4) QUANTIZATION OF DIRAC FIELD

• DIRAC LAGRANGIAN

↳ DIRAC EQ. $\underline{\underline{\left(i\hbar\,\gamma^\mu \partial_\mu - m_0 c\right)\psi = 0}}$

WITH $\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2 g^{\mu\nu}$

NOTE $\begin{cases} \gamma^{0\dagger} = \gamma^0 \\ \gamma^{i\dagger} = -\gamma^i \end{cases}$ $\qquad \gamma^0 = \begin{pmatrix} \mathbb{1} & \\ & -\mathbb{1} \end{pmatrix}$ $\qquad \gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix}$

$\downarrow$

$$\gamma^{\mu\dagger} = \gamma^0 \gamma^\mu \gamma^0$$

↳ ADJOINT FIELD

$$\bar{\psi}(x) \equiv \psi^\dagger(x)\,\gamma^0$$

TAKE † OF DIRAC EQ. :

$$-i\hbar\left(\partial_\mu \psi^\dagger\right)\gamma^{\mu\dagger} - \psi^\dagger m_0 c = 0$$

$$\Updownarrow$$

$$i\hbar\left(\partial_\mu \psi^\dagger\right)\gamma^0\gamma^\mu\gamma^0 + \psi^\dagger m_0 c = 0$$

$\Downarrow$ MULTIPLY BY γ^0 ON RIGHT

$$\underline{\underline{i\hbar\left(\partial_\mu \bar{\psi}\right)\gamma^\mu + \bar{\psi}\, m_0 c = 0.}}$$

LAGRANGIAN

TREAT ψ & $\bar{\psi}$ AS INDEPENDENT FIELDS (COMPLEX VALUED)

DIRAC EQS. FOR ψ & $\bar{\psi}$ CAN BE DERIVED FROM LAGRANGIAN

$$\boxed{\mathcal{L} = c\,\bar{\psi}\left[i\hbar\gamma^{\mu}\partial_{\mu} - m_0 c\right]\psi}$$

⤳ EULER-LAGRANGE EQ. FOR $\bar{\psi}$:

$$\frac{\partial \mathcal{L}}{\partial \psi} = c\,\bar{\psi}\,(-m_0 c)$$

$$\frac{\partial \mathcal{L}}{\partial(\partial_{\mu}\psi)} = c\,\bar{\psi}\,(i\hbar\gamma^{\mu})$$

$\Downarrow$

EL. EQ. $$\frac{\partial \mathcal{L}}{\partial \psi} - \partial_{\mu}\frac{\partial \mathcal{L}}{\partial(\partial_{\mu}\psi)} = 0$$

$\Downarrow$

$$c\,\bar{\psi}(-m_0 c) - c\left(\partial_{\mu}\bar{\psi}\right)i\hbar\gamma^{\mu} = 0$$

$$\therefore \quad i\hbar\left(\partial_{\mu}\bar{\psi}\right)\gamma^{\mu} + \bar{\psi}m_0 c = 0 \quad ✓$$

⤳ EULER-LAGRANGE EQ. FOR ψ:

$$\frac{\partial \mathcal{L}}{\partial \bar{\psi}} = c\left(i\hbar\gamma^{\mu}\partial_{\mu} - m_0 c\right)\psi$$

$$\frac{\partial \mathcal{L}}{\partial(\partial_{\mu}\bar{\psi})} = 0 \qquad \therefore \quad i\hbar\gamma^{\mu}\partial_{\mu}\psi - m_0 c\,\psi = 0 \quad ✓$$

↳ CONJUGATE MOMENTA :

$$\pi = \frac{\partial \mathcal{L}}{\partial \dot{\psi}} = i\hbar \bar{\psi} \gamma^0 = i\hbar \psi^+$$

$$\bar{\pi} \equiv \frac{\partial \mathcal{L}}{\partial \dot{\bar{\psi}}} = 0$$

↳ HAMILTONIAN

$$H = \int d^3\bar{x} \left(\pi \dot{\psi} + \dot{\bar{\psi}} \underbrace{\cancel{\bar{\pi}}}_{0} - \mathcal{L} \right)$$

$$= \int d^3\bar{x} \left(i\hbar \psi^+ \frac{\partial \psi}{\partial t} - c i\hbar \bar{\psi} \gamma^\mu \partial_\mu \psi + m_0 c^2 \bar{\psi}\psi \right)$$

$$= c \int d^3\bar{x}\, \bar{\psi} \left(i\hbar \gamma^0 \partial_0 - i\hbar \gamma^\mu \partial_\mu + m_0 c \right) \psi$$

$$= c \int d^3\bar{x}\, \bar{\psi} \left(-i\hbar \gamma^i \frac{\partial}{\partial x^i} + m_0 c \right) \psi$$

