Lecture 11

DGLAP Evolution of PDFs

Bjorken scaling: $F_{1,2}(x)$ are independent of Q^2 → prediction of parton model

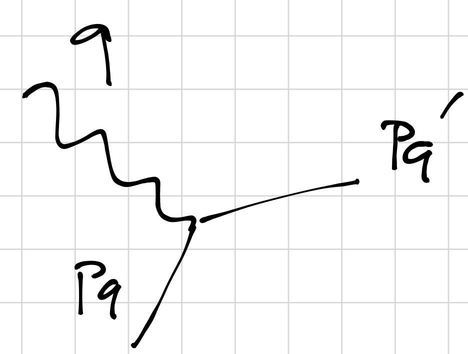
This scaling is only approximate!
QCD radiative corrections

$$W_0^{\mu\nu} = \sum_q e_q^2 \int_0^1 \frac{dx}{x}\, q(x)\, w^{\mu\nu}_{q\to q}$$

$\frac{1}{x}$ from initial flux $\frac{1}{2E_q} = \frac{1}{2Ex}$

$$w^{\mu\nu}_{q\to q} = \frac{1}{2\pi}\,\frac{1}{2}\sum_{s,s'}\int \frac{d^3\vec{p}_q^{\,\prime}}{(2\pi)^3 2E_q'}(2\pi)^4\,\delta^4(p+q-p_q')$$
$$\times\, J^{\mu *}_{q\to q}\, J^{\nu}_{q\to q}$$

$$J^{\mu}_{q\to q} = \bar{u}(p_q')\,\gamma^{\mu}\,u(p_q)$$



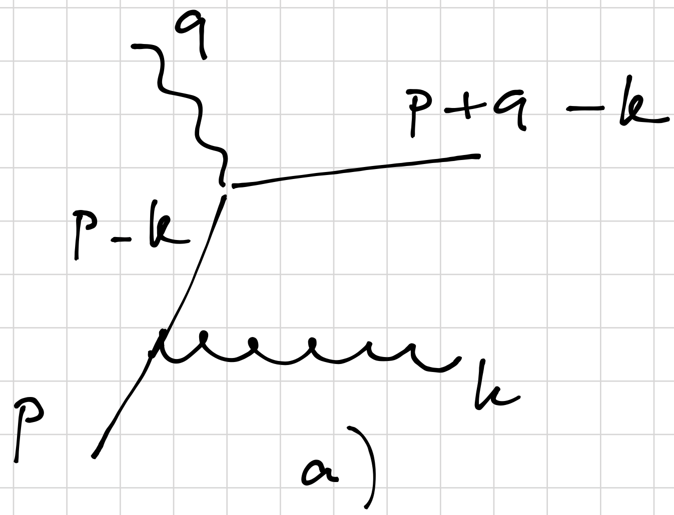
$$\int \frac{d^3 p_q'}{(2\pi)^3 2E_q'} = \int \frac{d^4 p_q'}{(2\pi)^4}\, 2\pi\,\delta(p_q'^2)\,\Theta(E_q')$$

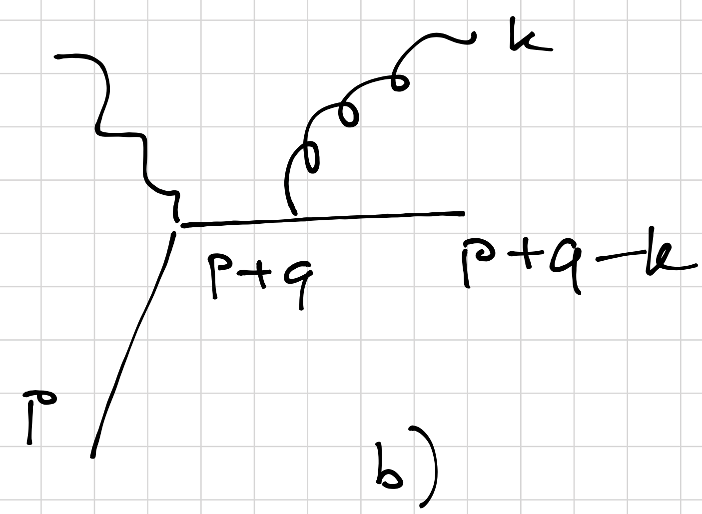
$$\Rightarrow\; w^{\mu\nu}_{q\to q} = \delta\left((p_q+q)^2\right)\Theta(E_q')\; 2\Big[(p_q+q)^{\mu} p_q^{\nu} + (p_q+q)^{\nu} p_q^{\mu}$$
$$- (p_q+q, p_q)\, g^{\mu\nu}\Big]$$

$$= 2pq\,\delta(2pq - Q^2)\,\theta(E_q')$$

$$\left[-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} + \frac{2}{pq}\left(p - \frac{pq}{q^2}q\right)^\mu \left(p - \frac{pq}{q^2}q\right)^\nu\right]$$

Now energetic gluon emission





$$W^{\mu\nu}_{q\to qg} = \frac{1}{2\pi}\,\frac{1}{2}\sum_{s,s'}\sum_{\lambda}\int \frac{d^4k}{(2\pi)^4}\, 2\pi\,\delta(k^2)\,\theta(k^0)$$

$$\times \int \frac{d^4p'}{(2\pi)^4}\, 2\pi\,\delta(p'^2)\,\theta(E')\cdot(2\pi)^4\,\delta(p+q-p'-k)$$

$$J^{\mu *}_{q\to qg}\cdot J^{\nu}_{q\to qg}$$

$$J^{\nu}_{q\to qg\,(a)} = \bar{u}(p+q-k)\,\gamma^\nu\,\frac{i(\not p - \not k)}{(p-k)^2}\,ig\,T^a\,\not\epsilon^*_\lambda\,u(p)$$

