Lamb shift in ordinary H-atom

PHYSICAL REVIEW VOLUME 72, NUMBER 3 AUGUST 1, 1947

Fine Structure of the Hydrogen Atom by a Microwave Method* **

WILLIS E. LAMB, JR. AND ROBERT C. RETHERFORD
Columbia Radiation Laboratory, Department of Physics, Columbia University, New York, New York

(Received June 18, 1947)

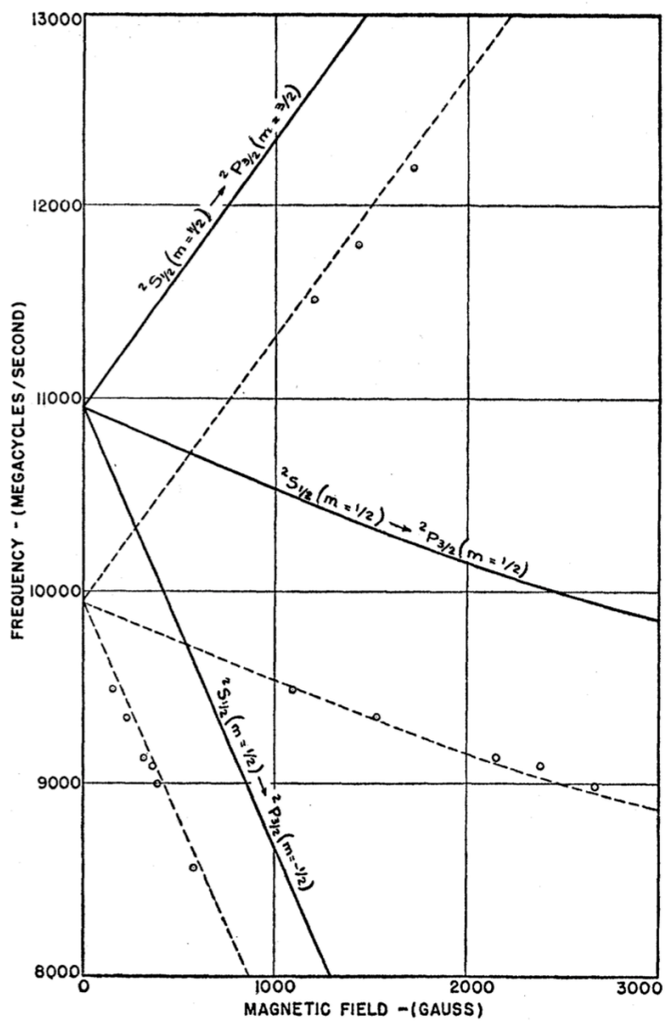
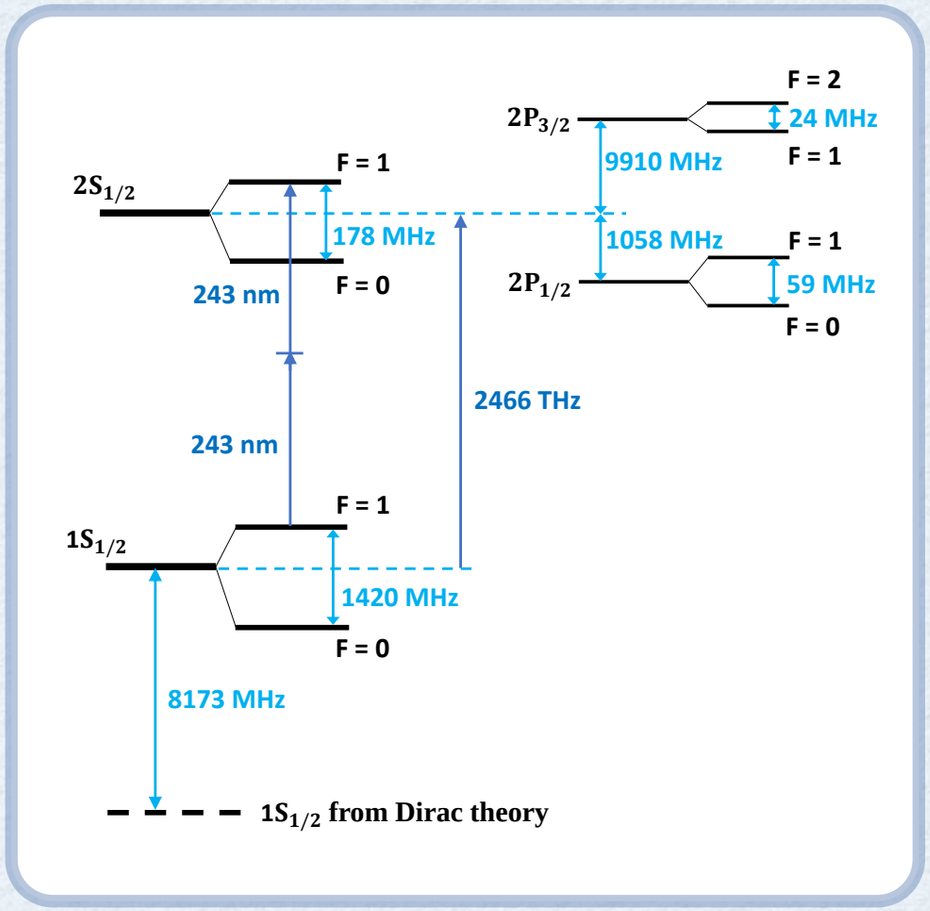


FIG. 2. Experimental values for resonance magnetic fields for various frequencies are shown by circles. The solid curves show three of the theoretically expected variations, and the broken curves are obtained by shifting these down by 1000 Mc/sec. This is done merely for the sake of comparison, and it is not implied that this would represent a "best fit." The plot covers only a small range of the frequency and magnetic field scale covered by our data, but a complete plot would not show up clearly on a small scale, and the shift indicated by the remainder of the data is quite compatible with a shift of 1000 Mc.



Lamb shift in ordinary H-atom: reported at Shelter Island conference, June 2-4 1947

From left to right, I.I. Rabi, Pauling, J. Van Vleck, W.E. Lamb, Gregory Breit, Duncan MacInnes, Karl Darrow, G.E. Uhlenbeck, Julian Schwinger, Edward Teller, Bruno Rossi, Arnold Nordsieck, John von Neumann, J.A. Wheeler, Hans Bethe, R. Serber, R.E. Marshak, Abraham Pais, J. Robert Oppenheimer, David Bohm, Richard Feynman, Victor F. Weisskopf, Herman Feshbach; The First Shelter Island Conference.

PHYSICAL REVIEW VOLUME 72, NUMBER 4 AUGUST 15, 1947

The Electromagnetic Shift of Energy Levels

H. A. Bethe
Cornell University, Ithaca, New York

(Received June 27, 1947)

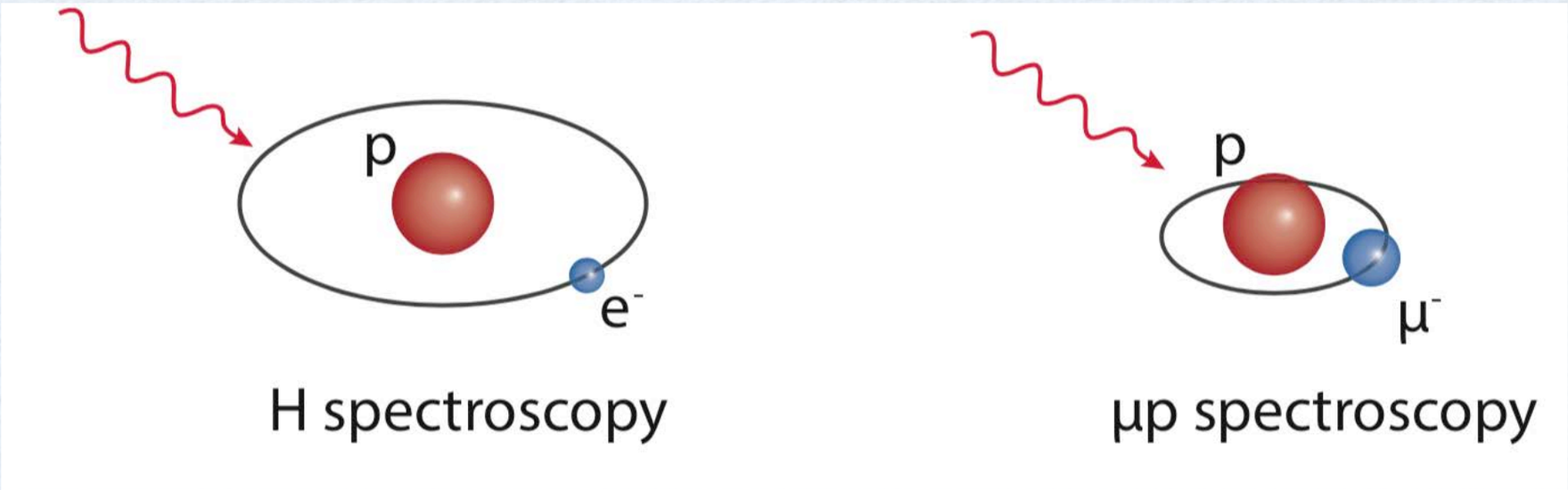
Inserting (10) and (9) into (6) and using relations between atomic constants, we get for an S state