↳ MOMENTUM

4 MOMENTUM $\quad c P^\nu \equiv \int d^3\bar{x} \left\{ c\, \pi\, \partial^\nu \psi - \mathcal{L} g^{0\nu} \right\}$

$\underline{\nu = i}$

$$c P^i = c \int d^3\bar{x}\; \pi (\partial^i \psi)$$

$$P^i = \int d^3\bar{x}\; \psi^+ (-i\hbar \nabla^i) \psi$$

NOTE $\quad \partial^i = \frac{\partial}{\partial x_i} = -\nabla^i = -\frac{\partial}{\partial x^i}$

↳ CHARGE

~> CONSIDER GLOBAL PHASE TRANSFORMATION

$$\psi \to e^{i\alpha}\psi$$
$$\bar{\psi} \to \bar{\psi}\, e^{-i\alpha}$$

α CONSTANT REAL NUMBER

~> FOR α INFINITESIMAL

$$\psi \to \psi + \underbrace{i(\delta\alpha)\psi}_{\delta\psi}$$
$$\bar{\psi} \to \bar{\psi} \underbrace{- i(\delta\alpha)\bar{\psi}}_{\delta\bar{\psi}}$$

~> $\mathcal{L} = c\,\bar{\psi}\left[i\gamma^\mu \partial_\mu - m_0 c\right]\psi$

$\mathcal{L}$ IS INVARIANT UNDER GLOBAL PHASE TF.

$$\delta\mathcal{L} = \frac{\partial\mathcal{L}}{\partial\psi}\,\delta\psi + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)}\,\delta(\partial_\mu\psi)$$
$$+ \delta\bar{\psi}\, \underbrace{\cancel{\frac{\partial\mathcal{L}}{\partial\bar{\psi}}}}_{0 \text{ (DIRAC EQ.)}}$$
$$= \partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)}\,\delta\psi\right)$$
$$= 0$$
$$\Updownarrow$$

CONSERVED CURRENT :

$$\partial_\mu \left(\bar{\psi} (i\hbar \gamma^\mu) \cdot (i (\delta\alpha) \psi) \right) = 0$$

$$\partial_\mu J^\mu = 0$$

$$J^\mu = \bar{\psi} \gamma^\mu \psi$$

⇝ CONSERVED CHARGE (q : ELECTRIC CHARGE OF FERMION)
SPIN 1/2

$$Q = q \int d^3\bar{x} \quad J^0(x)$$

$$= q \int d^3\bar{x} \quad \psi^+(x) \, \psi(x)$$

• SECOND QUANTIZATION OF DIRAC FIELD

↳ TREAT $\psi(x)$ AND $\bar{\psi}(x)$ AS FIELD OPERATORS WHICH CAN CREATE OR ANNIHILATE A DIRAC PARTICLE AT POSITION x.

⇝ WE LIKE TO MAKE A NORMAL MODE EXPANSION OF $\psi, \bar{\psi}$

EACH MODE CORRESPONDS WITH PARTICLE WITH MOMENTUM $\bar{p}$ & $\left\{ \begin{array}{l} \text{SPIN PROJECTION ALONG } \pm s \text{ (IN REST FRAME)} \\ \text{OR HELICITY } \pm s \end{array} \right.$

$$u(p,s) = A \begin{pmatrix} \chi_s \\ \dfrac{c\,\bar{\sigma}.\bar{p}}{E_p + m_0c^2}\chi_s \end{pmatrix} \rightsquigarrow (\not{p} - m_0 c)\, u(p,s) = 0$$

$$v(p,s) = A \begin{pmatrix} \dfrac{c\,\bar{\sigma}.\bar{p}}{E_p + m_0c^2}\chi'_s \\ \chi'_s \end{pmatrix} \rightsquigarrow (\not{p} + m_0 c)\, v(p,s) = 0$$

