Lecture 17

Consistency of the Electroweak Model

→ HE behavior (unitarity)

→ Gauge anomaly (cancellation)

→ Renormalization

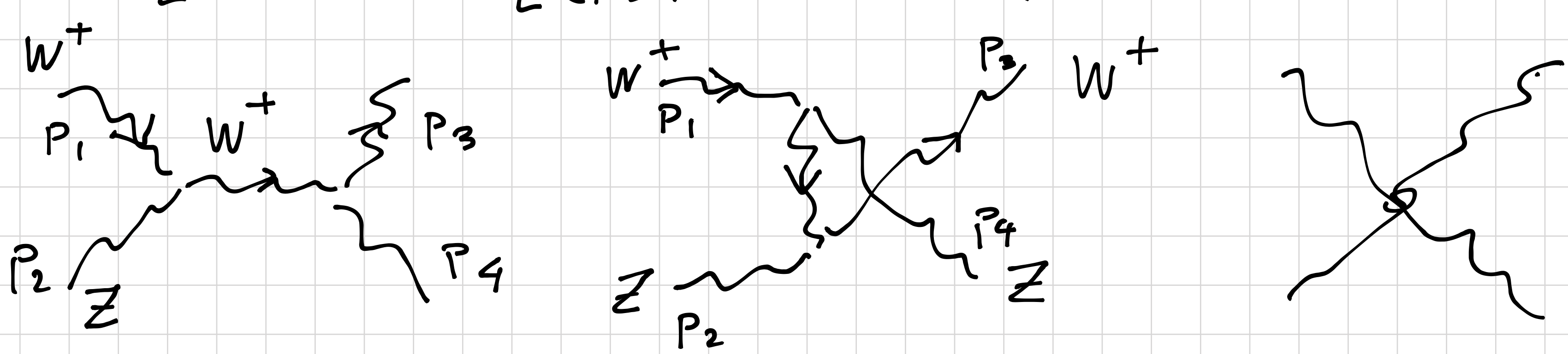
Consider elastic $W^+Z \to W^+Z$ scattering in the longitudinal mode

Polarization vectors

$$\varepsilon_L^{\mu}(p) = \frac{1}{M}\left(|\vec{p}|, E\hat{p}\right)$$

$$\varepsilon_L^{\mu} p_{\mu} = 0 \;; \quad \varepsilon_L^2 = -1$$

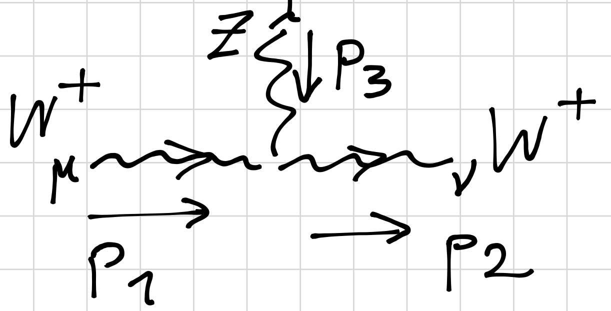
$$W_L^+(p_1) + Z_L(p_2) \longrightarrow W_L^+(p_3) + Z_L(p_4)$$



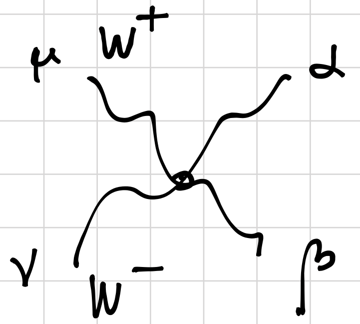
a) b) c)

Few ingredients needed:

Feynman rules for 3- and 4- gauge boson vertices (Exercise – derivation)



$$-ie\cot\theta_W\left[g^{\mu\nu}(p_1+p_2)^\lambda + g^{\nu\lambda}(-p_2-p_3)^\mu + g^{\lambda\mu}(p_3-p_1)^\nu\right]$$



$$ie^2\cot^2\theta_W\left[g^{\mu\alpha}g^{\nu\beta} + g^{\nu\alpha}g^{\mu\beta} - 2g^{\alpha\beta}g^{\mu\nu}\right]$$