$$W_{ns}' = \frac{8}{3\pi}\left(\frac{e^2}{\hbar c}\right)^3 \mathrm{Ry}\frac{Z^4}{n^3}\ln\frac{K}{\langle E_n - E_m\rangle_{\mathrm{Av}}}, \quad (11)$$

where Ry is the ionization energy of the ground state of hydrogen. The shift for the $2p$ state is negligible; the logarithm in (11) is replaced by a value of about -0.04. The average excitation energy $\langle E_n - E_m\rangle_{\mathrm{Av}}$ for the $2s$ state of hydrogen has been calculated numerically[7] and found to be 17.8 Ry, an amazingly high value. Using this figure and $K = mc^2$, the logarithm has the value 7.63, and we find

$$W_{ns}' = 136 \ln[K/(E_n - E_m)]$$
$$= 1040 \text{ megacycles.} \quad (12)$$

Precision hadronic structure from muonic atom spectroscopy program



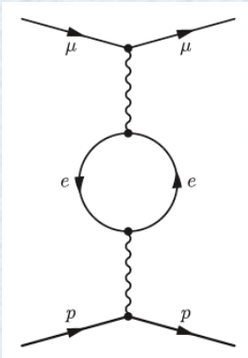
Lamb shift in muonic H: QED corrections

- Calculated by several groups

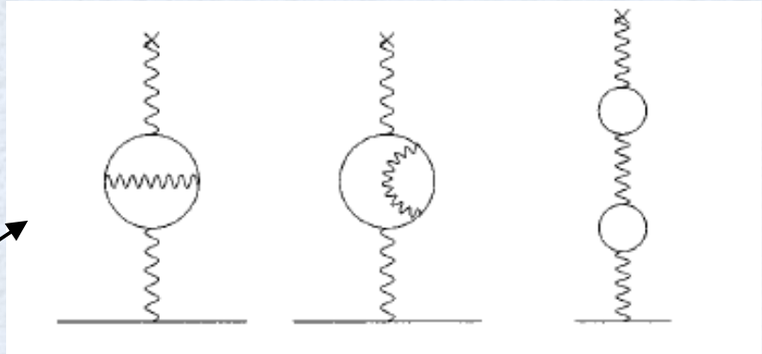
Pachucki (1996, 1999)

Borie (1976, 2005)

- 1 loop electron

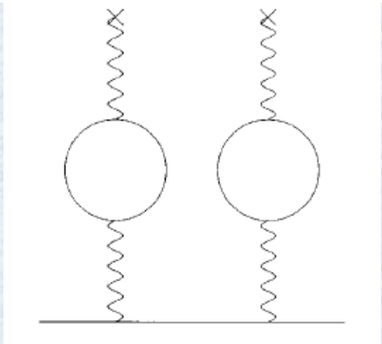


ΔE = 205.0282 meV



ΔE = 1.5081 meV

- 2 loop electron



ΔE = 0.1509 meV

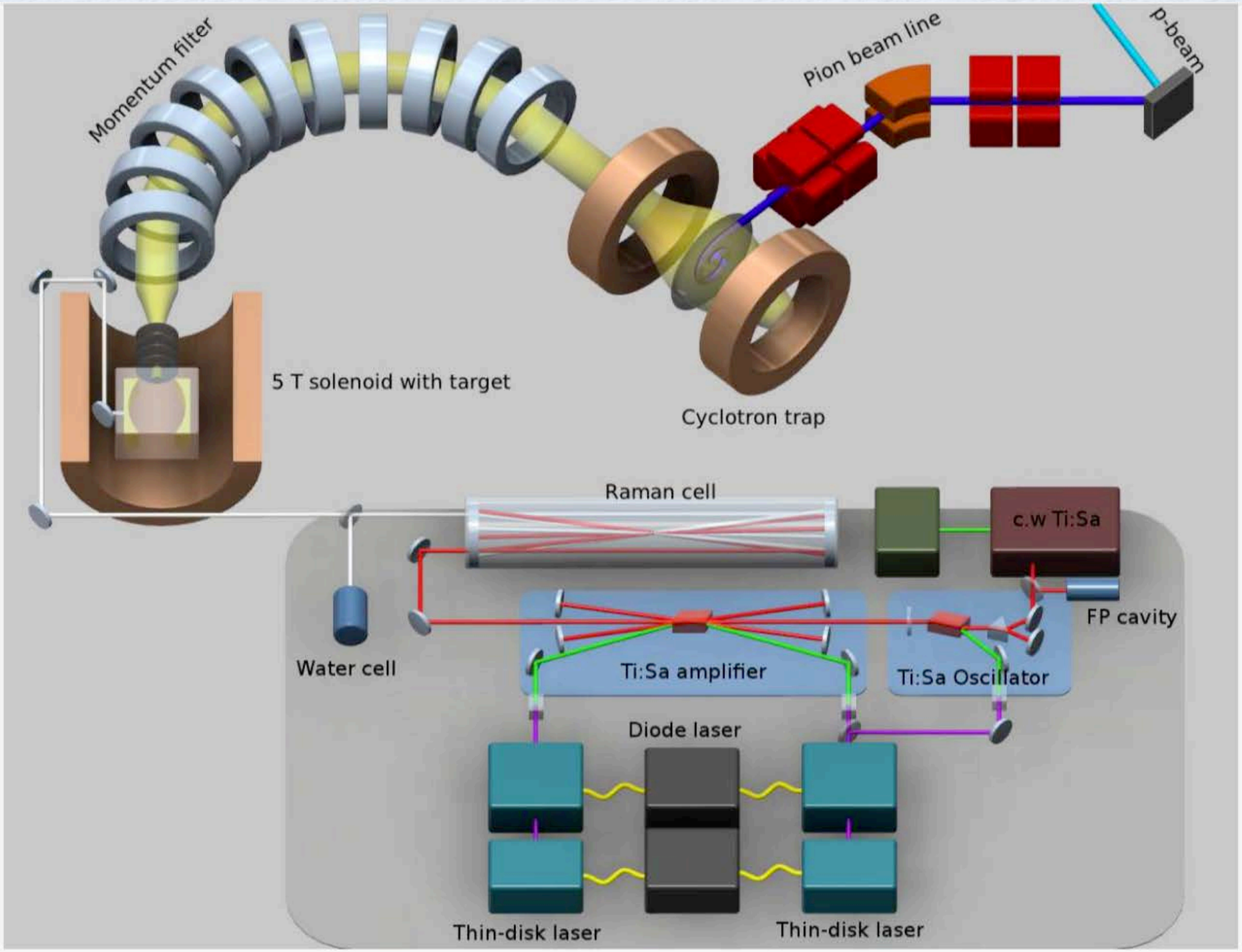
- Muon self-energy, vacuum polarization ΔE = -0.6677 meV

- other QED corrections calculated : all of size 0.005 meV or smaller << 0.3 meV

Muonic Hydrogen (μH) spectroscopy program at the Paul Scherrer Institute PSI (Villigen, Switzerland)

The μH Lamb shift setup (2010…)

p-beam
↓
π (MeV)
↓
μ (MeV)
↓
μ (keV)
↓
μp(2S)
↓
Laser
↓
X-ray



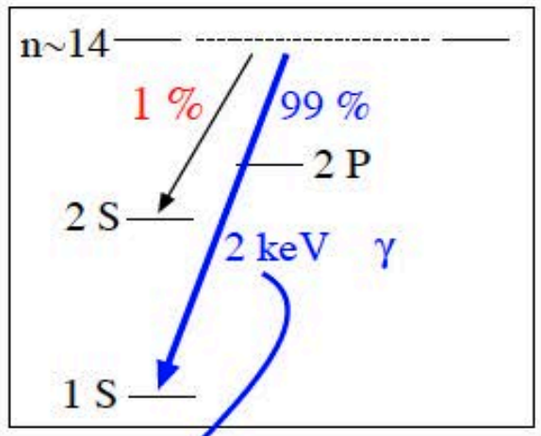
940 nm
↓
1030 nm
↓
515 nm
↓
708 nm
↓
1.0 μm
↓
1.6 μm
↓
6.0 μm