⇝ WE WILL SIMPLIFY NOTATION AND DENOTE SECOND ARGUMENT AS VALUE $(\pm \frac{1}{2})$ OF $\left\{ \begin{array}{l} \text{HELICITY OR} \\ \text{SPIN PROJECTION} \\ \text{IN REST FRAME} \end{array} \right.$ (s) ALONG AXIS z, i.e. $s = \pm \frac{1}{2}$ (IN UNITS $\hbar$)

$$u(p,s) \quad \frown \quad \chi_{s=+\frac{1}{2}} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\frown \quad \chi_{s=-\frac{1}{2}} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$v(p, s) \rightsquigarrow \quad \chi'_{s=+\frac{1}{2}} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\rightsquigarrow \quad \chi'_{s=-\frac{1}{2}} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

NOTE : OPPOSITE SPIN LABELING
FOR v (ANTI-PARTICLE) AS FOR u (PARTICLE)

$\rightsquigarrow$ NORMALIZATION (A) OF SPINORS
CHOSEN SUCH THAT

$$\bar{u}(p, s)\, u(p, s') = \delta_{s s'}$$

$$\bar{v}(p, s)\, v(p, s') = -\delta_{s s'}$$

<u>CHECK</u> :
$$A^2 \left(\chi_s^+ \quad -\chi_s^+ \frac{c\,\bar{\sigma}.\bar{p}}{E_p + m_0 c^2} \right) \begin{pmatrix} \chi_{s'} \\ \frac{c\,\bar{\sigma}.\bar{p}}{E_p + m_0 c^2}\chi_{s'} \end{pmatrix}$$

$$= A^2 \chi_s^+ \left(1 - \frac{c^2 (\bar{\sigma}.\bar{p})(\bar{\sigma}.\bar{p})}{(E_p + m_0 c^2)^2} \right) \chi_{s'}$$

$$= A^2 \chi_s^+ \chi_{s'} \left(1 - \frac{c^2 \bar{p}^2}{(E_p + m_0 c^2)^2} \right)$$

$$= A^2 \chi_s^+ \chi_{s'} \left(1 - \frac{E_p^2 - m_0^2 c^4}{(E_p + m_0 c^2)^2} \right)$$

$$= A^2 \chi_s^+ \chi_{s'} \left(1 - \frac{E_p - m_0 c^2}{E_p + m_0 c^2} \right)$$

$$= A^2 \chi_s^+ \chi_{s'} \frac{2 m_0 c^2}{E_p + m_0 c^2}$$

$$\Downarrow$$

USING $\chi_s^+ \chi_{s'} = \delta_{ss'}$

$$\bar{u}(p,s)\, u(p,s') = \delta_{ss'}\, A^2 \frac{2 m_0 c^2}{E_p + m_0 c^2}$$

$$= \delta_{ss'}$$

$$\Updownarrow$$

$$\boxed{A = \sqrt{\frac{E_p + m_0 c^2}{2 m_0 c^2}}}$$

NORMALIZATION CHOICE

⇝ CHECK THAT WITH THE ABOVE NORMALIZATION

$$u^+(p,s)\, u(p,s') = \frac{E_p}{m_0 c^2}\, \delta_{ss'}$$

$$v^+(p,s)\, v(p,s') = \frac{E_p}{m_0 c^2}\, \delta_{ss'}$$

⇝ NORMAL MODES ⇝ FREE PROPAGATING DIRAC PARTICLE

PLANE WAVE (IN VOLUME V) $\dfrac{e^{-\frac{i}{\hbar} p.x}}{\sqrt{V}}$

NORMAL MODES $\psi^+(x) \sim \dfrac{e^{-\frac{i}{\hbar} p.x}}{\sqrt{V}}\, u(p,s)$ POS. ENERGY SOLUTION

OR

$\psi^-(x) \sim \dfrac{e^{+\frac{i}{\hbar} p.x}}{\sqrt{V}}\, v(p,s)$ NEG. ENERGY SOLUTION

$$\left(i\hbar\,\gamma^\mu \partial_\mu - m_0 c\right)\psi^+ = 0 \iff (\not{p} - m_0 c)\,u = 0$$

$$\left(i\hbar\,\gamma^\mu \partial_\mu - m_0 c\right)\psi^- = 0 \iff (\not{p} + m_0 c)\,v = 0$$

EXPANSIONS OF DIRAC FIELDS ψ & $\bar{\psi}$

$$\psi(x) = \sum_{\bar{p}} \sum_{s} \left(\frac{m_0 c^2}{E_p V}\right)^{1/2} \left\{ b(\bar{p}, s)\, u(\bar{p}, s)\, e^{-\frac{i}{\hbar} p.x} + d^+(\bar{p}, s)\, v(\bar{p}, s)\, e^{+\frac{i}{\hbar} p.x} \right\}$$

$$\bar{\psi}(x) = \sum_{\bar{p}} \sum_{s} \left(\frac{m_0 c^2}{E_p V}\right)^{1/2} \left\{ b^+(\bar{p}, s)\, \bar{u}(\bar{p}, s)\, e^{+\frac{i}{\hbar} p.x} + d(\bar{p}, s)\, \bar{v}(\bar{p}, s)\, e^{-\frac{i}{\hbar} p.x} \right\}$$