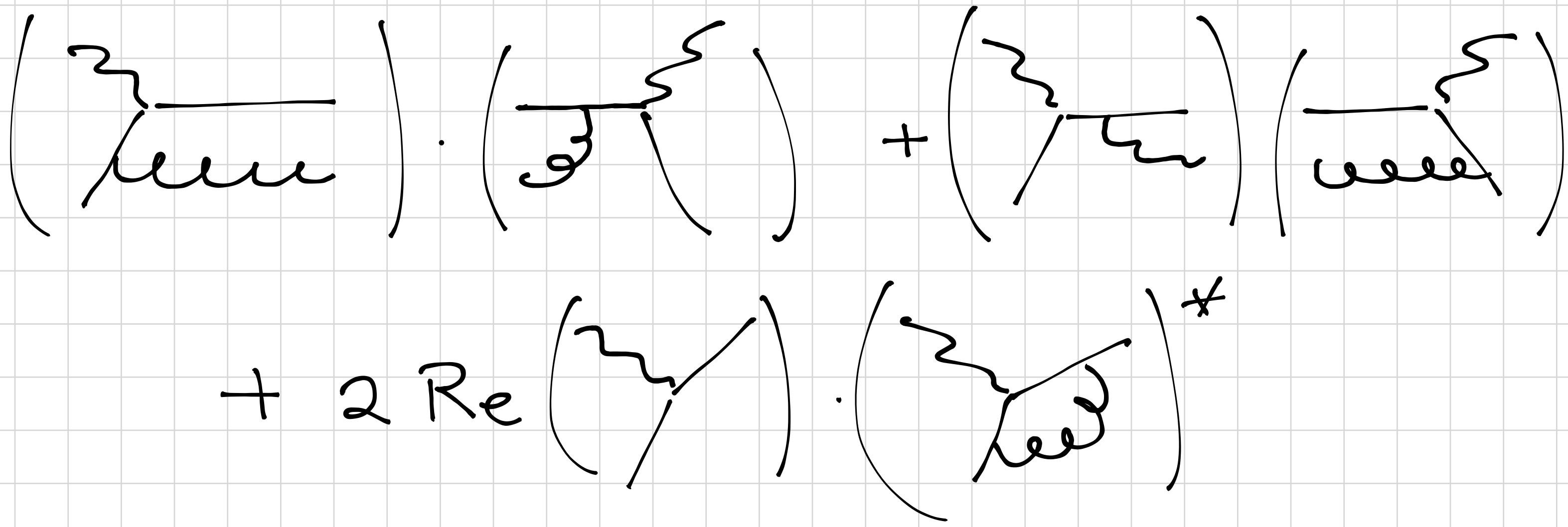
$$J^{\nu}_{q\to qg\,(b)} = \bar{u}(p+q-k)\,ig\,T^a\,\not\epsilon^*_\lambda\,\frac{i(\not p + \not q)}{(p+q)^2}\,\gamma^\nu\,u(p)$$

Color factor $C_F = \frac{4}{3}$

We will be looking for large logarithms

! IR singularity will cancel in the

sum



Look for collinear singularity

$$\sim \frac{1}{(p-k)^2} \sim \frac{1}{-2|p||k|(1-\cos\theta)} \Big|_{\cos\theta \to 1}$$

This singularity is regularized by quark mass μ

$$\frac{1}{(p-k)^2-\mu^2} = - \frac{1}{2|p||k|(1-\cos\theta)+\mu^2}$$

$$\int_{-1}^{1} \frac{dz}{2pk(1-z)+\mu^2} = \frac{1}{2pk} \ln\left(1+\frac{2pk}{\mu^2}\right)$$

Note that graph b) has no singularity for $(p+q)^2 = s$

$\hookrightarrow$ account for graph a) only

$$\frac{1}{2}\sum_{s,s',\lambda} \left[J^{\mu *}_{q\to q\gamma(a)} J^{\nu}_{q\to q\gamma(a)} \right]$$

$$= \frac{1}{2} \Delta_{\alpha\beta} \, \mathrm{Tr} \left\{ \gamma^\alpha (\not{p}-\not{k}) \gamma^\mu (\not{p}+\not{q}-\not{k}) \gamma^\nu (\not{p}-\not{k}) \gamma^\beta \not{p} \right\}$$

$$\Delta_{\alpha\beta} = \sum_{\lambda} \varepsilon_\alpha(k,\lambda)\, \varepsilon^*_\beta(k,\lambda)$$

Work out in the axial (light-cone) gauge

For $p \sim E \cdot \bar{n}$ $\quad (E = (p\cdot n))$

Choose $\quad \Delta_{\alpha\beta} = -g_{\alpha\beta} + \dfrac{k_\alpha n_\beta + k_\beta n_\alpha}{(kn)}$

$$k_\alpha \Delta^{\alpha\beta} = n_\alpha \Delta^{\alpha\beta} = 0$$

$\Downarrow$

$$w^{\mu\nu}_{q\to qg\,(a)} = \frac{1}{2}\frac{1}{(2\pi)^3} C_F g^2 \int \frac{d^3k}{2k}\, \delta((p+q-k)^2)\, \theta(p^0+q^0-k^0)$$

$$\cdot \frac{1}{\left[(p-k)^2-\mu^2\right]^2}$$

$$\cdot \Big\{ -\mathrm{Tr}\left(\gamma^\mu(\not{p}+\not{q}-\not{k})\gamma^\nu(\not{p}-\not{k})\gamma^\alpha \not{p}\gamma_\alpha(\not{p}-\not{k})\right)$$

$$+ \frac{1}{(kn)}\mathrm{Tr}\left(\gamma^\mu(\not{p}+\not{q}-\not{k})\gamma^\nu(\not{p}-\not{k})\not{n}\not{p}\not{k}(\not{p}-\not{k})\right)$$

$$+ \frac{1}{(kn)}\mathrm{Tr}\left(\gamma^\mu(\not{p}+\not{q}-\not{k})\gamma^\nu(\not{p}-\not{k})\not{k}\not{p}\not{n}(\not{p}-\not{k})\right)\Big\}$$