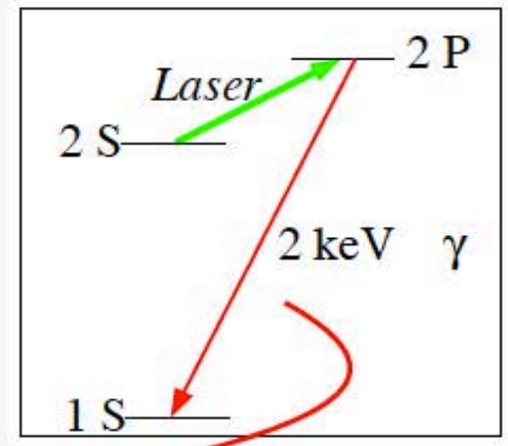
Principle of μH Lamb shift experiment

PSI experiment

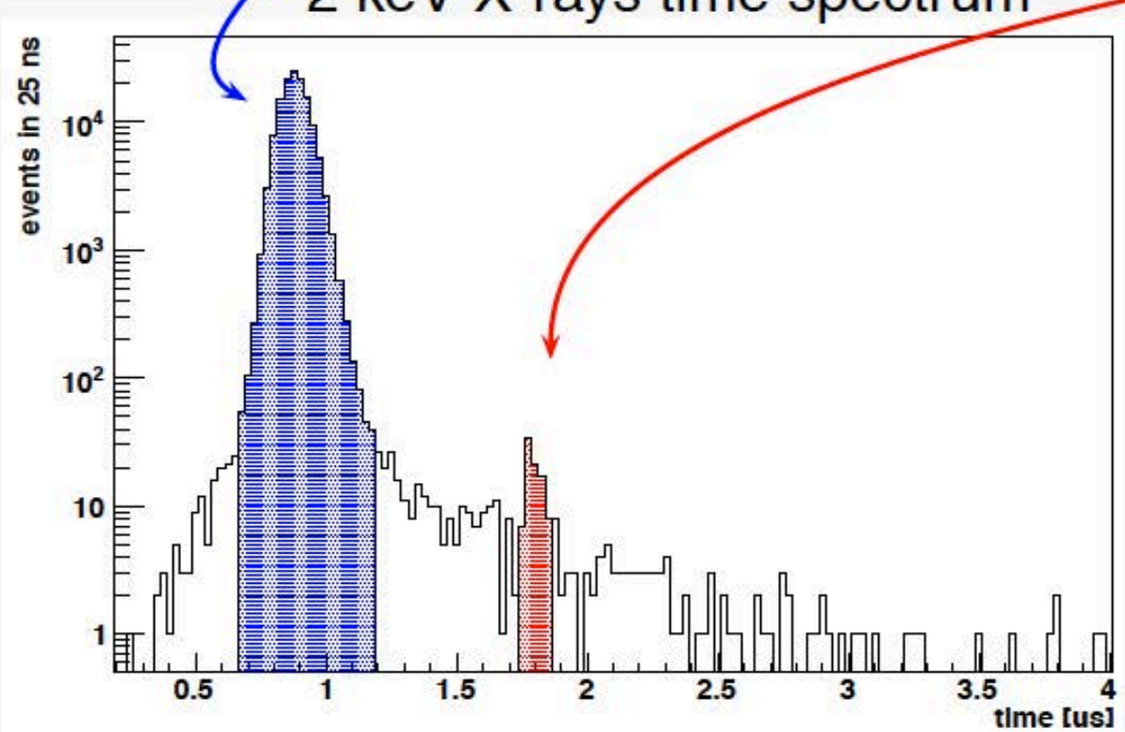
Pohl et al.(2010)

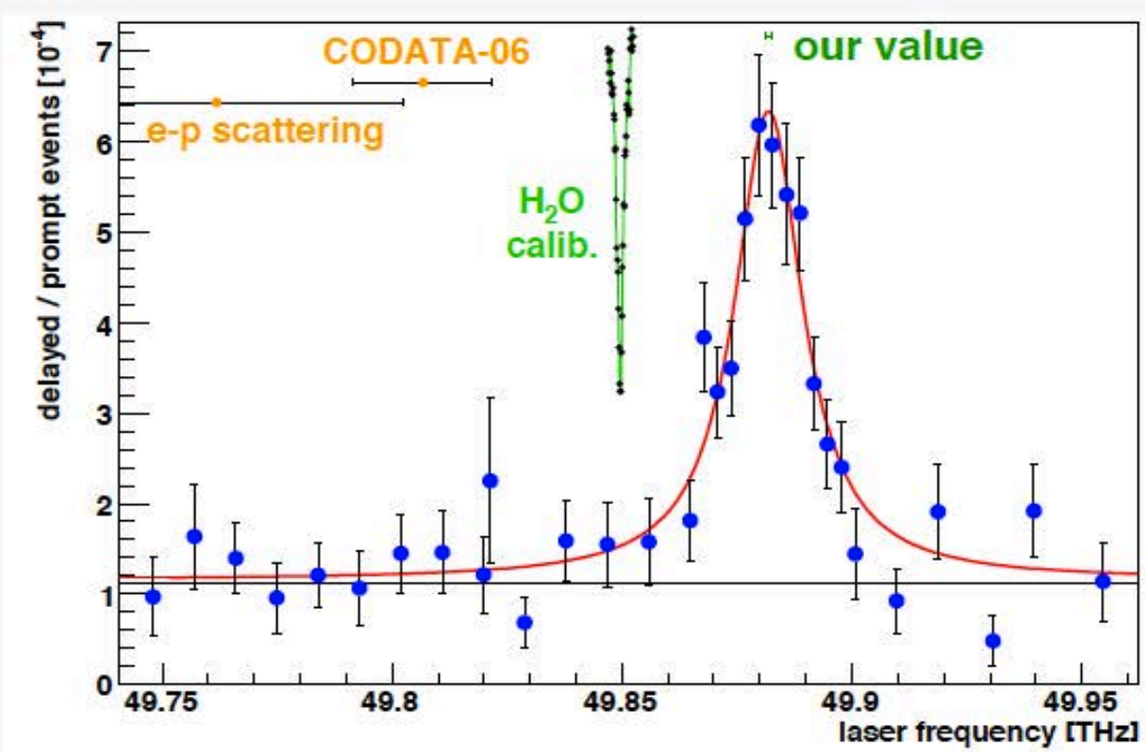
Antognini et al.(2013)