$b(\bar{p}, s)$ & $d^+(\bar{p}, s)$ ARE EXPANSION COEFFICIENTS WHICH WILL BECOME OPERATORS UPON SECOND QUANTIZATION

NOTE : NORMALIZATION FACTOR $\left(\frac{m_0 c^2}{E_p}\right)^{1/2}$

IS INTRODUCED TO GET SIMPLE

ANTI-COMMUTATORS FOR b, d AFTER SECOND QUANTIZATION

SECOND QUANTIZATION

IMPOSE ANTI-COMMUTATION RELATIONS FOR EXPANSION COEFFICIENTS b & d

$$\{ b(\bar{p},s), b^{+}(\bar{p}',s') \} = \delta_{\bar{p}\bar{p}'}\,\delta_{ss'}$$

$$\{ d(\bar{p},s), d^{+}(\bar{p}',s') \} = \delta_{\bar{p}\bar{p}'}\,\delta_{ss'}$$

$$\{ b(\bar{p},s), b(\bar{p}',s') \} = 0$$

$$\{ d(\bar{p},s), d(\bar{p}',s') \} = 0$$

$$\{ b(\bar{p},s), d(\bar{p}',s') \} = \{ b(\bar{p},s), d^{+}(\bar{p}',s') \} = 0$$

$$\{ d(\bar{p},s), b(\bar{p}',s') \} = \{ d(\bar{p},s), b^{+}(\bar{p}',s') \} = 0$$

$\left\{ \begin{matrix} b(\bar{p},s) \\ b^{+}(\bar{p},s) \end{matrix} \right\}$ IS INTERPRETED AS $\left\{ \begin{matrix} \text{ANNIHILATION} \\ \text{CREATION} \end{matrix} \right\}$ OPERATOR

OF DIRAC PARTICLE WITH MOMENTUM $\bar{p}$ & s

$\left\{ \begin{matrix} d(\bar{p},s) \\ d^{+}(\bar{p},s) \end{matrix} \right\}$ IS INTERPRETED AS $\left\{ \begin{matrix} \text{ANNIHILATION} \\ \text{CREATION} \end{matrix} \right\}$ OPERATOR

OF DIRAC ANTI-PARTICLE WITH MOMENTUM $\bar{p}$ & s

$\rightsquigarrow$ VACUUM $|0\rangle$

$$b(\bar{p},s)\,|0\rangle = 0$$

$$d(\bar{p},s)\,|0\rangle = 0$$

$\rightsquigarrow$ NUMBER OPERATORS

$b^+(\bar{p},s)\,b(\bar{p},s)$: # PARTICLES WITH $\bar{p},s$

$d^+(\bar{p},s)\,d(\bar{p},s)$: # ANTI-PARTICLES WITH $\bar{p},s$

$\rightsquigarrow$ ANTI-COMMUTATION RELATION FOR FIELD OPERATORS

FROM ANTI-COMMUTATION RELATIONS FOR b, d

$$\left\{ \psi_\alpha(\bar{x},t)\,,\ \psi_\beta^+(\bar{x}',t) \right\}$$

AT EQUAL TIME t !

$$\left\{ \begin{array}{l} x^\mu = (ct,\bar{x}) \\ x'^\mu = (ct,\bar{x}') \end{array} \right.$$

$$= \sum_{\bar{p}s}\sum_{\bar{p}'s'} \frac{m_0 c^2}{V\,(E_p E_{p'})^{1/2}}$$

$$\cdot \Big\{ b(\bar{p},s)\,u_\alpha(\bar{p},s)\,e^{-\frac{i}{\hbar}p.x} + d^+(\bar{p},s)\,v_\alpha(\bar{p},s)\,e^{+\frac{i}{\hbar}p.x}\,,$$

$$b^+(\bar{p}',s')\,u_\beta^+(\bar{p}',s')\,e^{+\frac{i}{\hbar}p'.x'} + d(\bar{p}',s')\,v_\beta^+(\bar{p}',s')\,e^{-\frac{i}{\hbar}p'.x'} \Big\}$$