Trace: $\quad 4(pk)\,\mathrm{Tr}\,\gamma^\mu(\not{p}+\not{q}-\not{k})\gamma^\nu\not{k}$

$$+\frac{2(pk)}{(kn)}\mathrm{Tr}\,\gamma^\mu(\not{p}+\not{q}-\not{k})\gamma^\nu\underbrace{\left[(\not{p}-\not{k})\not{n}\not{p} + \not{p}\not{n}(\not{p}-\not{k})\right]}_{\displaystyle = 2(pn)(\not{p}-\not{k}) + 2(p-k,n)\not{p} + 2(pk)\not{n}}$$

$$= 16(pk)\Big[(p+q-k)^\mu g^{\nu\alpha} + (p+q-k)^\nu g^{\mu\alpha} - g^{\mu\nu}(p+q-k)^\alpha\Big]$$

$$\cdot \left\{ k_\alpha + \frac{(pn)}{(kn)}(p-k)_\alpha + \frac{(p-k,n)}{(kn)} p_\alpha + \dots \right\}$$

Evaluate in the frame

$$p = \frac{Q}{2x}(1,0,0,1)$$

$$q = Q\,(0,0,0,-1)$$

In the numerator: enough to take

$k \parallel p$: $\quad k = \beta p$

! $\Theta(k^0)\,\Theta(p^0+q^0-k^0) \Rightarrow 0<\beta<1$

$$\hookrightarrow \{\ \}_\alpha = \left(\beta p + 2\frac{1-\beta}{\beta} p\right)_\alpha = \frac{1+(1-\beta)^2}{\beta} p_\alpha$$

$$[\ \ ]^{\mu\nu\alpha} = [(1-\beta)p+q]^\mu g^{\nu\alpha} + [(1-\beta)p+q]^\nu g^{\mu\alpha} - g^{\mu\nu}[(1-\beta)p+q]^\alpha$$

$$\Rightarrow \{\ \}\{\ \}[\ \ ] = \frac{1+(1-\beta)^2}{\beta}\Big\{-g^{\mu\nu}(pq) + 2(1-\beta)p^\mu p^\nu + p^\mu q^\nu + p^\nu q^\mu\Big\}$$

$$W^{\mu\nu}_{q+qg(a)} = \frac{1}{2}\frac{g^2 C_F}{(2\pi)^3}\cdot 4\int \frac{d^3k}{k}\frac{2pk}{[2pk+\mu^2]^2}$$

$$\cdot\, \theta(k^0)\,\theta(p^0+q^0-k^0)\;\delta((p+q-k)^2)$$

$$\cdot\, \frac{1+(1-\beta)^2}{\beta}$$

$$\cdot\left[-g^{\mu\nu}(pq) + 2(1-\beta)p^\mu p^\nu + p^\mu q^\nu + p^\nu q^\mu\right]$$

$$\delta((p+q-k)^2) = \delta(\underbrace{\cancel{(p-k)^2}}_{0} - Q^2 + 2(q, p-k))$$

$$= \delta\left(-Q^2 + \frac{Q^2}{x}(1-\beta)\right)$$

$$= \frac{x}{Q^2}\,\delta(1-\beta-x)$$

$$q_\mu[\quad] = \cancel{q^\nu(pq)} + 2(1-\beta)(pq)p^\nu + \cancel{(pq)q^\nu} - Q^2 p^\nu$$

$$= 0 \quad \underline{\text{OK}}$$

$$p_\mu[\quad] = -p^\nu(pq) + (pq)p^\nu = 0 \quad \text{OK}$$

$$\Rightarrow [\quad] = (pq)(-g_\perp^{\mu\nu}) = \frac{Q^2}{2x}(-g_\perp^{\mu\nu})$$

$$\int\frac{d^3k}{k} = \int k\,dk\,d\cos\theta\,d\varphi = \left(\frac{Q}{2x}\right)^2\int_0^1 \beta\,d\beta\,2\pi\int_{-1}^{1} d\cos\theta$$

$$\Downarrow$$

$$W^{\mu\nu}_{q\to qg(a)} = \frac{\alpha}{2\pi} C_F \left(-g_\perp^{\mu\nu}\right) \int_0^1 d\beta \int_{-1}^{1} dz \, \delta(1-\beta-x) \frac{1+(1-\beta)^2}{\beta(1-z)+\frac{4x^2\mu^2}{Q^2}}$$

$$\int_{-1}^{1} dz \;\longrightarrow\; \frac{1}{\beta} \ln \frac{2\beta Q^2 + 4x^2\mu^2}{4x^2\mu^2}$$

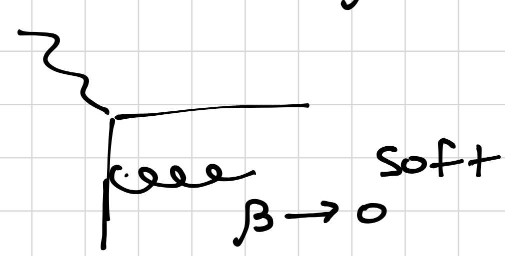
$$\boxed{W^{\mu\nu}_{q\to qg(a)} \simeq -g_\perp^{\mu\nu} \cdot \frac{\alpha}{2\pi} C_F \int_0^1 \frac{d\beta}{\beta}\left[1+(1-\beta)^2\right] \ln\frac{Q^2}{\mu^2}}$$

LLA

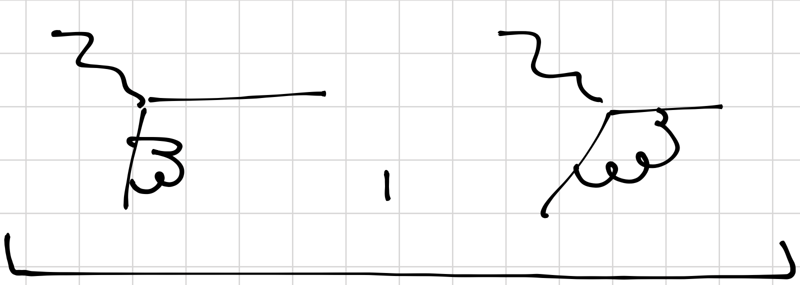
$\Downarrow$

$$f_2(x,Q^2) = 2x f_1(x,Q^2) = C_F \frac{\alpha_s}{2\pi} \times \int_0^1 d\beta \, \frac{1+(1-\beta)^2}{\beta} \ln\left(\frac{Q^2}{\mu^2}\right)$$

Attn: additionally to collinear log $\ln\frac{Q^2}{\mu^2}$ there is IR singularity $\frac{1}{\beta}$.