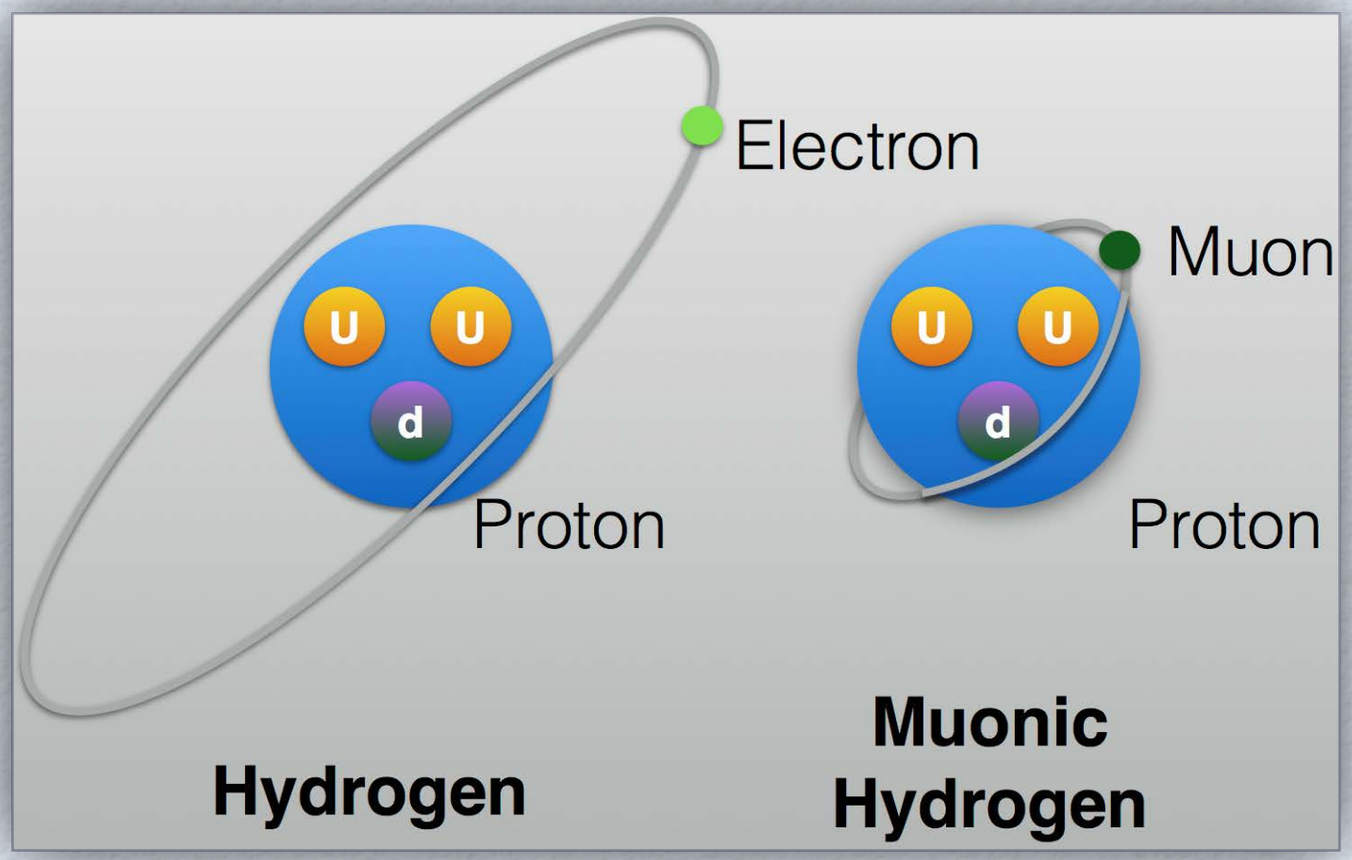


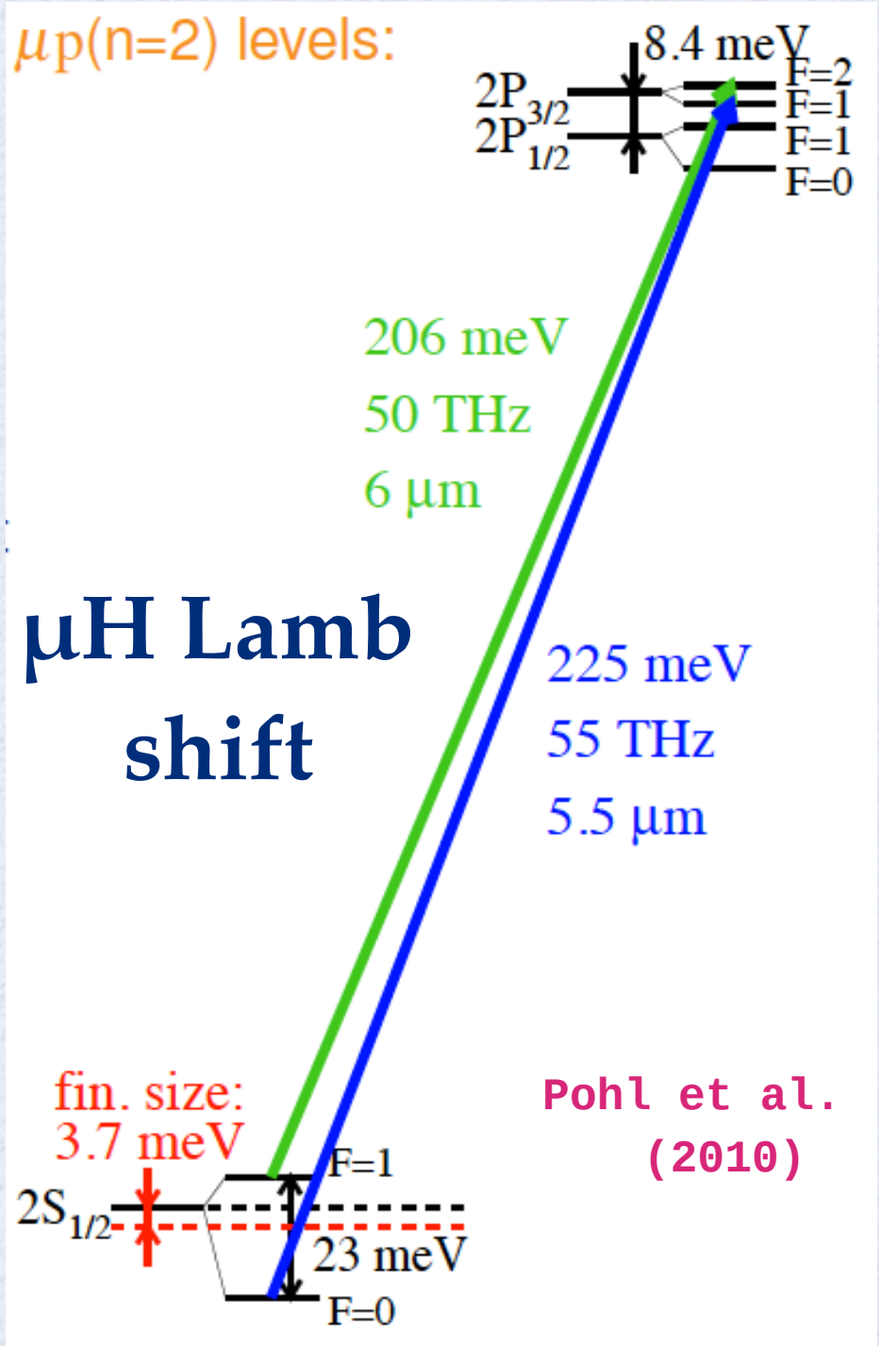
2 keV X-rays time spectrum





Proton radius from Hydrogen spectroscopy





$$\Delta E_{LS} = 206.0336\,(15) \;-\; 5.2275\,(10)\; R_E^2 \;+\; \Delta E_{TPE} \quad \text{meV}$$

Antognini et al.(2013)

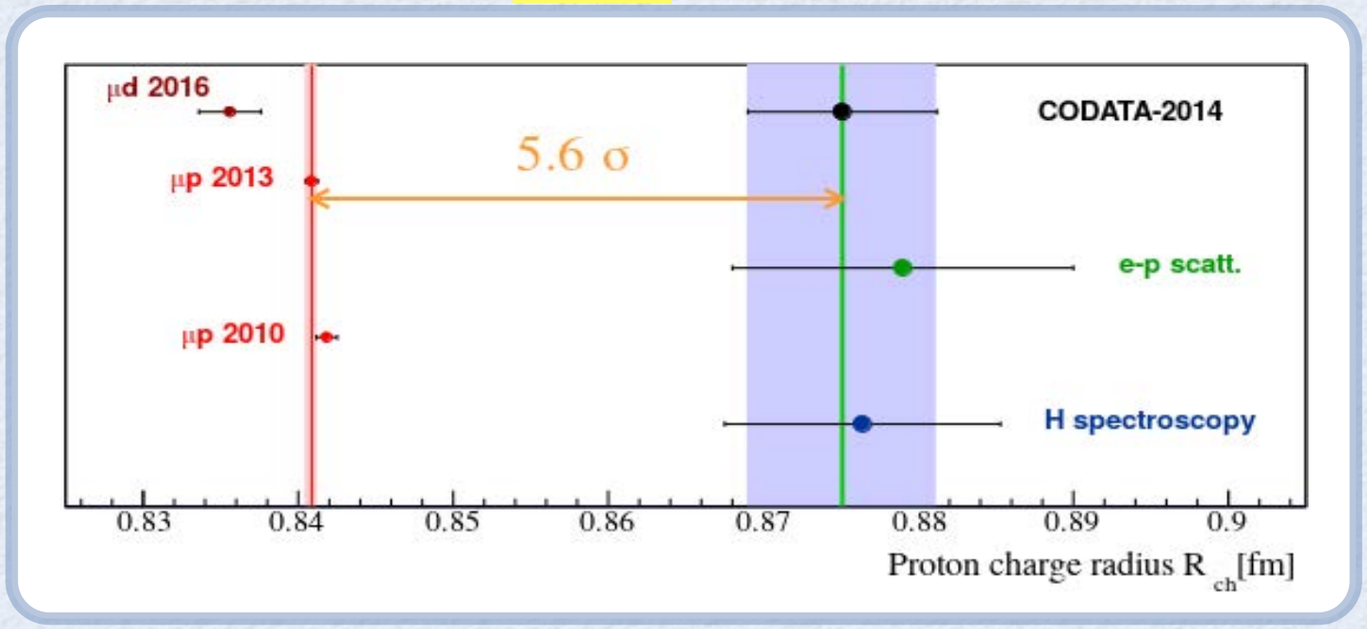
3.70 meV

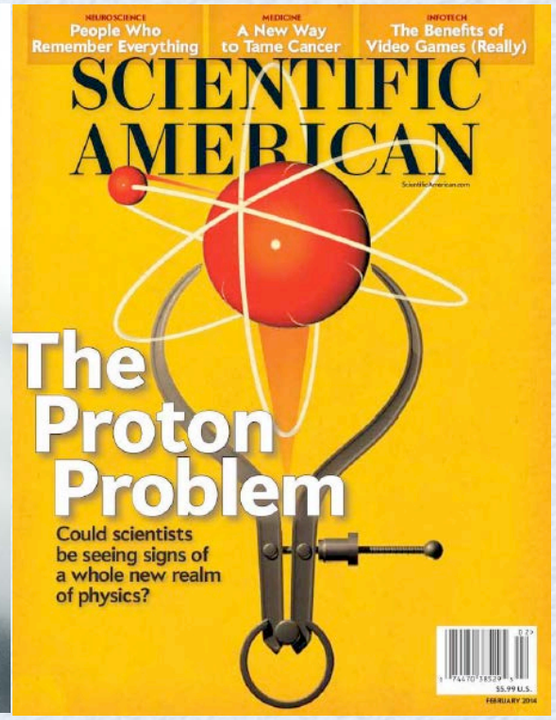
$O(\alpha^5)$ correction

0.0332 (20) meV

Proton radius puzzle

2016





μH data: $R_E = 0.8409 \pm 0.0004$ fm

Pohl et al.(2010)