Mandelstam variables

$$s=(p_1+p_2)^2 \quad t=(p_1-p_3)^2 \quad u=(p_2-p_3)^2 \qquad s+t+u=2M_W^2+2M_Z^2$$

Rewrite
$$\varepsilon_L^\mu(p_1) = \left[\frac{p_1^\mu}{M_W} + \frac{2M_W}{t-2M_W^2}p_3^\mu\right]\cdot\frac{t-2M_W^2}{\sqrt{t(t-4M_W^2)}}$$

$$\varepsilon_L^\alpha(p_3) \quad \text{—}\,//\,\text{—} \quad (p_1 \leftrightarrow p_3)$$

$$\varepsilon_L^\nu(p_2) = \left[\frac{p_2^\nu}{M_Z} + \frac{2M_Z}{t-2M_Z^2}p_4^\nu\right]\cdot\frac{t-2M_Z^2}{\sqrt{t(t-4M_W^2)}}$$

$$\varepsilon_L^\beta(p_4) = \quad \text{—}\,//\,\text{—} \quad (p_2 \leftrightarrow p_4)$$

Upon computing 3 diagrams (somewhat cumbersome)

$$\mathcal{M} = \frac{e^2\cot^2\theta_W}{4M_W^2M_Z^2}\Big\{\cancel{2su+s^2} - \cancel{2M_W^2\frac{3su+u^2}{s+u}} + \cancel{2M_Z^2\frac{s^2-3su-2u^2}{s+u}} - \frac{M_Z^4}{M_W^2}s$$

$$+\cancel{2su+u^2} - \cancel{2M_W^2\frac{3su+s^2}{s+u}} + \cancel{2M_Z^2\frac{u^2-3su-2s^2}{s+u}} - \frac{M_Z^4}{M_W^2}u$$

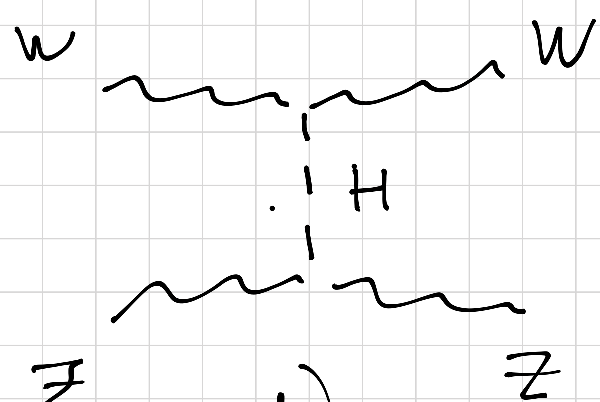
$$-\cancel{s^2-4su-u^2} + 2(\cancel{M_W^2}+\cancel{M_Z^2})\frac{s^2+6su+u^2}{s+u} + O(1)\Big\}$$

$$= \frac{e^2 \cot^2\theta_W}{4M_Z^2 M_W^2}\left(-\frac{M_Z^4}{M_W^2}\right)(s+u) = \frac{t}{v^2} + O(1)$$

using $M_W = M_Z \cos\theta_W$, $v = 2M_W \frac{\sin\theta_W}{e}$

The amplitude grows for $t \to \infty$ violating unitarity!

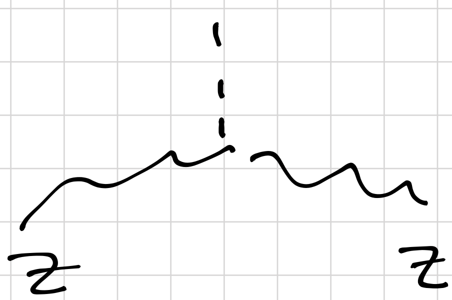
Situation is saved by Higgs exchange



d)

Feynman rules:

$$\frac{ie}{\sin\theta_W} M_W g^{\mu\nu}$$



$$\frac{ie}{\sin\theta_W \cos^2\theta_W} M_W g^{\alpha\beta}$$

$$\hookrightarrow d) \sim -\frac{e^2}{4M_Z^2 \sin^2\theta_W \cos^2\theta_W} \frac{t^2}{t - M_h^2} = -\frac{t}{v^2} + O(1).$$

$\to$ Divergence is exactly cancelled in the sum

Additionally, Higgs mass cannot be too large. By the discovery of Higgs in 2012 the actual mass window was relatively well constrained.

