Introduction to Theoretical Particle Physics: WS 2022/2023: Exercise sheet 2

18.11.2022

Exercise 1: Path integral quantization (100+25 points)

(0)(0 points) How much time did you spend in solving this exercise sheet?

(a)(30 points) Since Grassmann variables anticommute, does it mean that the term $\mathcal{L}_{int} = \bar{\psi}\psi\bar{\psi}\psi$ always vanish in the path integral? What about higher order interactions, for example $\left(\bar{\psi}\psi\right)^5$?

Hint: remember that ψ is not a single variable, but a multicomponent object. Space-time dimension is set to $D = 1 + 3$.

(b)(40 points) Would you get the same tree level amplitude for $e^+e^- \to 4e^+e^-$ process from the $\left(\bar{\psi}\psi\right)^5$ term in the canonical formalism and with the path integral?

Hint: think about the operator ordering and Wick theorem.

(c)(30 points) In $D = 1+3$ the only super-renormalizable theory is a scalar field with cubic interaction:

$$\mathcal{L} = \frac{1}{2}\left(\partial\phi\right)^2 - \alpha\phi^3$$

Prove that this theory is not self-consistent since the path integral does not converge. *Note*: curiously, even though all diagrams are finite, the perturbation series is anyway divergent.

Hint: the easiest way to see this is to perform Wick rotation.

Also consider the theory with an admixture of ϕ^4 interaction:

$$\mathcal{L} = \frac{1}{2}\left(\partial\phi\right)^2 - \alpha\phi^3 - \beta\phi^4$$

Why this theory does not face similar problems compared to the pure ϕ^3?

(d*)(Bonus - 25 points) List all renormalizable interactions with scalars, spinors and massless vector fields in $D = 1{+}3$. Why massive vector field is non-renormalizable? Also prove that in $D = 1 + 1$ an arbitrary polynom of scalar fields and massless vector fields appears to be renormalizable, but spinor field is still restricted with the mass term only.

Note: spin 3/2 field leads to solutions which propagate with faster than light velocities. Gravity is also non-renormalizable because $[G_{Newton}] = -2$. And so on - only a few options are actually available.

Literature

1. Quantum Field Theory and the Standard Model, Schwartz M.D. - chapter 14.