$$= \sum_{\bar{p}}\sum_{s} \frac{m_0 c^2}{E_p\,V} \Big(u_\alpha(\bar{p},s)\,u_\beta^+(\bar{p},s)\ e^{+\frac{i}{\hbar}\bar{p}.(\bar{x}-\bar{x}')}$$

$$+ v_\alpha(\bar{p},s)\,v_\beta^+(\bar{p},s)\ e^{-\frac{i}{\hbar}\bar{p}.(\bar{x}-\bar{x}')} \Big)$$

$$\left.\begin{array}{l} u^{+} = \bar{u}\,\gamma^{0} \\ v^{+} = \bar{v}\,\gamma^{0} \end{array}\right.$$

$$\sum_{s} u(\bar{p},s)\, u^{+}(\bar{p},s) = \underbrace{\sum_{s} u(\bar{p},s)\,\bar{u}(\bar{p},s)}_{\frac{(\not{p}+m_0 c)}{2 m_0 c}}\,\gamma^{0}$$

CHECK !

$$\sum_{s} v(\bar{p},s)\, v^{+}(\bar{p},s) = \underbrace{\sum_{s} v(\bar{p},s)\,\bar{v}(\bar{p},s)}_{-\frac{(-\not{p}+m_0 c)}{2 m_0 c}}\,\gamma^{0}$$

CHECK !

$$\therefore \quad \left\{ \psi_{\alpha}(\bar{x},t)\,,\, \psi_{\beta}^{+}(\bar{x}',t) \right\}$$

$$= \sum_{\bar{p}} \frac{m_0 c^2}{E_p V} \left(\left[\frac{(\not{p}+m_0 c)}{2 m_0 c}\gamma^{0} \right]_{\alpha\beta} e^{\frac{i}{\hbar}\bar{p}\cdot(\bar{x}-\bar{x}')} + \left[\frac{(\not{p}-m_0 c)}{2 m_0 c}\gamma^{0} \right]_{\alpha\beta} e^{-\frac{i}{\hbar}\bar{p}\cdot(\bar{x}-\bar{x}')} \right)$$

$$\downarrow \quad \sum_{\bar{p}} e^{\frac{i}{\hbar}\bar{p}\cdot(\bar{x}-\bar{x}')} = \frac{V}{(2\pi)^3\hbar^3}\int d^3\bar{p}\; e^{\frac{i}{\hbar}\bar{p}\cdot(\bar{x}-\bar{x}')}$$

$$= V\,\delta^{3}(\bar{x}-\bar{x}')$$

ONLY p^0 TERM SURVIVES $\qquad p^0 = E_p/c$

$$= \frac{m_0 c^2}{E_p}\,\delta^{3}(\bar{x}-\bar{x}')\,\frac{2E_p/c}{2 m_0 c}\,\delta_{\alpha\beta}$$

$$\therefore \quad \left\{ \psi_{\alpha}(\bar{x},t)\,,\, \psi_{\beta}^{+}(\bar{x}',t) \right\} = \delta_{\alpha\beta}\,\delta^{3}(\bar{x}-\bar{x}')$$

ANTI - COMMUTATION RELATIONS FOR FIELDS

ANALOGOUSLY

$$\left\{ \psi_\alpha(\bar{x},t) ,\ \psi_\beta(\bar{x}',t) \right\} = 0$$

$$\left\{ \psi_\alpha^+(\bar{x},t) ,\ \psi_\beta^+(\bar{x}',t) \right\} = 0$$

↳ NORMAL ORDERING

WHEN CALCULATING PHYSICAL QUANTITIES, SUCH AS ENERGY, MOMENTUM, CHARGE, ... WE WANT TO EXPRESS THEM RELATIVE TO VACUUM.

CAN BE ACHIEVED BY '$\underline{\text{NORMAL ORDERING}}$' (N) OF OPERATORS

$\downarrow$

IN NORMAL ORDERED PRODUCT : ALL CREATION OPERATORS STAND TO LEFT OF ANNIHILATION OPERATORS

e.g.
$$N(b^+ b) = b^+ b$$
$$N(b\, b^+) = -b^+ b$$
$$N(b\, d^+) = -d^+ b$$
$$\vdots$$