Antognini et al.(2013)

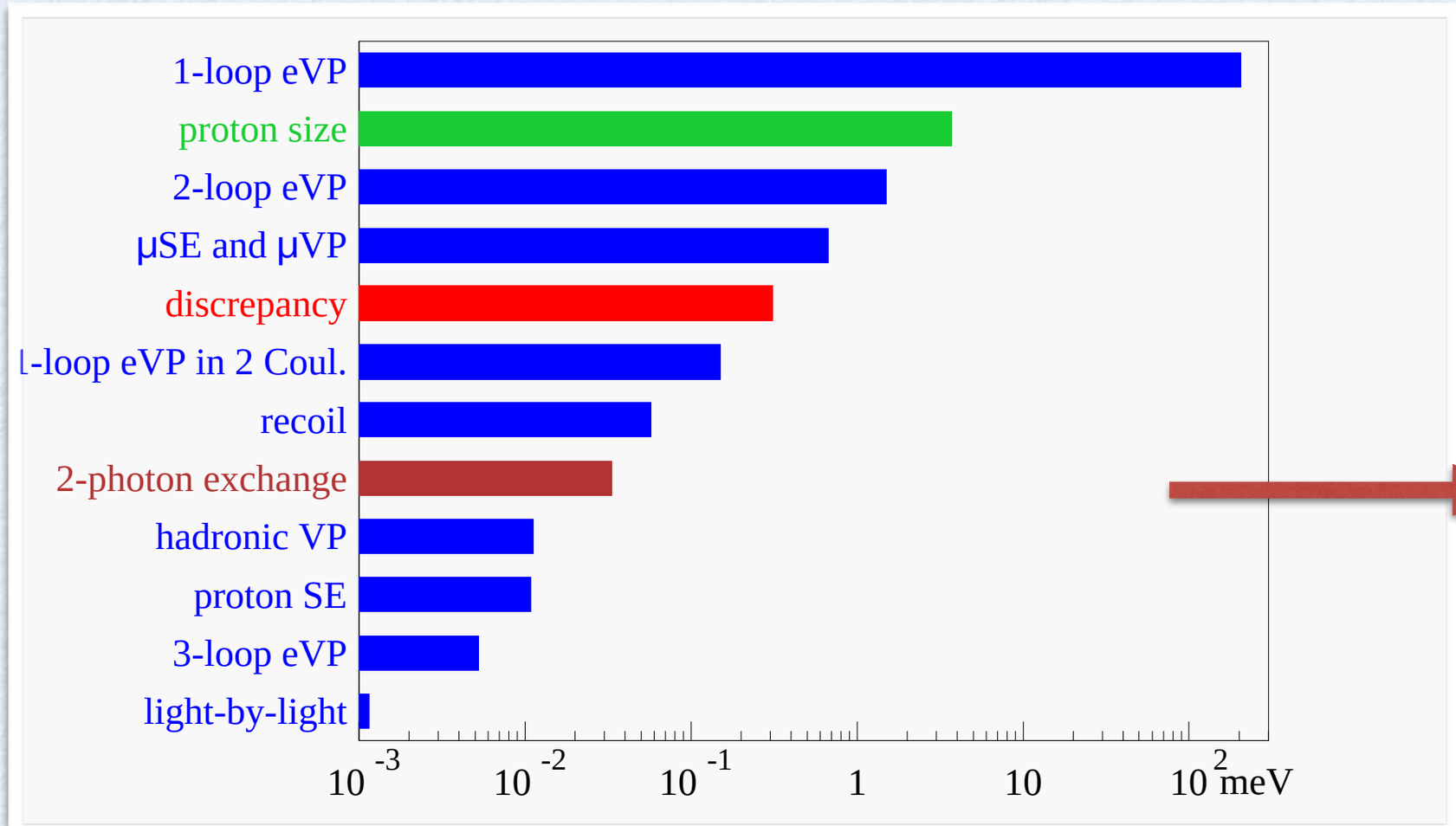
5.6 σ difference

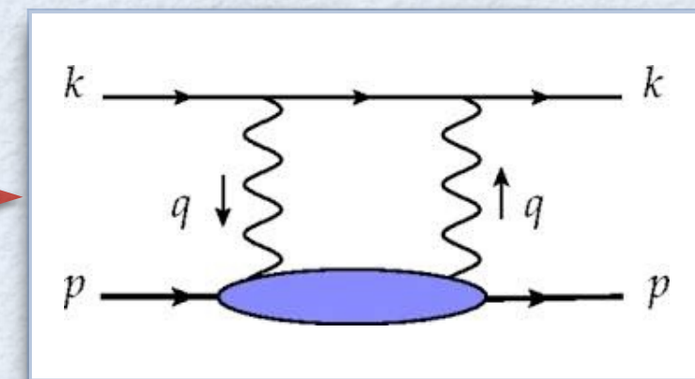
ep data: $R_E = 0.8775 \pm 0.0051$ fm

CODATA(2014)

Lamb shift: status of theory

μH Lamb shift: summary of corrections





largest theoretical uncertainty

- elastic contribution on 2S level: ΔE_{2S} = -23 μeV
- inelastic contribution: Carlson, Vdh (2011) + Birse, McGovern (2012)
- **Current experimental precision on ΔE_{LS} : 2.3 μeV**

total hadronic correction on Lamb shift

ΔE_{TPE} (2P-2S) = (33 ± 2) μeV

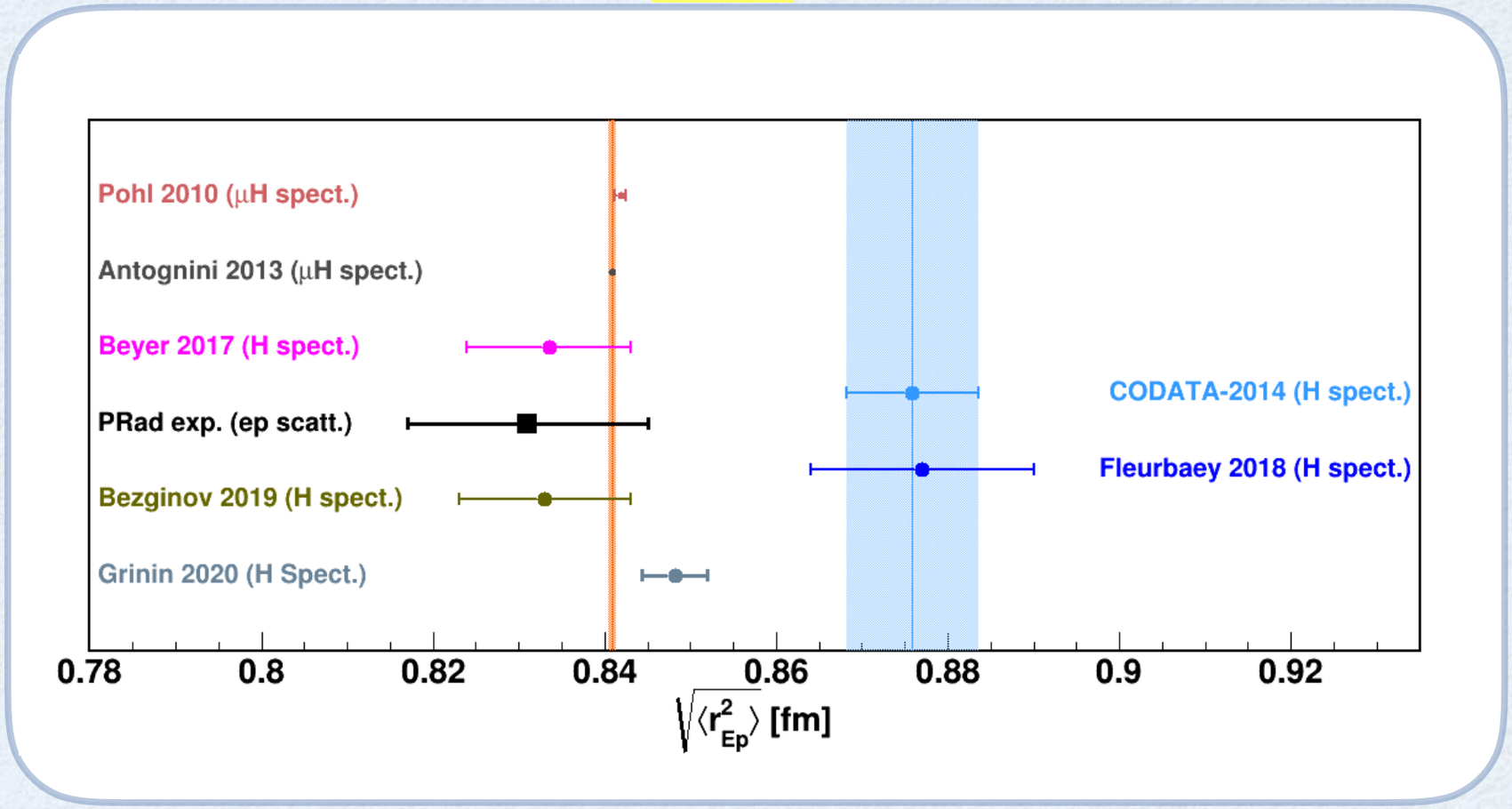
Proton charge radius: present experimental status

