

$$a_2 = -2g_V^e \frac{F_3^{\gamma Z}}{F_2^\gamma} = (1 - 4S_w^2) \frac{9}{5} \frac{u - \bar{u} + d - \bar{d}}{u + \bar{u} + d + \bar{d}}$$

Because $S_w^2 \approx 0.23$ the 2nd term $\ll a_1$

$$\Rightarrow A^{PV} \approx - \frac{G_F Q^2}{4\sqrt{2}\pi d} \left(\frac{9}{5} - 4S_w^2 \right)$$

$$\approx 0.9 \cdot 10^{-4} \left[\frac{Q^2}{\text{GeV}} \right] \left(\frac{9}{5} - 4S_w^2 \right)$$

Confirmed by Prescott exp at
SLAC 1979

Further tests with Charged Current DIS

$$\frac{F_2^{eD}}{F_2^{vD}} = \frac{\frac{1}{2}(F_2^{\bar{u}p} + F_2^{\bar{u}n})}{\frac{1}{2}(F_2^{\bar{d}p} + F_2^{\bar{d}n})} = \frac{\frac{5}{18} \times (u + \bar{u} + d + \bar{d})}{\frac{1}{2} \alpha \times (u + \bar{d} + d + \bar{u})} = \frac{5}{18}$$

Relative sign of $V - A$ on the quark side \longleftrightarrow relative sign of F_2 and F_3 for particles and antiparticles

γ -DIS : γ cross sections larger than $\bar{\gamma}$ cross sections

With $G_F = 1.1663787(6) \cdot 10^{-5} \text{ GeV}^{-2}$

$$\rightarrow v = \frac{1}{\sqrt{2G_F}} = 246.21965 \text{ GeV}$$

$$\rightarrow M_W = \frac{ev}{2 \sin \theta_W}$$

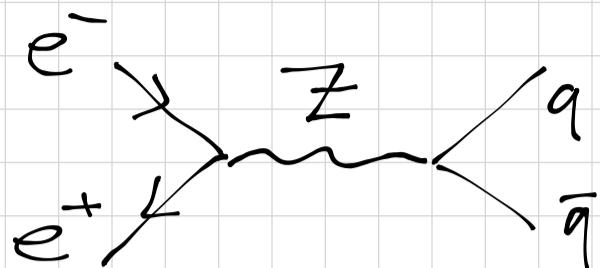
$$M_Z = \frac{ev}{2 \sin \theta_W \cos \theta_W}$$

With $\sin^2 \theta_W \approx 0.23$

$$M_W \approx 77.7 \text{ GeV}$$

$$\alpha^{-1} = 137.035999084(21) \quad M_Z^0 = 88.6 \text{ GeV}$$

LEP @ CERN was predominantly built to measure Z -mass



$$M_Z = 91.1876(21) \text{ GeV}$$

$$M_W = 80.379(12) \text{ GeV}$$

We see that measurements are very precise, and differ from naive tree-level expectations a lot: $M_Z - M_Z^0 \approx 2.6 \text{ GeV}$ $\Delta M_Z = 2.1 \text{ MeV}$

Need to include radiative corrections
1-loop accuracy should suffice

Formally, $\frac{\alpha}{2\pi} \approx 0.1\%$

but can be accompanied by large logs

$$\sim \ln \frac{M_Z^2}{M_e^2} \sim 25$$

need to resum

All quantities run \rightarrow need to decide on input parameters

QCD corrections can be significant

The program of Precision Tests of the St. Mod.

Basic parameters:

main

- $\left[\begin{array}{l} \text{SU}(2) \times U(1) \text{ couplings } g \text{ and } g' \\ \text{SSB scale } v \end{array} \right]$
- $\left[\begin{array}{l} \text{Higgs mass } m_h \end{array} \right]$

(Heavy) fermion masses m_t, m_b

Strong coupling α_s

→ enter via Rad. Corr.