Cancelled by loops



Have similar structure, but

$$\frac{1+(1-\beta)^2}{\beta} \longrightarrow \lambda\,\delta(\beta)$$

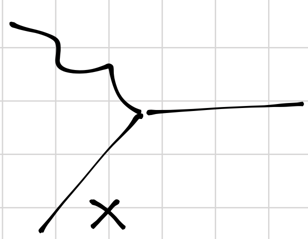
$\lambda \longrightarrow$ const.

$$\boxed{f_2(x,Q^2)_{LLA} = \frac{\alpha_s}{2\pi} C_F \ln\frac{Q^2}{\mu^2} \times \int_0^1 d\beta\, \delta(1-x-\beta)\,(1-\beta)\left[\frac{1+(1-\beta)^2}{\beta_+} + \lambda\,\delta(\beta)\right]}$$

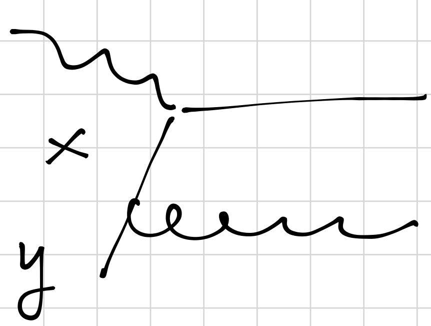
Here: PV integral $\displaystyle\int_0^1 d\beta \,\frac{f(\beta)}{\beta_+} \equiv \int_0^1 d\beta\, \frac{f(\beta)-f(0)}{\beta}$

The leading log correction to the non-singlet (valence) quark distribution

NS



$\longrightarrow \quad q_v(x) = q(x) - \bar{q}(x)$



$q_v(x,Q^2)$

Quark struck by virtual γ has fraction $z = \frac{x}{y}$ of the initial q momentum

$$f_2^{NS}\left(\frac{x}{y},Q^2\right) = \frac{\alpha_s}{2\pi}\ln\frac{Q^2}{\mu^2}\,\frac{x}{y}\, P^{NS}_{q\to qg}\left(\frac{x}{y}\right),$$

with NS $q-g$ splitting function

$$P^{NS}_{q\to qg}(z) \equiv C_F\left[\frac{1+z^2}{(1-z)_+} + \lambda\,\delta(1-z)\right]$$

Correction to NS PDF $q_v(x) \longrightarrow$ from $\frac{1}{x}F_2$

$$\delta q_v(x,Q^2) = \frac{\alpha_s}{2\pi}\ln\frac{Q^2}{\mu^2}\int_x^1 \frac{dy}{y}\,P^{NS}_{q\to qg}\left(\frac{x}{y}\right) q_v(y,Q^2)$$

Leading log evolution

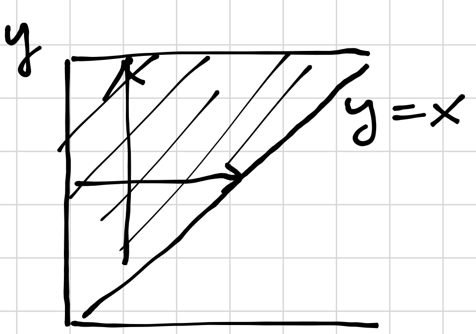
$$q_v(x,Q^2) = q_v(x) + \delta q_v(x,Q^2)$$

DGLAP evolution (Dokshitzer – Gribov – Lipatov, Altarelli – Parisi)

$$\frac{\partial}{\partial \ln Q^2} q_v(x,Q^2) = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y}\,P^{NS}_{q\to qg}\left(\frac{x}{y}\right) q_v(y,Q^2)$$

Can now determine λ: $\int_0^1 dx\, q_v(x,Q^2) = \begin{cases} 2, & q=u \\ 1, & q=d \end{cases}$

$$\Longrightarrow\quad 0 = \int_0^1 dx \int_x^1 \frac{dy}{y}\,P^{NN}_{q\to qg}\left(\frac{x}{y}\right) q_v(y,Q^2)$$



$$\int_0^1 dx \int_x^1 dy = \int_0^1 dy \int_0^y dx\ ; \quad x \longrightarrow yz$$

$$\Downarrow$$

$$0 = \int_0^1 dy\, q_v(y,Q^2) \left[\int_0^1 dz\, P^{NS}_{q\to qg}(z) \right]$$

$$\int_0^1 dz \left[\frac{1+z^2}{(1-z)_+} + \lambda\, \delta(1-z) \right] = \int_0^1 dz \left[\frac{z^2-1}{1-z} + \lambda \delta(1-z) \right]$$

$$= -\frac{3}{2} + \lambda = 0 \implies \lambda = \frac{3}{2}$$

$$\Downarrow$$

$$\boxed{P^{NS}_{q\to qg}(z) = C_F \left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z) \right]}$$

At next - leading orders :
also $\gamma^* g \to q\bar{q}$ subprocess
$g \to gg$ splitting viable

