Theoretical Physics 6a (QFT): SS 2022
Exercise sheet 10

27.06.2022

Exercise 1. (100+25 points): Spontaneous breaking of global symmetry + Feynman rules

(0)(0 points) How much time did you spend in solving this exercise sheet?

(a)(50 points) Consider the two-component real scalar theory:

$$\mathcal{L} = \frac{1}{2}\left(\partial^\mu \phi\right)^T \left(\partial_\mu \phi\right) - V$$
$$V = \frac{\lambda}{4}\left(\phi^T\phi - v^2\right)^2; \quad \phi(x) = \begin{pmatrix} \phi_1(x) \\ \phi_2(x) \end{pmatrix}$$

It is symmetric under global (with constant parameter) $O(2)$ symmetry:

$$\begin{pmatrix} \phi_1'(x) \\ \phi_2'(x) \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \phi_1(x) \\ \phi_2(x) \end{pmatrix}$$

First of all, prove that this potential has a wrong mass sign compared to the usual scalar theory, i.e. it describes tachyons - particles with purely imaginary mass. Then note that the potential has the form of "Mexican hat" which means it has infinitely many minima determined by the condition:

$$\phi^T\phi = \phi_1^2(x) + \phi_2^2(x) = v^2$$

All minima are obviously physically equivalent, i.e. they are not distinguishable from each other in any way and give the same action. Then we can choose:

$$\phi_{\text{min}} = \begin{pmatrix} v \\ 0 \end{pmatrix}$$

Prove that for small deviations from the equilibrium position:

$$\phi = \begin{pmatrix} v + \sigma(x) \\ \pi(x) \end{pmatrix}$$

The Lagrangian takes the following form:

$$\mathcal{L} = \frac{1}{2}\left(\partial\sigma\right)^2 + \frac{1}{2}\left(\partial\pi\right)^2 - \frac{m^2}{2}\sigma^2 - \lambda v\sigma^3 - \lambda v\sigma\pi^2 - \frac{\lambda}{4}\left(\sigma^2 + \pi^2\right)^2$$

The sign of the mass term of the field σ is now correct ($m^2 = 2\lambda v^2$), field π is massless. This Lagrangian exhibits the low-energy behaviour of the theory and has no $O(2)$ symmetry anymore.
Note: if we make the symmetry local, the situation will be very different. Moreover, it can be easily proven that local symmetry can't be broken at all.

(b)(50 points) Derive Feynman rules for the Lagrangian from the previous part. Write all first order loop corrections for the field π propagator and expressions for them (you don't need to calculate the integrals).
Hint: there are 5 diagrams of order λ.

(c*)(Advanced level problem for those who are interested - 25 points) Without explicit calculations prove that if particle π is on-shell ($p^2 = 0$), then sum of all loop corrections from the previous part is zero.
Hint: due to the space-time homogeneity, propagator can only depend on p^2. If $p^2 = 0$, then propagator has no p^μ dependence at all.

Literature

1. See exercise classes and lecture notes.