i.e. ANY CONSTANT TERM (VACUUM EXPECTATION VALUE) GETS SUBTRACTED

$\Downarrow$

$\therefore$ VACUUM EXPECTATION VALUE OF NORMAL ORDERED PRODUCT IS ZERO

$$N(A.B) = A.B - \langle 0 | A.B | 0 \rangle$$

CHARGE IN 2^0 QUANTIZATION

$$Q = \int d^3\bar{x} \quad N\left(\psi^+(\bar{x},t)\, \psi(\bar{x},t) \right)$$

$$= \sum_{\bar{p}} \sum_{\bar{p}'} \sum_{s} \sum_{s'} \frac{m_0 c^2}{V} \frac{1}{\sqrt{E_p E_{p'}}}$$

$$\cdot \int d^3\bar{x} \Big\{ b^+(\bar{p}\, s)\, b(\bar{p}' s')\, u^+(\bar{p}\, s)\, u(\bar{p}' s')\, e^{\frac{i}{\hbar}(p-p').x}$$

$$+ b^+(\bar{p}\, s)\, d(\bar{p}' s')\, u^+(\bar{p}\, s)\, v(\bar{p}' s')\, e^{\frac{i}{\hbar}(p+p').x}$$

$$+ d(\bar{p}\, s)\, b(\bar{p}' s')\, v^+(\bar{p}\, s)\, u(\bar{p}' s')\, e^{-\frac{i}{\hbar}(p+p').x}$$

$$- d^+(\bar{p}' s')\, d(\bar{p}\, s)\, v^+(\bar{p}\, s)\, v(\bar{p}' s')\, e^{-\frac{i}{\hbar}(p-p').x} \Big\}$$

DUE TO NORMAL ORDERING

$$\frac{1}{V} \int d^3\bar{x}\; e^{\frac{i}{\hbar}(p-p').x}$$

$$= \frac{1}{V} \int d^3\bar{x}\; e^{\frac{i}{\hbar}(E_p - E_p')t} \cdot e^{-\frac{i}{\hbar}(\bar{p}-\bar{p}').\bar{x}}$$

$$= \delta_{\bar{p}\bar{p}'}$$

$$= \sum_{\bar{p}} \sum_{s} \sum_{s'} \frac{m_0 c^2}{E_p}$$

$$\cdot \Big\{ b^+(\bar{p}\, s)\, b(\bar{p}\, s')\; u^+(\bar{p}\, s)\, u(\bar{p}\, s')$$

$$+ b^+(\bar{p}\, s)\, d(-\bar{p}\, s')\; u^+(\bar{p}\, s)\, v(-\bar{p}\, s')$$

$$+ \; d(\bar{p}\, s)\; b(-\bar{p}\, s')\; v^+(\bar{p}\, s)\; u(-\bar{p}\, s')$$
$$- \; d^+(\bar{p}\, s')\; d(\bar{p}\, s)\; v^+(\bar{p}\, s)\; v(\bar{p}\, s') \Big\}$$

USING NORMALIZATION CONVENTION

$$u^+(\bar{p}\, s)\; u(\bar{p}\, s') = \frac{E_p}{m_0 c^2}\, \delta_{s s'}$$
$$v^+(\bar{p}\, s)\; v(\bar{p}\, s') = \frac{E_p}{m_0 c^2}\, \delta_{s s'}$$
$$v^+(\bar{p}\, s)\; u(-\bar{p}\, s') = 0$$
$$u^+(\bar{p}\, s)\; v(-\bar{p}\, s') = 0$$

$$\therefore \quad \boxed{Q = \sum_{\bar{p}} \sum_{s} \Big\{ b^+(\bar{p}\, s)\, b(\bar{p}\, s) - d^+(\bar{p}\, s)\, d(\bar{p}\, s) \Big\}}$$

$b^+ b$: # PARTICLES

$d^+ d$: # ANTI - PARTICLES

<u>NOTE</u> : ANTI - PARTICLES HAVE OPPOSITE CHARGE AS PARTICLES.

ELECTRICAL CHARGE IS $(-e)\, Q$

$\hookrightarrow -e$: CHARGE OF e^- (WITH $e > 0$)

$+e$ IS CHARGE OF POSITRON e^+

↳ ENERGY / MOMENTUM IN 2° QUANTIZATION

ANALOGOUSLY ONE CAN SHOW THAT

$$H = \sum_{\bar{p}} \sum_{s} E_p \left(b^+(\bar{p}s)\, b(\bar{p}s) + d^+(\bar{p}s)\, d(\bar{p}s) \right)$$

$$\bar{P} = \sum_{\bar{p}} \sum_{s} \bar{p} \left(b^+(\bar{p}s)\, b(\bar{p}s) + d^+(\bar{p}s)\, d(\bar{p}s) \right)$$