$\Rightarrow$ DGLAP is a system of $\int$-∂ eqs:

$$\mu \frac{\partial}{\partial \mu} \begin{pmatrix} f_i(x,\mu) \\ f_g(x,\mu) \end{pmatrix} = \sum_j \frac{\alpha_s}{\pi} \int_x^1 \frac{d\xi}{\xi} \begin{pmatrix} P_{q_i q_j} & P_{q_i g} \\ P_{g q_j} & P_{gg} \end{pmatrix} \left(\frac{x}{\xi}\right) \begin{pmatrix} f_j(x,\mu) \\ f_g(x,\mu) \end{pmatrix}$$

with splitting functions

$$P_{qq}(z) = C_F \left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z) \right]$$

$$P_{qg}(z) = T_F \left[z^2 + (1-z)^2 \right]$$

$$P_{gq} = C_F \, \frac{1+(1-z)^2}{z}$$

$$P_{gg} = 2C_A \left[\frac{z}{(1-z)_+} + \frac{1-z}{z} + z(1-z) \right] + \frac{\beta_0}{2} \delta(1-z)$$

with $\beta_0 = \frac{11}{3} C_A - \frac{4}{3} T_F n_f$

With these evolution equations one performs global fits of DIS data.
Quark distribution functions are scale-dependent; identify the renorm. scale μ^2 with Q^2 $\longrightarrow$ parametrize $q(x, Q_0^2)$
e.g. as $q_v(x, Q_0^2) = N \, x^{\alpha}(1-x)^{\beta}(1 + a\sqrt{x} + bx)$;
usually choose $Q_0^2 \sim 1 \text{ GeV}^2$;
$N \rightarrow$ from normalization $\int_0^1 dx \, q_v = n_q$

Evolve $q_v(x, Q_0^2)$ to Q^2 by DGLAP $\longrightarrow$ obtain parameters α, β, a, b from best χ^2

Why does collinear gluon emission strongly modify scattering amplitudes/cross sec. at high energies?

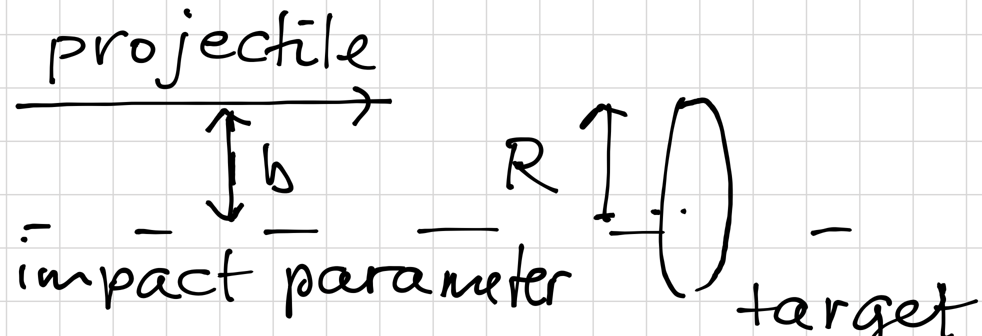
$\longrightarrow$

$\longleftarrow$

Two HE colliding particles have no time to interact; it is therefor preferrable to lose some energy by radiating a collinear gluon (collinear $\rightarrow$ enhanced by a large log, non-collinear is not). In fact, this process repeats many times untie the colliding particles slowed down enough to be able to "talk".

Peculiar property of high-energy scatterin of hadrons $\longrightarrow$ growing interaction radius.

Classically :



For $b \leq R$ the projectile hits the target, for $b < R$ it misses.

R is a geometrical characteristics of t.

Raise the velocity $\longrightarrow$ nothing changes!

Constant radius; Cross section remains constant $\sigma = \pi R^2$

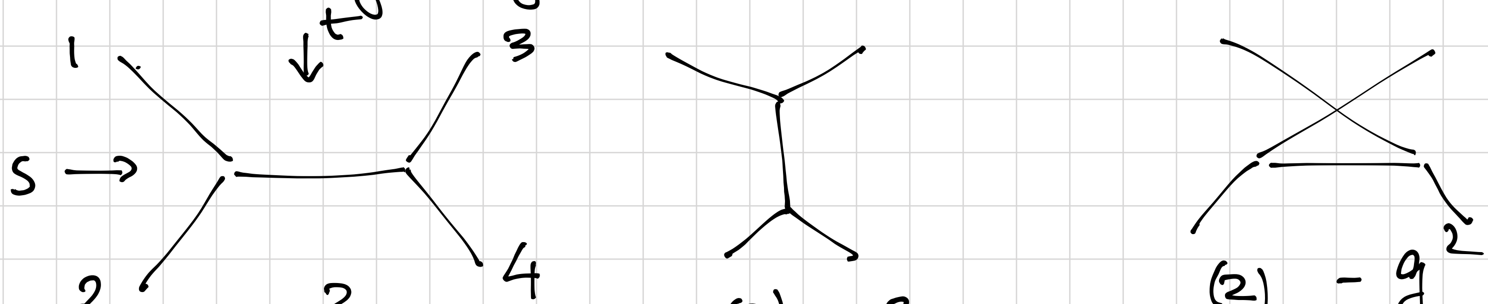
In QFT this is not true anymore!

Hadronic cross sections rise at HE!