E.g. global fit to Tevatron data:

$$115 < m_H < 140 \text{ GeV}$$

The problem: Higgs mass is not related to EW symmetry breaking!

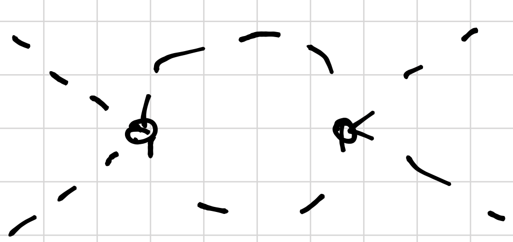
$$V(\phi) = -m^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2 + Y_{ij} \bar{L}^i H R^j$$

Only $v = \frac{m}{\sqrt{\lambda}}$, but not m itself

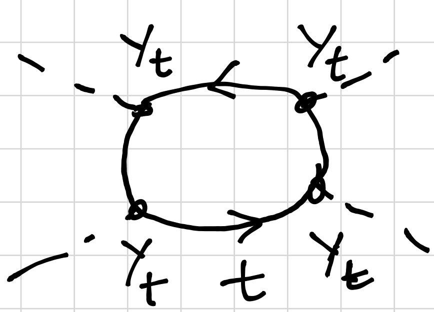
Quantum corrections?

1. v is determined as a minimum of the potential

Self-coupling runs (RGE)



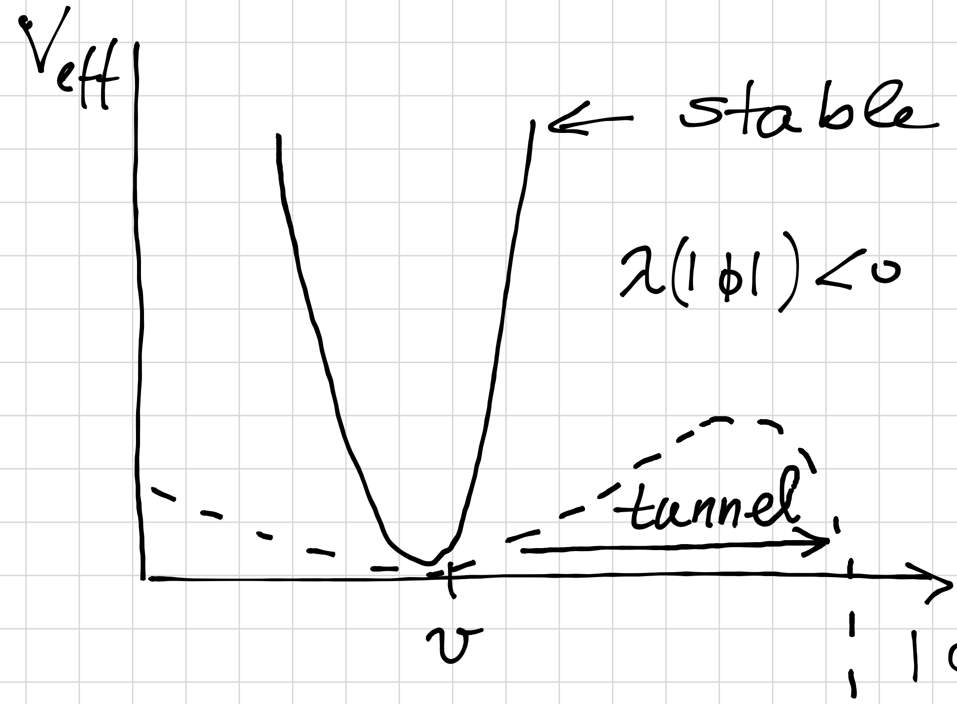
$\sim \frac{m_h^2}{v^2} \rightarrow$ increases λ



$\sim Y_t^4 \sim \left(\frac{m_t}{v}\right)^4 \rightarrow$ decreases λ

Interplay of quantum corrections may affect stability of vacuum of SM.