2020

Hydrogen 2S-4P

Hydrogen 2S-2P

Hydrogen 1S-3S



Hydrogen 1S-3S

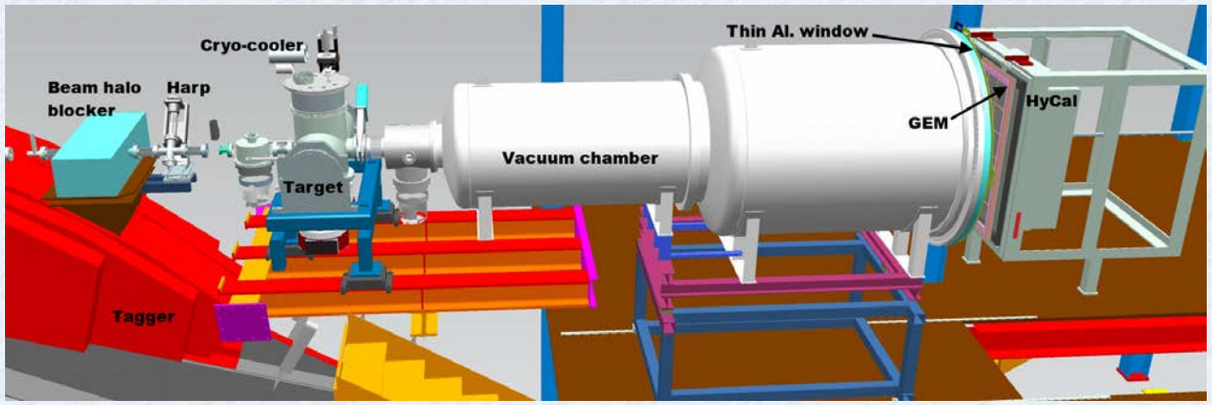
from recent compilation

H.Gao, M.Vdh

Rev. Mod. Phys. 94 (2022) 015002

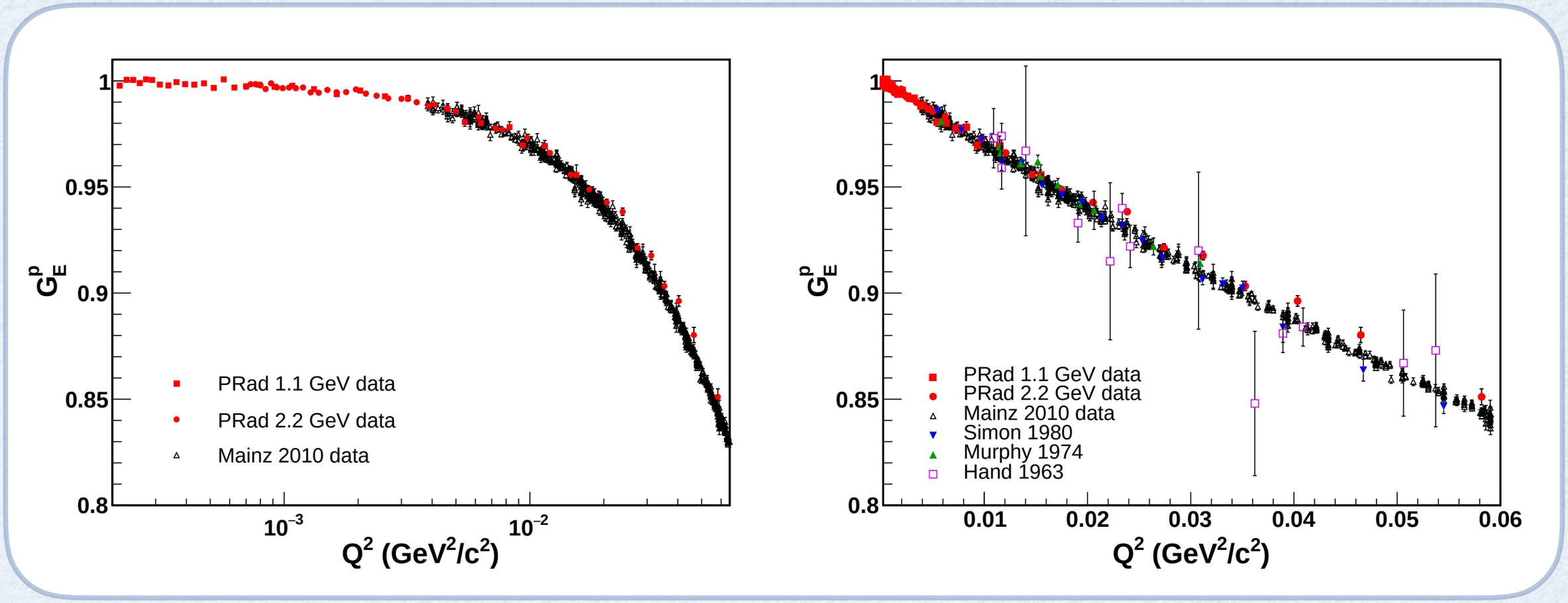
- 3 out of 5 new results are fully consistent with muonic hydrogen result
- inconsistency between Fleurbaey et al. (Paris) and Grinin et al. (Garching) results for 1S-3S H : Grinin et al.: factor 2 more precise, 2.1σ smaller than Fleurbaey et al., ~2σ larger than μH result

Proton radius from scattering: PRad experiment @ JLab



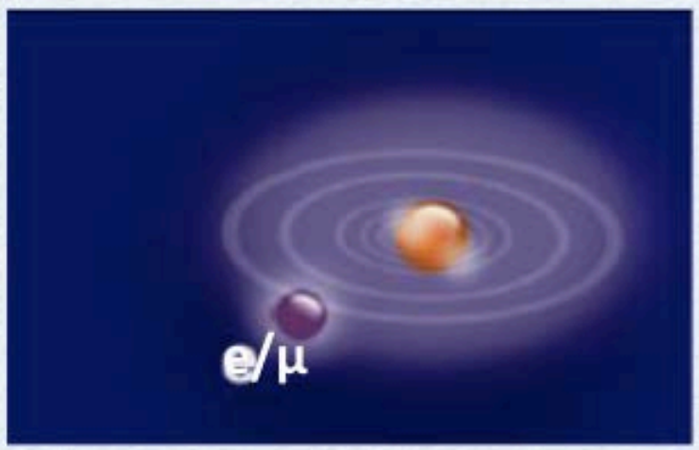
Xiong et al. (2019)

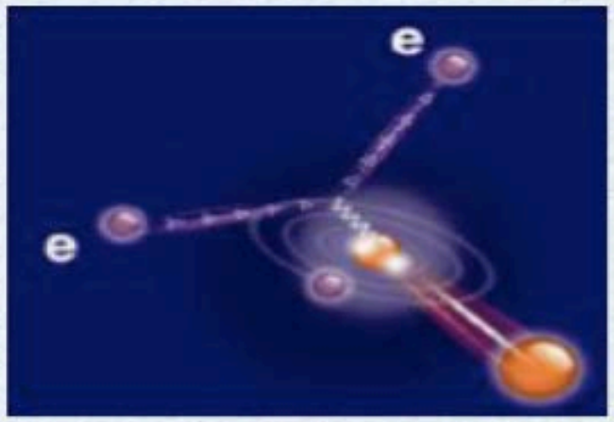
- High-resolution e.m. calorimeter (HyCal) instead of magnetic spectrometer
- unprecedented low Q^2
- to control background: windowless cryogenically cooled gas-jet target
- Luminosity control: e-p normalised to Moeller



