Theoretical Physics 6a (QFT): SS 2022
Exercise sheet 6

30.05.2022

Exercise 1. (100+25 points): Scalar QED

(0)(0 points) How much time did you spend in solving this exercise sheet?

(a)(50 points) Consider the charged Klein-Gordon Field:

$$\mathcal{L} = (D_\mu \phi)^\dagger (D^\mu \phi) - m^2 \phi^\dagger \phi$$
$$D_\mu \phi = (\partial_\mu + ieA_\mu)\,\phi$$
$$(D_\mu \phi)^\dagger = (\partial_\mu - ieA_\mu)\,\phi^\dagger$$

- Check that this Lagrangian is invariant under $U(1)$ local gauge transformation of the form $\phi(x) \to \phi'(x) = e^{i\alpha(x)}\phi(x)$;
- Write down the interaction Lagrangian and identify the terms. Sketch each one as a different diagram;
- Deduce the Feynman rules for this theory.

(b)(50 points) Calculate the amplitude for the Compton scattering in scalar QED.

(c*)(Advanced level problem for those who are interested - 25 points) Consider the abelian Maxwell-Chern-Simons theory:

$$\mathcal{L} = \frac{\eta}{4\pi}\varepsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho - \frac{1}{4} F^{\mu\nu} F_{\mu\nu}$$

Prove that equations of motion can be written in the following form:

$$(\partial^2 + g^2)\widetilde{F}^\mu = 0$$

We denote:

$$\widetilde{F}^\mu = \frac{1}{2}\varepsilon^{\mu\alpha\beta} F_{\alpha\beta}$$
$$g^2 = \frac{\eta^2}{4\pi^2}$$

Which means that due to the presence of the topological term the electromagnetic field obtained the mass without breaking the gauge invariance and renormalizability of the theory, completely independent of the standard Higgs mechanism for generating masses for gauge fields.

Literature

1. Quantum Field Theory. Zuber J.-B., Itzykson C. (chapters 6 and 3). This book contains a lot of wisdom, but be careful about the notation and possible typos.