Introduction to Theoretical Particle Physics: WS 2022/2023: Exercise sheet 8

27.01.2023

Exercise 1: The Standard Model (100+100 points)

(a)(100 points) Expand the electroweak sector Lagrangian:

$$\mathcal{L} = -\frac{1}{4}\left(W^a_{\mu\nu}\right)^2 - \frac{1}{4}B^2_{\mu\nu} + \left(D^\mu H\right)^+ \left(D_\mu H\right) + m^2\left(H^+ H\right) - \lambda\left(H^+ H\right)^2$$

To arrive to Schwartz's (29.9) upon following the steps (29.1-8). Then put back the dynamical Higgs as in (29.3) to arrive to (29.14). Read off the Feynman rules for gauge boson and Higgs interactions and check that the expanded Lagrangian is gauge invariant with respect to the massless field A^μ.

(b*)(Bonus - 50 points) Consider a massless QED with one fermion. Vector and axial symmetries together lead to two conservation laws:

$$\partial^\mu j_\mu = 0$$
$$\partial^\mu j^5_\mu = 0$$

And consequently two Ward identities. Prove by direct calculation that the correlation function $\langle j^5_\alpha(x)\, j_\mu(y)\, j_\nu(z)\rangle$ can't fulfill both of them at the same time.

(c*)(Bonus - 50 points) Prove that if a Lagrangian does not contain any dimensional parameters, it has a dilation symmetry:

$$x^\mu \to x^\mu e^\lambda$$
$$\Phi \to \Phi e^{-\Delta\lambda}$$

Where Φ is an arbitrary field (scalar, spinor, vector) and Δ refers to it's dimension ($[\Phi] = \Delta$). Prove that this symmetry must be broken by an anomaly and verify the expression for $\partial_\mu j^\mu_D$ in case of 4-dimensional massless spinor QED:

$$\partial_\mu j^\mu_D = \beta(e)\frac{\partial \mathcal{L}}{\partial e}$$

Where $\beta(e)$ is a beta-function.

An important note

You have two weeks to complete this exercise sheet.

Literature

1. Quantum Field Theory and the Standard Model, Schwartz M.D. - chapters 29 and 30.
2. An Introduction to Quantum Field Theory, Peskin M.E. and Schroeder D.V. - chapter 19.