Introduction to Theoretical Particle Physics: WS 2022/2023: Exercise sheet 7

13.01.2023

Exercise 1: Chiral Lagrangians (100+25 points)

(0)(0 points) How much time did you spend in solving this exercise sheet?

(a)(50 points) The chiral Lagrangian for nucleons and pions is given by:

$$\mathcal{L}_{\pi N} = \bar{N}\left(i\not\partial - M_N + \not v + g_A \not a \gamma^5\right)N,$$

Where the following definitions were adopted:

$$v_\mu = \frac{1}{2i}\left(u\partial_\mu u^+ + u^+\partial_\mu u\right)$$
$$a_\mu = \frac{1}{2i}\left(u^+\partial_\mu u - u\partial_\mu u^+\right)$$
$$u = \exp\left\{\frac{i\pi^a\tau^a}{2F_\pi}\right\},$$

π^a stand for the pion fields, τ^a are the SU(2) Pauli matrices, and $\gamma_5^\dagger = \gamma_5$. The electromagnetic interaction is included by a minimal substitution:

$$\partial_\mu N \to \partial_\mu N - ieA_\mu \frac{1+\tau_3}{2}N, \quad \partial_\mu \pi^a \to \partial_\mu \pi^a - eA_\mu \epsilon^{ab3}\pi^b$$

π/F_π is considered to be a parameter of perturbation theory. Derive Feynman rules for a theory with pions, nucleons and photons up to and including the order $1/F_\pi^2$ (start by expanding a, v to the needed order first).

(b)(50 points) Perform an axial rotation $N \to \xi N$ with:

$$\xi = \exp\left\{\frac{ig_A\pi^a\tau^a}{2F_\pi}\gamma^5\right\}$$

Check that $\bar{N} \to \bar{N}\xi$, $\xi\gamma^\mu\xi = \gamma^\mu$, $\xi\gamma^\mu\gamma_5\xi = \gamma^\mu\gamma_5$ by expanding ξ to the needed order. Check that the rotated Lagrangian reads:

$$\mathcal{L} = \bar{N}\left(i\not\partial - M_N\right)N + M_N\bar{N}\left(1-\xi^2\right)N +$$
$$+\bar{N}\left(i\xi\not\partial\xi - \xi\not v\xi + g_A\xi\not a\gamma^5\xi\right)N.$$

Derive the new Feynman rules up to and including order the $1/F_\pi^2$.
Hint: neglect all terms proportional to $g_A - 1,\ g_A^2 - 1$ as those are higher order in chiral counting $1/F_\pi$.

(c*)(Bonus - 25 points) Obtain the amplitude T^μ for pion photoproduction $\gamma(q) + N(p) \to \pi(q_\pi)N'(p')$ at tree level in the pseudovector and the pseudoscalar theories. Check the Ward identity $q_\mu T^\mu = 0$ in both cases. What is the advantage of the chirally rotated πN theory?

Literature

1. Predictive powers of chiral perturbation theory in Compton scattering off protons, V. Lensky and V. Pascalutsa, https://arxiv.org/abs/0907.0451.