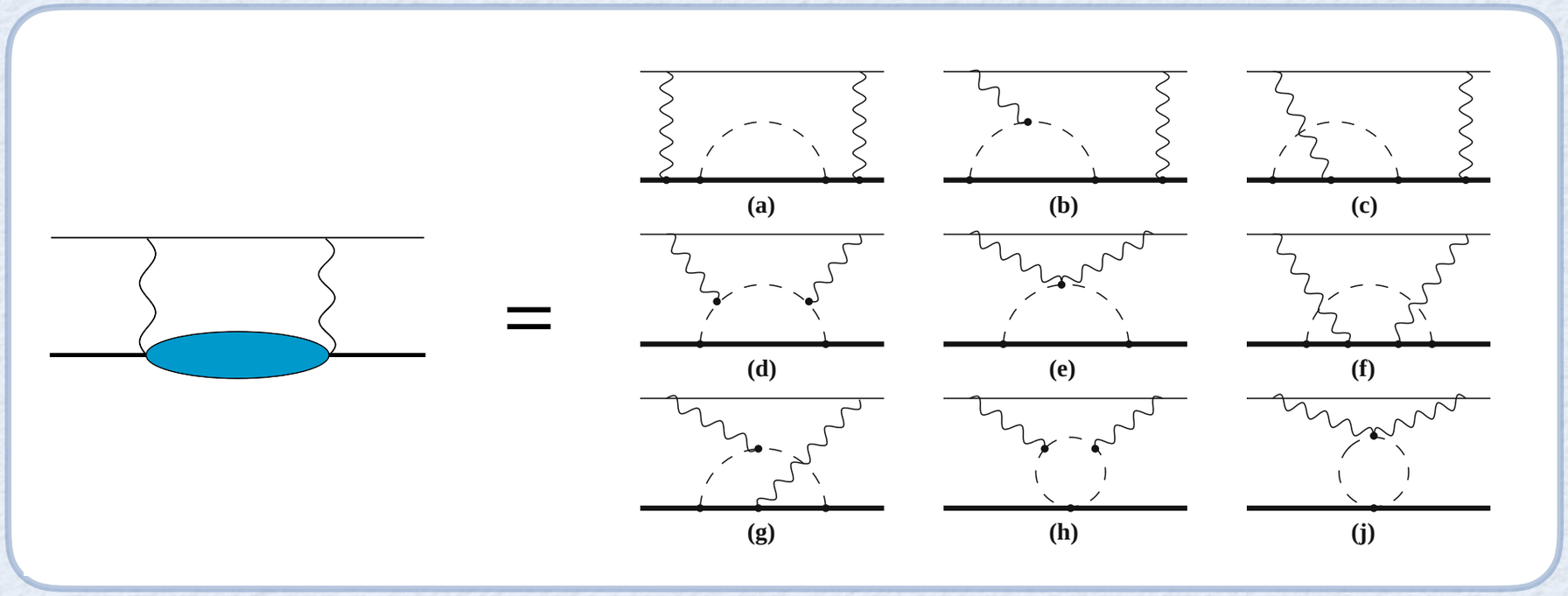
stat. uncertainty shown, syst. uncertainty ~ 0.1 to 0.6 %

Proton size
and
electromagnetic structure





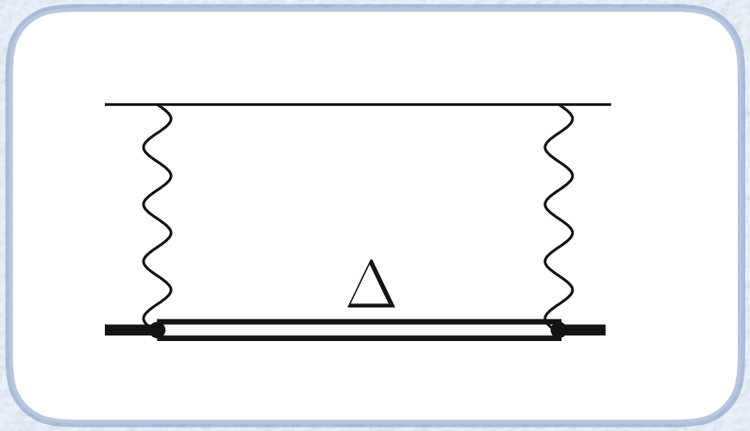
Chiral Perturbation Theory of Lamb shift



Alarcon, Lensky, Pascalutsa (2014)

- LO BChPT prediction:

$$E_{\rm LS}^{\langle {\rm LO}\rangle\ {\rm pol.}}(\mu{\rm H}) = 8^{+3}_{-1}\ \mu{\rm eV}$$



Lensky, Hagelstein, Pascalutsa, Vdh (2018)

- Δ prediction:

$$E_{\rm LS}^{\langle \Delta\text{-exch.}\rangle\ {\rm pol.}}(\mu{\rm H}) = -1.0(1.0)\ \mu{\rm eV}$$

Muonic atom spectroscopy needs nucleon/nuclear input

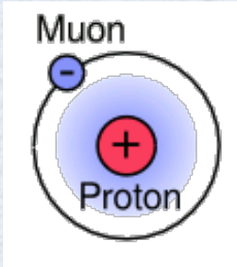
2S-2P Lamb Shift:

	THEORY		EXPERIMENT	
	$\Delta E_{TPE} \pm \delta_{theo}\ (\Delta E_{TPE})$	Ref.	$\delta_{exp}(\Delta_{LS})$	Ref.
μH	33 μeV $\pm$ 2 μeV	Antognini et al. (2013)	2.3 μeV	Antognini et al. (2013)
μD	1710 μeV $\pm$ 15 μeV	Krauth et al. (2015)	3.4 μeV	Pohl et al. (2016)
μ^3He$^+$	15.30 meV $\pm$ 0.52 meV	Franke et al. (2017)	0.05 meV	
μ^4He$^+$	9.34 meV $\pm$ 0.25 meV -0.15 meV $\pm$ 0.15 meV (3PE)	Diepold et al. (2018) Pachucki et al. (2018)	0.05 meV	Krauth et al. (2020)

μH: present accuracy comparable with experimental precision
Future: factor 5 improvement in LS planned

μD, μ³He⁺, μ⁴He⁺: present accuracy factor 5-10 worse than experimental precision

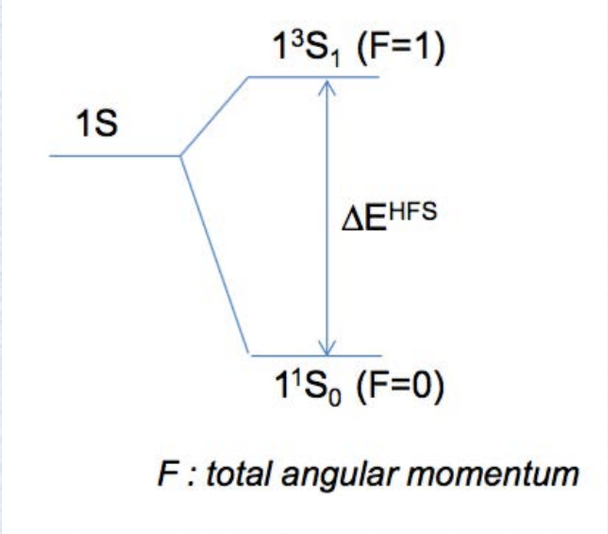
Next frontier: Hyperfine splitting in muonic Hydrogen