At large field values $V_{eff}(|\phi|) \sim \lambda(|\phi|)\,|\phi|^4$



← stable if $\lambda > 0$ for all $|\phi|$

$\lambda(|\phi|) < 0$ at $|\phi| \gg$: Metastable : has a deeper min. at higher $|\phi|$

↳ will eventually decay!

May be realized
if lifetime long enough (> age of Universe)

Absolute stability condition:

$$m_h > 129.4 \text{ GeV} + 2\left[m_t - 173.1 \text{ GeV}\right] - \frac{1}{2}\,\frac{\alpha_s(M_Z) - 0.1184}{0.0007} \pm (1 \text{ GeV})_{th}.$$

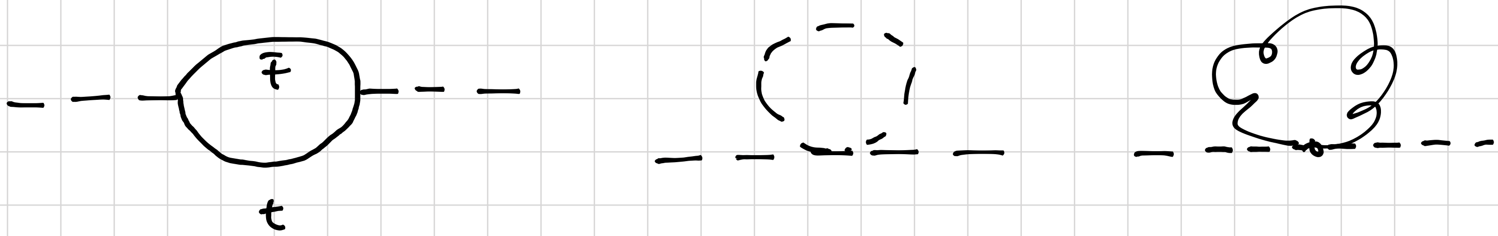
With actual $m_h = 125$ GeV the SM vacuum is metastable!

That is not all: <u>every</u> term in $V(|\phi|)$ is problematic!!!

$$V(|\phi|) = -m^2|\phi|^2 + \lambda|\phi|^4 + Y_{ij}\,\bar{L}^i \phi R^j$$

Higgs mass quadratically divergent!

First hint: $m_h^2 = \lambda v^2$ ↗ UV scale of EW!

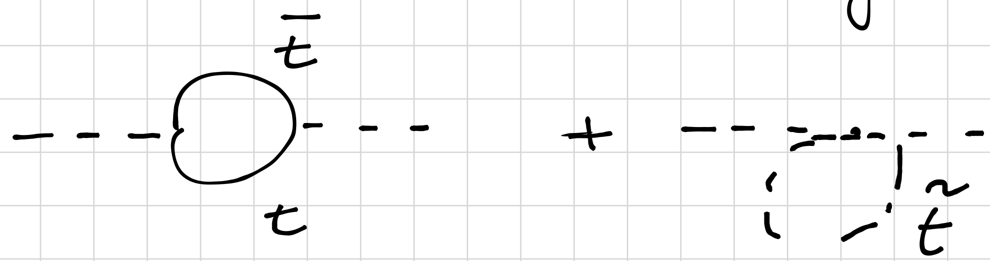


$$\delta M_h^2 \sim \Lambda_{UV}^2 > v^2$$

Quantum corrections to Higgs mass in SM are larger than tree-level.
If no new physics exists between EW scale $v = 246$ GeV and $M_{Pl} \sim 10^{19}$ GeV then why is Higgs mass finite?

Solutions: 1) some heavy new particles set the scale $< M_{Pl}$
$\longrightarrow$ searches at LHC

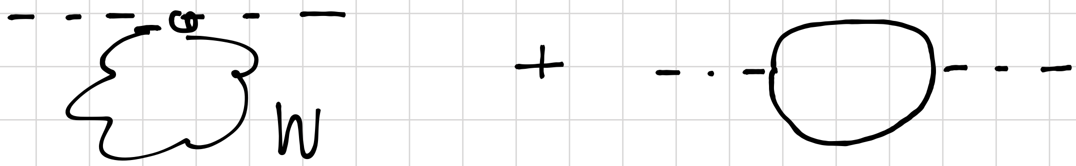
2) a (spontaneously broken) symmetry between fermions and bosons leads to cancellation of divergences at each order
(fermion loop $\rightarrow$ "$-$" sign;
if a boson with the same coupling exists $\rightarrow$ they cancel)

e.g. 