This feature can be understood qualitatively in φ^3 theory

$2 \to 2$ scattering, $\mathcal{L}_{int} = \frac{g}{3!}\varphi^3$

At leading order:



$A_s^{(2)} = \frac{-g^2}{s-m^2}$ $\quad A_t^{(2)} = \frac{-g^2}{t-m^2}$ $\quad A_u^{(2)} = \frac{-g^2}{u-m^2}$

$s + u + t = 4m^2 \, ; \qquad s \gg \quad t \sim -m^2 \ll s$

$u \sim -s$

Choose frame: lightlike vectors

$n = \frac{\sqrt{s}}{2}(1,0,0,1) \, ; \; \bar{n} = \frac{\sqrt{s}}{2}(1,0,0,-1) \rightarrow 2 n\bar{n} = s$

Notation: $\quad a^\mu = \alpha n^\mu + \beta \bar{n}^\mu + \vec{a}_\perp \equiv (\alpha, \beta, \vec{a}_\perp)$

$p_1^\mu \cong (1, \gamma, \vec{0}_\perp) \, ; \; \gamma = \frac{m^2}{s} \ll 1$

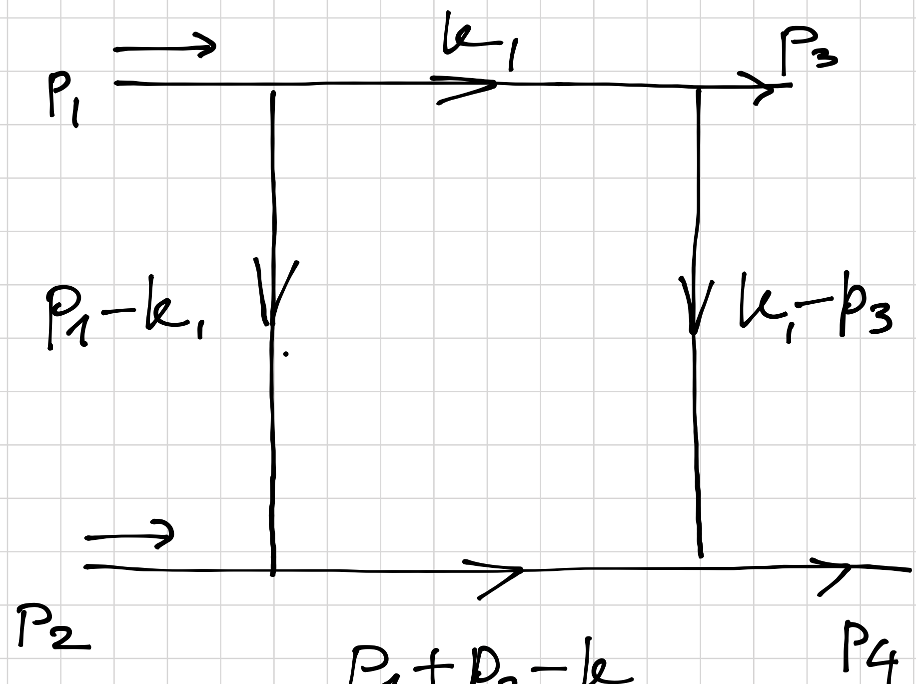
$p_2^\mu = (\gamma, 1, \vec{0}_\perp) \qquad\qquad (p_1+p_2)^2 = s + O(m^2)$

Momentum transfer $\quad t = (p_1 - p_3)^2 = -\vec{\Delta}_\perp^2 \ll s$

$p_3^\mu = (1, \gamma, \vec{\Delta}_\perp) \qquad u = (p_2 - p_3)^2 \cong -s$

$p_4^\mu = (\gamma, 1, -\vec{\Delta}_\perp)$

at NLO:



$k_1 = (\alpha_1, \beta_1, \vec{k}_{1\perp})$

Evaluate for $t = 0$

$(p_1 = p_3 \, , \; p_2 = p_4)$

$$J_2 = \int \frac{d^4k_1}{(2\pi)^4} \frac{1}{k_1^2 - m_+^2} \frac{1}{(p_1-k_1)^2 - m_+^2} \frac{1}{(p_1+p_2-k_1)^2 - m_+^2} \frac{1}{(p_3-k)^2 - m_+^2}$$

$$\int d^4k = \frac{s}{2} \int_{-\infty}^{\infty} d\alpha \int_{-\infty}^{\infty} d\beta \int d^2\vec{k}_\perp$$

$$k_1^2 - m_+^2 = \alpha_1\beta_1 s - m^2 - k_\perp^2 + i\varepsilon$$

$$(p_1-k_1)^2 - m_+^2 = (1-\alpha_1)(\gamma-\beta_1)s - m^2 - \vec{k}_\perp^2 + i\varepsilon = (p_3-k_1)^2 - m_+^2$$

$$(p_1+p_2-k)^2 - m_+^2 = (1+\gamma-\alpha_1)(1+\gamma-\beta_2)s - \vec{k}_\perp^2 - m^2 + i\varepsilon$$

$$A_s^{(4)} = -\frac{g^2}{s} \frac{g^2}{16\pi^2 m^2}\left[\ln\frac{s}{m^2} - i\pi\right]$$

Collinear log $\longrightarrow$ from $\alpha_1 \sim 1$, $\gamma \lesssim \beta_1 \lesssim 1$

Vertical lines $\longrightarrow$ $\displaystyle\int \frac{d^2\vec{k}_\perp}{(k_\perp^2+m^2)^2} = \frac{\pi}{m^2}$

Upper line $\rightsquigarrow$ $\displaystyle\int_\gamma^1 \frac{d\beta_1}{m^2+k_\perp^2 - \beta_1 s - i\varepsilon} = \frac{1}{s}\ln\left(-\frac{s}{m^2} - i\varepsilon\right)$

$k_\perp^2$ and m^2 regularize UV and IR

α_1 picks a pole at $\alpha_1 \simeq 1+$

$\beta_1 \simeq \gamma \longrightarrow$ Im part (not log-enhanced)

$\int_\gamma^1 d\beta_1 \longrightarrow$ log enhancement for Re part.
due to large phase space

Similarly, $A_u^{(4)} = -\frac{g^2}{u}\frac{g^2}{16\pi^2 m^2}\ln\left(\frac{-u}{m^2}\right)$! no Im for $s > 4m^2$