Trade g, g', σ for constants that are most precisely known

$$\rightarrow \alpha^{-1}(0) = 137,035\,999\,139(31)$$

from $(g-2)_e$

To connect to EW observables:

extrapolate to M_Z (via RGE)

$$\alpha^{-1}(M_Z) \equiv \frac{1 - \Delta \alpha(M_Z^2)}{\alpha}$$

Here, renormalization scheme enters the game:

$$\text{on-shell } \alpha^{-1}(M_Z) = 128.527(17)$$

$$\overline{\text{MS}} \quad \alpha^{-1}(M_Z) = 127,950(17)$$

Uncertainty mainly $\Delta \alpha_{\text{had}}^{(5)} = 0.02764(13)$
with $\alpha_s(M_Z) = 0.118(2)$

→ Fermi constant from γ_e :

$$\frac{1}{\sqrt{2} V^2} = G_F = G_\mu = 1.1663787(6) \cdot 10^{-5} \text{ GeV}^{-2}$$

! G_μ does not ren (anom. dim. = 0)

$$*) \quad \sin^2 \theta_w = \frac{g'^2}{g^2 + g'^2}$$

↓

$$M_Z = 91.1876(21) \text{ GeV}$$

↑

preferable choice due to high precision

$$\text{Then, } M_Z = \frac{ev}{2\sin \theta_w \cos \theta_w}.$$

Renormalized $\sin^2 \theta_w$

At tree-level $\sin^2 \theta_w$ can be taken from any of the following def:
Then, each def. is enforced at arbitrary loop order

$$\sin^2 \theta_w = 1 - \frac{M_W^2}{M_Z^2} \quad (\text{on-shell})$$

$$\sin^2 \theta_w = \frac{g'^2}{g^2 + g'^2} \quad (\overline{\text{MS}})$$

$$\sin^2 \theta_w \cos^2 \theta_w = \frac{\tau_1 \lambda}{\sqrt{2} G_F m_Z^2} \quad (\text{Z-mass})$$

$$g_V^e = -\frac{1}{2} + 2\sin^2 \theta_w \quad (\text{effective})$$

To distinguish — introduce notation

On-shell	S_W^2	Can all be related
$\overline{\text{MS}}$	\hat{S}_Z^2	by known corrections
Z-mass	$S_{M_Z}^2$	
effective	\bar{S}_e^2	$O(d)$

On-shell scheme uses exp. measured pole masses $M_Z, M_W \rightarrow$ based on SSB

Problems: because m_t is close to M_Z , $Z f\bar{f}$ vertices receive large corrections $\sim m_t, M_H$

$S_W^2 = 0.22336(10)$ has larger unc. than other schemes
mixed QCD-EW RC large

Z-mass scheme: tree-level $M_Z - S_{M_Z}^2$ relation to all orders

Low. uncertainty: $S_{M_Z}^2 = 0.23105(5)$, no m_t, M_H dep.

Uncertainty dominated by $\delta(M_Z)$

Problems: similar to on-shell scheme

+ m_t, M_H dependence reemerges in other observables

$$\overline{\text{MS}} \text{ scheme: } \hat{S}_Z^2(\mu^2) = \frac{\hat{g}^2}{\hat{g}^2 + \hat{g}'^2}$$

Cures most of the problems of other schemes

$$\hat{S}_Z^2(M_Z) = 0.23129(5)$$

Running $\hat{S}_Z^2(\mu^2)$ can be tested experimentally

Useful to define

$$\hat{S}_0^2 = \hat{S}_Z^2(0) = 0.23865(8)$$

— relevant for low-energy tests (PVES, Atomic PU)

Problems: "theorist" definition: all masses and couplings run, and this M_Z is not the measured pole mass

Effective: \bar{S}_f^2 is defined so that

$Z \rightarrow f\bar{f}$ widths and asym.,

e.g. $A_{fb} = \frac{2g_A^f g_V^f}{(g_A^f)^2 + (g_V^f)^2}$ retain

their tree-level form

Couplings

$$\bar{g}_A^f = \sqrt{\bar{\rho}_f} t_{fL}^3$$

$$\bar{g}_V^f = \sqrt{\bar{\rho}_f} (t_{fL}^3 - 2\bar{S}_f^2 q_f)$$

Directly related to Z -pole measurements \rightarrow very precise

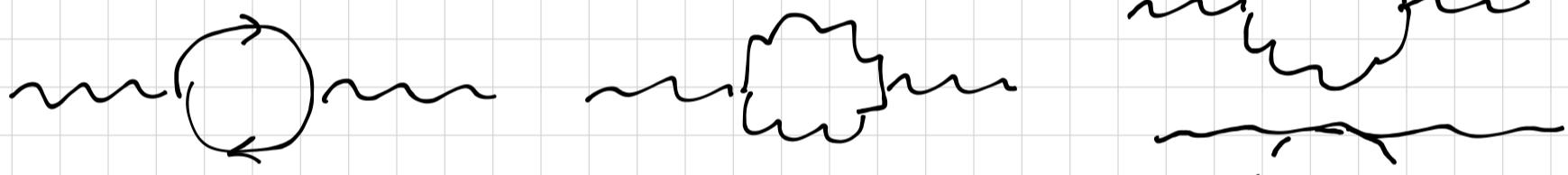
$$\overline{S}_e^2 = 0.23152(5) \text{ for } l^\pm$$

Problems :

- phenomenological def.
- depends on RC applied to 2-pole measurements
- \overline{S}_f^2 different for all fermions
- not easily translatable to non-2 pole observables.