$\sim \ln \Lambda$

top

s-top
(superquark - boson)



Wino — s-fermion

S-fermions:
Higgsino
photino
Zino

$\left.\begin{array}{l}\text{photino}\\ \text{Zino}\end{array}\right\} = \text{neutralino}$

S-Bosons
s-électron
s-quark
s-neutrino

Very appealing idea: SUSY in ~300 GeV range also solution to Dark Matter problem

Apart from explaining duality fermion - boson...

Can accommodate spin-2 graviton
→ SuperGravity (SUGRA)

Allows for Strong-EW (Grand) unification in SU(5) GUT

Alas, no SUSY found at the LHC.
Origin of fermion families? Strong CP problem?

Finally, Yukawas

$$Y_{ij} \bar{L}^i \phi R^j$$

SM flavor hierarchy problem:
Y_{ij} span over 5 orders of magnitude
strongly hierarchical structure

Anomalies

Symmetry of the SM

$$SU(2)\times U(1) = SU(2)_V \times SU(2)_A \times U(1)_V \times U(1)_A$$

$U(1)_V \to \psi \to e^{i\alpha}\psi$

$U(1)_A \to \psi \to e^{i\alpha\gamma_5}\psi$

$\mathcal{L} = \bar\psi\,(i\not\partial - e\not A - m\psi)$ $\quad m\to 0$ $U(1)_{A,V}$ good.

Noether currents : $J^\mu = \bar\psi\gamma^\mu\psi$

$J_5^\mu = \bar\psi\gamma^\mu\gamma_5\psi$

Equations of motion: $\partial_\mu J^\mu = 0$

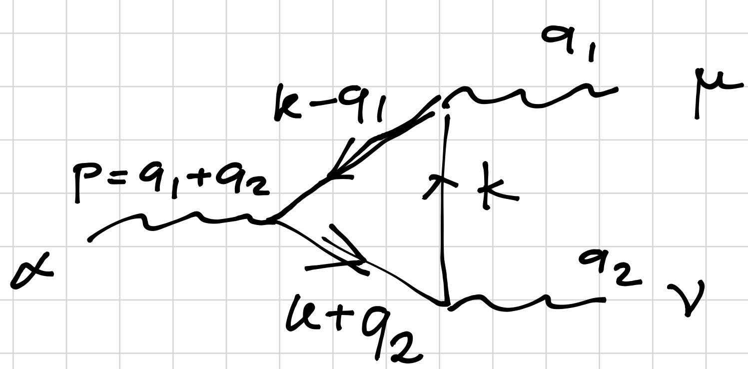
$\partial_\mu J_5^\mu = 2im\,\bar\psi\gamma_5\psi$

Classically, for $m=0$ $\partial_\mu J_5^\mu = 0$

In quantum theory $\partial_\mu J_5^\mu \neq 0$ even for $m=0$

Consider

$M_5^{\alpha\mu\nu} =$



$+$ crossed $(q_1\leftrightarrow q_2,\ \mu\leftrightarrow\nu)$

$$= \int \frac{d^4k}{(2\pi)^4}\left[\frac{\mathrm{Tr}[\gamma^\mu \not k \gamma^\nu (\not k + \not q_2)\gamma^\alpha\gamma_5(\not k - \not q_1)]}{k^2(k+q_2)^2(k-q_1)^2} + \begin{matrix}\mu\leftrightarrow\nu\\ q_1\leftrightarrow q_2\end{matrix}\right]$$

We expect $q_{1\mu} M_5^{\alpha\mu\nu} = q_{2\nu} M_5^{\alpha\mu\nu} = 0$

$p_\alpha M_5^{\alpha\mu\nu} \neq 0$

$$p_\alpha M_5^{\alpha\mu\nu} = 4i\varepsilon^{\mu\nu}{}_{\rho\sigma} \int \frac{d^4k}{(2\pi)^4} \left[\frac{k^\rho q_2^\sigma}{k^2(k+q_2)^2} + \frac{k^\rho q_1^\sigma}{k^2(k-q_1)^2} \right]$$

$$+ \begin{pmatrix} \mu \leftrightarrow \nu \\ q_1 \leftrightarrow q_2 \end{pmatrix}$$