$\Downarrow$ LO + NLO

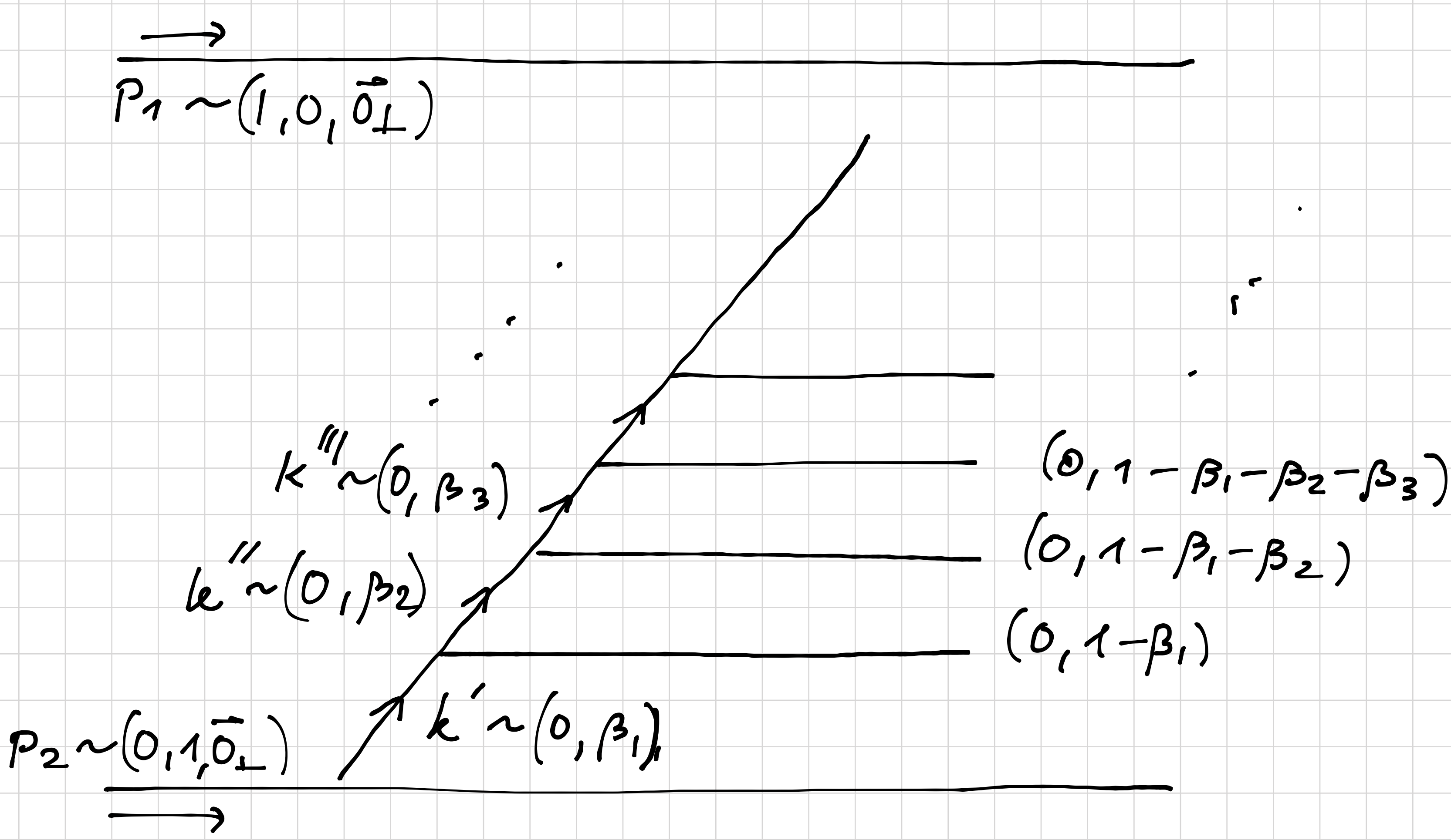
$$-\frac{g^2}{t} - \frac{g^2}{s}\left(1+\frac{g^2}{16\pi^2 m^2}\left(\ln\frac{s}{m^2}-i\pi\right)\right)$$
$$- \frac{g^2}{u}\left(1+\frac{g^2}{16\pi^2 m^2}\left(\ln\frac{-u}{m^2}-i\pi\right)\right)$$

For sufficiently high energy the correction may become large!

The structure of the result:

$p_{1,3}-k$: high energy ($\sim\beta\sqrt{s}$) but small virtuality $\gamma\beta s\sim m^2$

Particle 2 travels at $v\sim c$, emits a collinear particle that carries away a fraction β_1 of its energy. In turn, this collinear satellite can split its energy further $\to \beta_2$ and so on!



The large contribution $\to$ from strong ordering

$$\frac{m^2}{s} \ll \beta_1 \ll \beta_2 \ll \dots \ll \beta_n \ll 1, \quad \text{or}$$