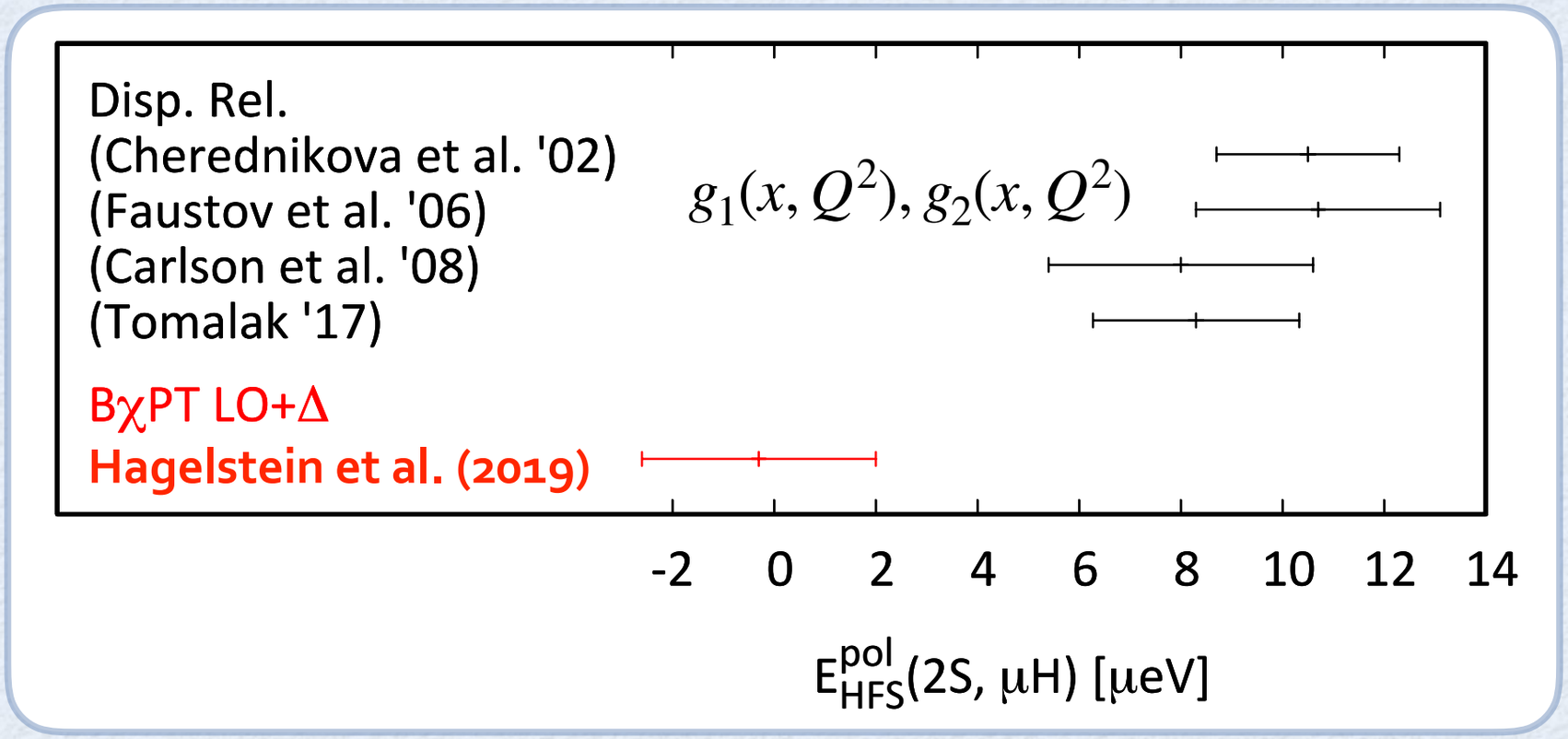


Measurements of the μH ground-state HFS planned by CREMA, FAMU, J-PARC collaborations **precision goal: 1ppm !**

Currently: theory has precision of only around 160 ppm

In addition: theory shows disagreement between data-driven evaluations and chiral perturbation theory

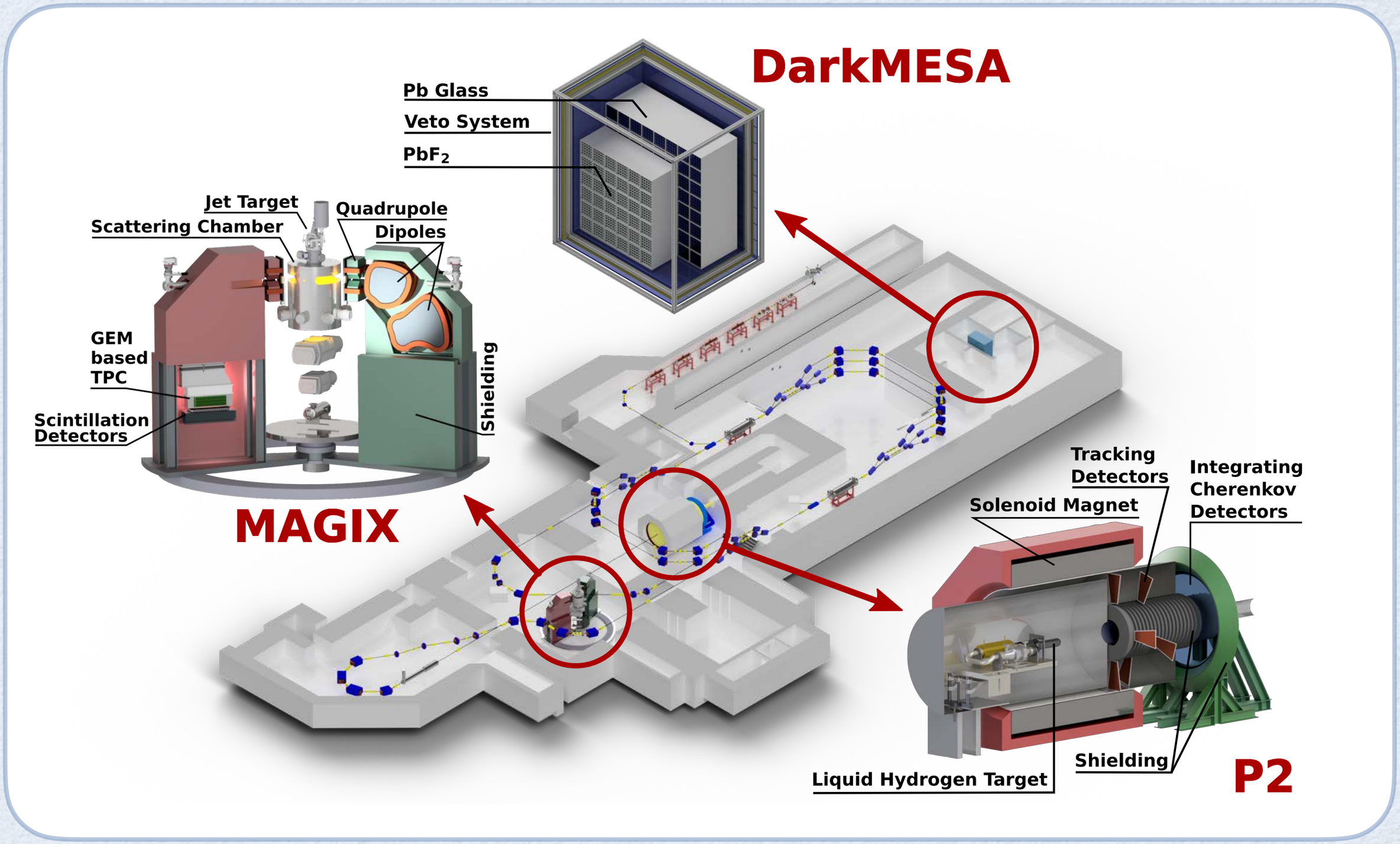




Calls for re-evaluation of empirical parametrizations of nucleon structure functions

compilation by Hagelstein, Pascalutsa (2019)

Next frontier in electron scattering: Form factor program at MAGIX@MESA (Mainz Energy-Recovering Superconducting Accelerator)

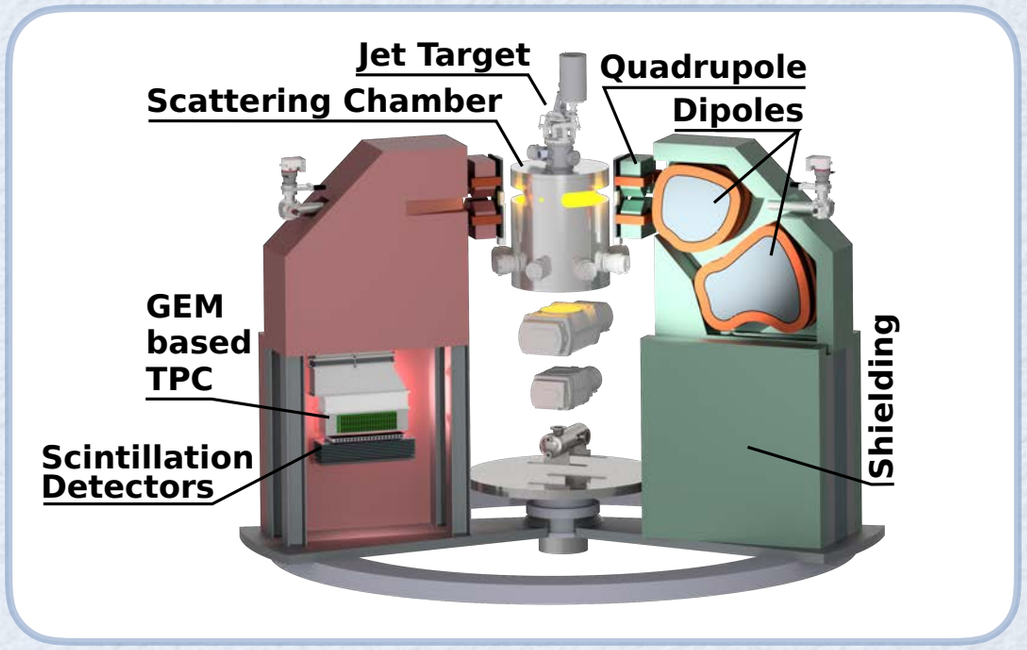


Low-Q^2 proton FF: MAGIX@MESA

Operation of a high-intensity (polarized) ERL beam in conjunction with light internal target
→ a novel technique in nuclear and particle physics

High resolution spectrometers MAGIX:

- double arm, compact design
- momentum resolution: $\Delta p/p < 10^{-4}$
- acceptance: 18.1 msr
- GEM-based focal plane detectors
- Gas Jet or polarized T-shaped target



- G_{Ep} forward measurement at **MAGIX** with **beam dump** (105 MeV, 10 μA)
- **MAGIX** with **ERL mode** full experimental campaign to measure G_{Ep} and G_{Mp}