Theoretical Physics 6a (QFT): SS 2022
Exercise sheet 3

09.05.2022

Exercise 1 (100+25 points): Correlation functions

(0)(0 points) How much time did you spend in solving this exercise sheet?

(a)(100 points) As it was shown during lectures and exercises, Green's functions and correlators of field operators can be expressed in terms of the $\Delta(x)$ and related functions:

$$i\Delta(x) = \int \frac{e^{-iE_p t + i\vec{\mathbf{p}}\vec{\mathbf{x}}}}{2E_p} \frac{d^3p}{(2\pi)^3}$$

Evaluate function $\Delta(x)$ explicitly and do the following:

1) Identify the asymptotic for the case $x = (0, \vec{\mathbf{x}})$, $|\vec{\mathbf{x}}| \to \infty$ - i.e. correlation between two causally disconnected points.

2) Identify the asymptotic for the case $x = (t, 0)$, $t \to \infty$ - this case can be treated as a particle at rest.

3) Write explicitly Feynman propagator and commutation function.

Hint: after integration over angles define a new function $f(t, |\vec{\mathbf{x}}|)$:

$$\Delta(x) = const \times \frac{1}{|\vec{\mathbf{x}}|} \frac{\partial}{\partial |\vec{\mathbf{x}}|} f(t, |\vec{\mathbf{x}}|)$$

The function f can be calculated using the following parametrization for 4-momentum:

$$|\vec{\mathbf{p}}| = m \sinh \psi$$
$$E_p = m \cosh \psi$$

For the 4-coordinate we have to distinguish 4 different cases, related to the signs of t and x^2:

$$\begin{cases} t > 0; t > |\vec{\mathbf{x}}| \to t = \sqrt{x^2} \cosh \psi_0; |\vec{\mathbf{x}}| = \sqrt{x^2} \sinh \psi_0 \\ t > 0; t < |\vec{\mathbf{x}}| \to t = \sqrt{-x^2} \sinh \psi_0; |\vec{\mathbf{x}}| = \sqrt{-x^2} \cosh \psi_0 \\ t < 0; t > |\vec{\mathbf{x}}| \to t = -\sqrt{x^2} \cosh \psi_0; |\vec{\mathbf{x}}| = \sqrt{x^2} \sinh \psi_0 \\ t < 0; t < |\vec{\mathbf{x}}| \to t = -\sqrt{-x^2} \sinh \psi_0; |\vec{\mathbf{x}}| = \sqrt{-x^2} \cosh \psi_0 \end{cases}$$

After these substitutions you will be able to express f in terms of Bessel functions. Don't forget to get back to $\Delta(x)$ afterwards!

(c*)(Additional problem for those who are interested - 25 points) Express $\Delta(x)$, Feynman propagator and commutation function in case of zero mass.

Literature

1. Introduction to the Theory of Quantized Fields, N. Bogoliubov and D. Shirkov.
§16.1 - Evaluation of D^+ and D^- function.
Note - in our notation $D^-(x) = -\Delta(x)$ and $D^+(x) = \Delta(-x)$.
2. A Treatise on the Theory of Bessel Functions, G. Watson.
3. Bessel Functions on Wolfram Mathworld
4. A Note on analytic formulas of Feynman propagators in position space. Hong-Hao Zhang, Kai-Xi Feng, Si-Wei Qiu, An Zhao, Xue-Song Li, https://arxiv.org/abs/0811.1261v1