Introduction to Theoretical Particle Physics: WS 2022/2023: Exercise sheet 5

09.12.2022

Exercise 1: One-Loop Renormalization of Non-Abelian Gauge Theories (100+100 points)

(0)(0 points) How much time did you spend in solving this exercise sheet?

(a)(50 points) Consider an arbitrary $SU(N)$-invariant theory in an arbitrary gauge with n_f massive fermions:

$$\mathcal{L} = -\frac{1}{4}F_{a;\mu}F_a^{\mu\nu} - \frac{1}{2\xi}\left(\partial_\mu A_a^\mu\right)^2 + \left(\partial^\mu \bar{c}_a\right)\left(\delta_{ac}\partial_\mu + g f_{abc}A_{b;\mu}\right)c_c +$$
$$+\sum_i^{n_f} \bar{\psi}_i \left[\delta_{ij}\left(i\partial\gamma\right) + g\left(A_a\gamma\right)\left(T_a\right)_{ij} - m\delta_{ij}\right]\psi_j$$

Prove that the lowest-order contribution to the running coupling is gauge-invariant and has the following form:

$$\alpha_s = \frac{2\pi}{\beta_0}\frac{1}{\ln\frac{\mu}{\Lambda_{QCD}}};\quad \beta_0 = \frac{11}{3}N - \frac{4}{3}n_f T_f$$

Where T_f refers to the trace normalization (i.e. it contains the information about the color representation of spinor fields).

(b)(50 points) Note that the coupling constant appears in four different places in the Lagrangian. Due to the charge universality any of those terms must give the same expression for the beta-function.

As an example - check that we could equally well have computed the beta-function from the A^3 term and $\bar{\psi}A\psi$ (or choose any two you like).

(c*)(Bonus - 100 points) Add n_s color-charged massive scalar fields to the Lagrangian from the previous section. Prove that in this case one obtains:

$$\beta_0 = \frac{11}{3}N - \frac{4}{3}n_f T_f - \frac{1}{3}n_s T_s$$

Hint: it is advised (but not necessary) to obtain this formula from the renormalization of A^3 interaction.

An important note

You have two weeks to complete this exercise sheet. You can use programs you like for the suggested problems (project files to be sent by email).

Literature

1. Quantum Field Theory and the Standard Model, Schwartz M.D. - chapters 25 and 26.
2. https://feyncalc.github.io/