Radiative corrections in $\overline{\text{MS}}$ scheme

Gauge boson self energy

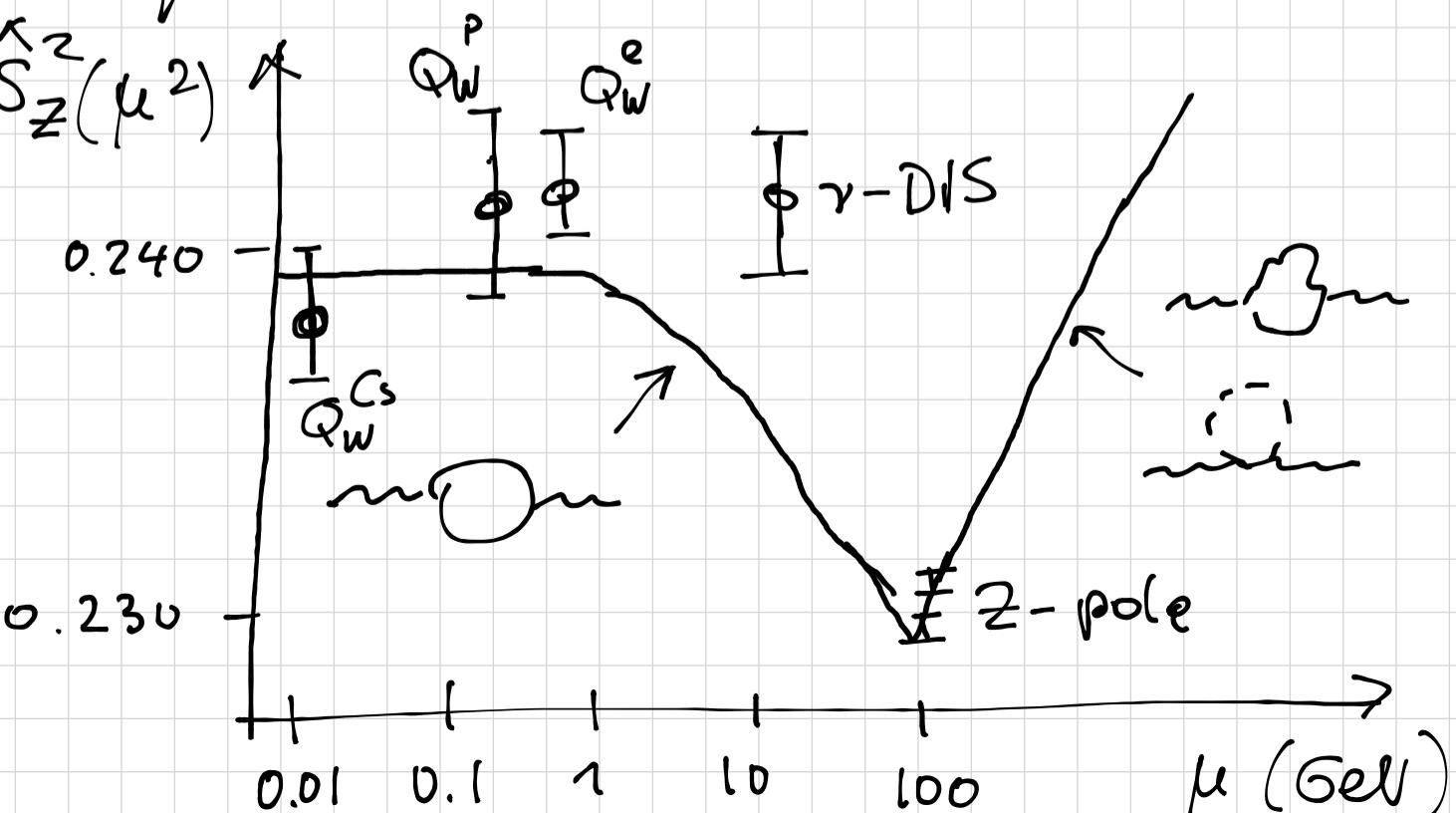


$\gamma\gamma, \gamma Z, ZZ, WW$ propagators

Main effects : $O(\alpha m_t^2)$, $O(\alpha \ln M_H)$

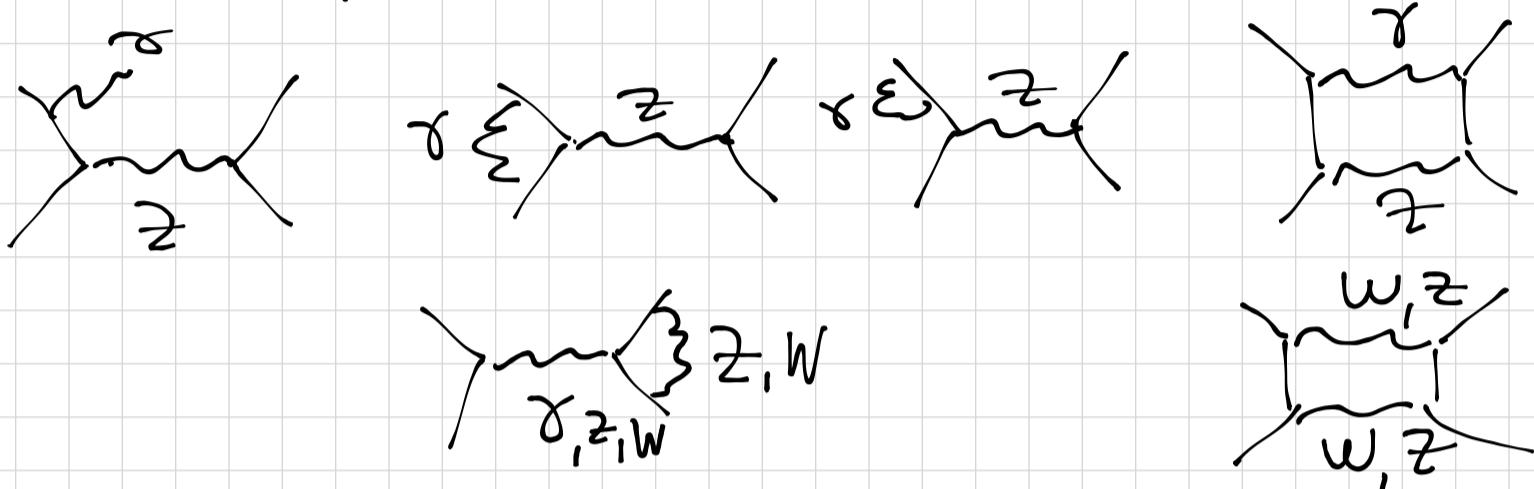
A.k.a. : Oblique corrections

Running $\overline{S}_Z^2(\mu^2)$



The oblique corrections are universal
(and specific for SM)

Other corrections depend on kinematics
(process-specific)



Renormalized
W-mass

$$M_W = \frac{\sqrt{\pi \alpha / \sqrt{2} G_F}}{\hat{s}_Z (1 - \Delta \hat{r}_W)^{1/2}}$$

where $\Delta \hat{r}_W \approx 1 - \frac{\alpha}{2 \cdot (M_Z)} \simeq 0.06630(13)$

with other small corr.

$$\Delta \hat{F}_W = 0.06952(13)$$

—

Renormalized
Z-mass

$$M_Z = \frac{M_W}{\hat{\rho}^{1/2} \hat{c}_Z}$$

$$\hat{\rho} \approx 1 + \rho_t \quad \rho_t = \frac{3 G_F m_t^2}{8 \sqrt{2} \pi^2} \simeq 0.00940 \left(\frac{m_t}{173.34} \right)^2$$

To compare: in on-shell scheme

$$M_w = \frac{\sqrt{\tau_1 \alpha / \sqrt{2G_F}}}{S_w (1 - \Delta r)^{1/2}}$$

$$\eta_2 = \frac{M_w}{C_w}$$

$$\Delta r = 1 - \frac{\alpha}{\alpha(n_2)} - c f g^2 \theta_w \rho_t = 0.03648(31)$$

≡

Enhanced sensitivity to m_t ; lower precision