# Theoretical Physics 6a (QFT): SS 2022
# Exercise sheet 11

04.07.2022

## Exercise 1. (100+25 points): Loops and renormalization

**(0)(0 points)** How much time did you spend in solving this exercise sheet?

**(a)(50 points)** Prove that the self-energy of an "electron" is scalar QED has the following form:

$$-i\Sigma\left(p^2\right) = -e^2 \int_0^1 dx \int \frac{p^2\left(4\left(1-x\right)+x^2\right)+q^2}{\left(q^2+p^2x\left(1-x\right)-xm^2+i\varepsilon\right)^2}\frac{d^4q}{\left(2\pi\right)^4}$$

Calculate the integral over $dq$ in dimensional regularization. You don't need to calculate the integral over $dx$.
*Hint*: there are two diagrams, but one of them gives zero contribution.

**(b)(50 points)** Prove that two-dimensional photon self-energy in massless spinor QED is not divergent and has the following form:

$$\Pi^{\mu\nu}(k) = \frac{e^2}{\pi k^2}\left(k^2 g^{\mu\nu} - k^\mu k^\nu\right)$$

And thus the exact propagator has non-trivial poles:

$$\mathcal{D}^{\mu\nu}(k) = -\frac{g^{\mu\nu}}{k^2 - \frac{e^2}{\pi} + i\varepsilon}$$

Which means loop corrections produced a photon mass equal to $\frac{e}{\sqrt{\pi}}$. Also prove that this mass can't be removed by any gauge-invariant counterterms.
*Note*: this means that instead of 2-dimensional Coulomb potential we now have screened Coulomb potential, i.e. 2-dimensional Yukawa.

**(c*)(Advanced level problem for those who are interested - 25 points)** Which problems were the hardest/easiest in this course? Suggest your ideas for possible exam questions for this course.

## Literature

1. See exercise classes and lecture notes.
2. Quantum Field Theory and the Standard Model (Schwartz M.D.), chapter 16.
3. An Introduction To Quantum Field Theory (Peskin M. and Schroeder D.V.), chapter 19.