In terms of energies $E_i = \frac{\sqrt{s}}{2}\beta_i$

$$\frac{m^2}{2\sqrt{s}} \ll E_1 \ll E_2 \ll \dots E_n \ll \frac{\sqrt{s}}{2}$$

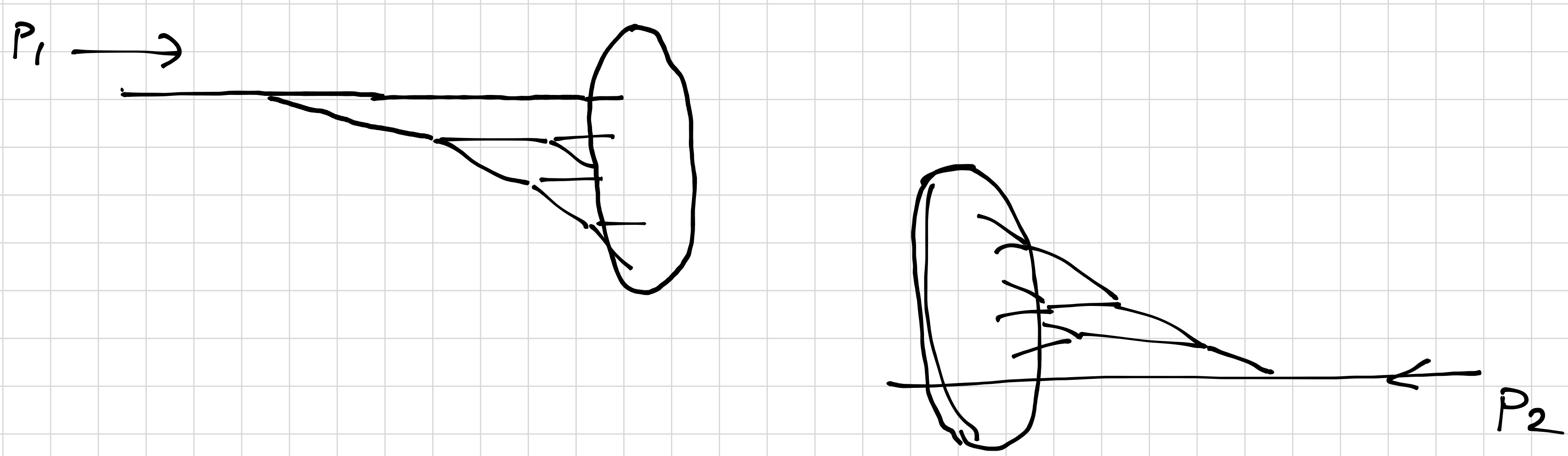
Can be resummed: $A_s^{(n)} \sim -\frac{g^2}{s}\frac{1}{(n-1)!}\left[\frac{g^2}{16\pi^2 M^2}\right]^{n-1} \ln^{n-1}\left(-\frac{s}{M^2} - i\varepsilon\right)$

$$\hookrightarrow \sum_{n=1}^{\infty} \longrightarrow -\frac{g^2}{s}\cdot s^{\alpha}, \quad \alpha = \left(\frac{g}{4\pi M}\right)^2$$

$\boxed{\text{Perturbative log corrections combine to change the HE asymptotics in a significant way!}}$

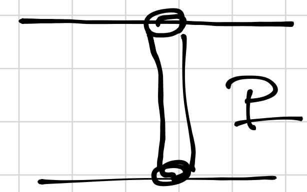
A quantum particle never walks alone

What interacts at high energies are not the particles but the "showers" or multi-particle jets:



The higher the energy $\to$ the more collinear particles it can emit $\to$ the transverse size of the jet grows $\to$ the cross sections go up

Phenomenon of rising cross section is called "pomeron" after Pomeranchuk who determined that this behavior is consistent with a t-channel exchange of an object that has q. n. of vacuum 0^{++}



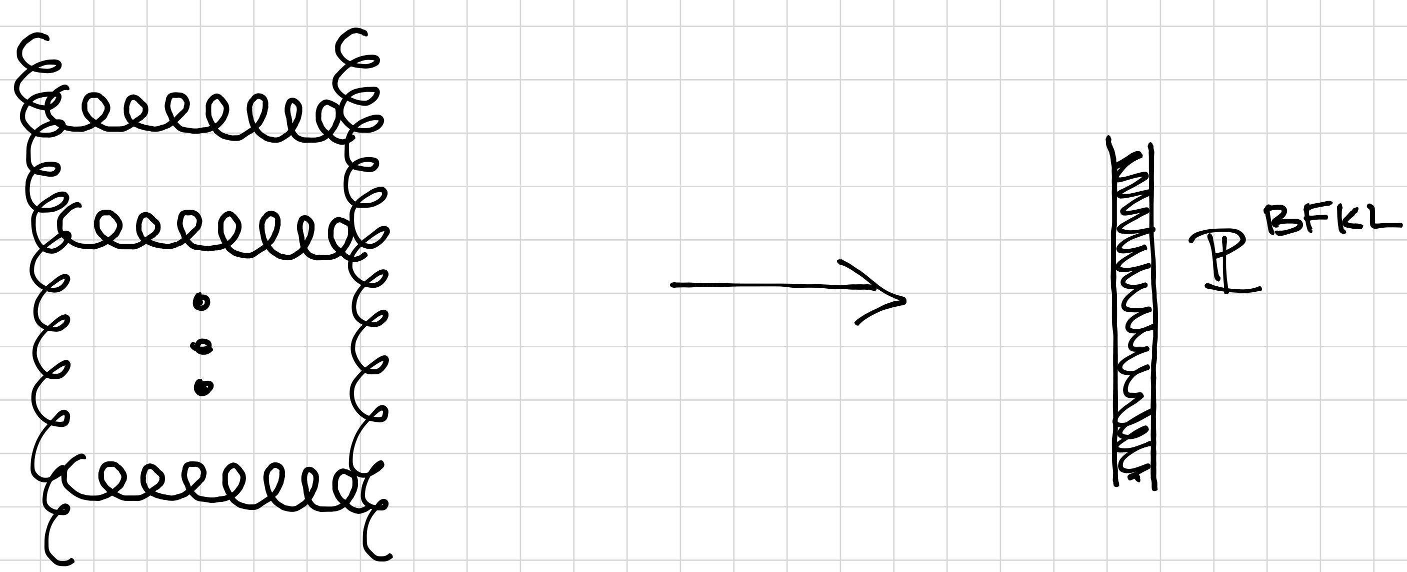
All processes where 0^{++} exchange is viable show $\sigma(s\to\infty) \sim s^{0.1}$

What can give such behavion in QCD?

φ^3 theory is unrealistic in many ways

However, for gluons in QCD the same mechanism works in the same way!

Resummation of gluon ladders



"BFKL pomeron"

Balitsky - Fadin - Kuraev - Lipatov

$$\sigma \sim s^{4 N_c \frac{\alpha_s}{\pi} \ln 2}$$

Collinear logarithms in QCD predict rising cross sections