The integrals seem to be zero

[Feynman param. $\frac{k^\rho q_2^\sigma}{k^2(k+q_2)^2} = \int_0^1 dx \frac{k^\rho q_2^\sigma}{\left[(k+xq_2)^2 + x(1-x)q_2^2\right]^2}$

↳ shift $\int$ $k \to \tilde{k} = k + xq_2$;

$\int d^4\tilde{k}\ \tilde{k}^\rho q_2^\sigma = 0$ by Lorentz inv.

$q_2^\rho q_2^\sigma \longrightarrow 0$ by antisym. of ε]

→ in accord with classical expectation

But this is wrong.

The integral is linearly divergent and applying a shift does not leave the integral unchanged

Trick: 1-D $\quad \Delta(a) = \int_{-\infty}^{\infty} dx \left[f(x+a) - f(x) \right]$

$\int dx f(x)$ linearly divergent

$\rightarrow f(\pm\infty) = \text{const.}$

Taylor expansion at $a=0$

$$\Delta(a) = \int_{-\infty}^{\infty} dx \left[a f'(x) + \frac{a^2}{2} f''(x) + \dots \right]$$

$$= a\left[f(\infty) - f(-\infty) \right] \neq 0$$

$$f^{(n)}(\pm\infty) = 0 \,, \; n \geq 1 \quad (\text{linear div.})$$

Similarly, in 4D

$$\Delta^{\alpha}(a^{\mu}) = \int \frac{d^4 k}{(2\pi)^4} \left[F^{\alpha}(k+a) - F^{\alpha}(k) \right]$$

$$\underset{k\to\infty}{F^{\alpha}(k)} = A \frac{k^{\alpha}}{k^4}$$

Same trick

$$\Delta^{\alpha}(a^{\mu}) = \int \frac{d^4 k}{(2\pi)^4} \left[a_{\mu} \frac{\partial}{\partial k_{\mu}} F^{\alpha}(k) + \dots \right]$$

Wick rotation + Gauss theorem

$$\Delta^{\alpha}(a^{\mu}) = i a_{\mu} \int \frac{d^3 S^{\mu}}{(2\pi)^4} F^{\alpha}(k_E)$$

$d^3 S^{\mu} = k_E^2 k_E^{\mu} d\Omega_4 \rightarrow$ surface element

$$\Rightarrow \Delta^{\alpha}(a^{\mu}) = i a_{\mu} \int \frac{d\Omega_4}{(2\pi)^4} A \frac{k_E^2 k_E^{\mu} k_E^{\alpha}}{k_E^4} = i \frac{A a^{\mu}}{32\pi^2}$$

Accounting for the shift ambiguity,

$$p_{\alpha} M_5^{\alpha\mu\nu} = \frac{1}{4\pi^2} \varepsilon^{\mu\nu\rho\sigma} q_{1\rho} q_{2\sigma}$$

$$q_{1\mu} M_5^{\alpha\mu\nu} = q_{2\nu} M_5^{\alpha\mu\nu} = 0$$

One consequence of EM gauge invariance: electric charges of L and R fermions should be the same

Importantly, chiral anomaly is not renormalized by higher order corrections (1-loop exact)

Chiral anomaly is responsible for $\pi^0 \to \gamma\gamma$ decay (as a result, π^0 lifetime is much shorter than that of $\pi^\pm$)

$$\partial_\mu J_5^\mu = -\frac{e^2}{16\pi^2} \varepsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta}$$

$$\Gamma(\pi^0 \to \gamma\gamma) = \frac{\alpha^2}{64\pi^3} \frac{m_\pi^3}{F_\pi^2} = 7.77\,\text{eV}$$

Exp. 7.81(12) eV

Anomalies associated with gauged symmetries are dangerous: symmetries may be broken by quantum corrections, Ward identities violated → the theory would be inconsistent.

A gauge-field theory must be gauge anonomaly-free.