A glimpse of the $H$ dibaryon from a lattice QCD perspective

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Perhaps a Stable Dihyperon*

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In the quark bag model, the same gluon-exchange forces which make the proton lighter than the Δ(1236) bind six quarks to form a stable, flavor-singlet (with strangeness of −2) \( J^P = 0^+ \) dihyperon \((H)\) at 2150 MeV. Another isosinglet dihyperon \((H^*)\) with \( J^P = 1^+ \) at 2335 MeV should appear as a bump in \( \Lambda \Lambda \) invariant-mass plots. Production and decay systematics of the \( H \) are discussed.

* MIT bag model predicts dihyperon state \((H)\) with

\[
I = 0, \quad S = -2, \quad J^P = 0^+
\]

and a mass of \( m_H = 2150 \) MeV

* \( H \) dibaryon must decay weakly
Observation of a $^{\Lambda\Lambda}_6$He Double Hypernucleus (E373@KEK):

A double-hyperfragment event has been found in a hybrid-emulsion experiment. It is identified uniquely as the sequential decay of $^{\Lambda\Lambda}_6$He emitted from a $\Xi^-$ hyperon nuclear capture at rest. The mass of $^{\Lambda\Lambda}_6$He and the $\Lambda-\Lambda$ interaction energy $\Delta B_{\Lambda\Lambda}$ have been measured for the first time devoid of the ambiguities due to the possibilities of excited states. The value of $\Delta B_{\Lambda\Lambda}$ is $1.01 \pm 0.20_{-0.11}^{+0.18}$ MeV. This demonstrates that the $\Lambda-\Lambda$ interaction is weakly attractive.

“Nagara” event

Observation of a $^{\Lambda\Lambda}_6$He double-hypernucleus

Binding energy:

$$B_{\Lambda\Lambda} = 7.25 \pm 0.19 \,(+0.18)_{-0.11} \text{ MeV}$$

Interpreted as sequential weak decay of $^{\Lambda\Lambda}_6$He

$$m_H > 2m_\Lambda - B_{\Lambda\Lambda} = 2223.7 \text{ MeV} \quad @ \, 90\% \, \text{CL}$$
The \( H \) dibaryon as a dark matter candidate

\* **udsuds** bound state as dark matter candidate:

\[ m_H < 2(m_p + m_e) = 1877.6 \text{ MeV} \Rightarrow H \text{ dibaryon absolutely stable} \]

\[ m_H > 2(m_p + \text{B.E.}) = 1860 \text{ MeV} \Rightarrow \text{Nuclei absolutely stable} \]

\* Recall: \( 2m_\Lambda = 2230 \text{ MeV} \)

\[ m_H = 2150 \text{ MeV} \quad (\text{Jaffe’s bag model estimate}) \]

\* Scenario requires very large binding energy of \( \approx 360 \text{ MeV} \)
Current Status

- $H$ dibaryon not firmly established experimentally
- Is a bound $H$ dibaryon a consequence of QCD?
- Try “ab initio” technique: Lattice QCD
**Beyond Perturbation Theory: Lattice QCD**

* Non-perturbative treatment; regularised Euclidean functional integrals

Lattice spacing: \( a, \ x_\mu = n_\mu a, \ a^{-1} = \Lambda_{\text{UV}} \)

Finite volume: \( L^3 \cdot T, \ N_s = L/a, \ N_t = T/a \)

\[
\langle \Omega \rangle = \frac{1}{Z} \int \prod_{x,\mu} dU_\mu(x) \ \Omega \prod_{f=u,d,s,...} \det(\mathcal{D}_{\text{lat}} + m_f) \ e^{-S_G[U]}
\]

* Stochastic evaluation of \( \langle \Omega \rangle \) via Markov process

Strong growth of numerical cost near physical \( m_u, m_d \)

* Pion mass, i.e. lightest mass in pseudoscalar channel:

\[
\approx 500 \text{ MeV} \quad \longrightarrow \quad \approx 130 \text{ MeV}
\]

\( (2001) \quad \longrightarrow \quad (\gtrsim 2015) \)
**Hadron spectrum in Lattice QCD**

- Spectral information contained in correlation functions

\[
\sum_{x,y} e^{i p \cdot (y-x)} \left\langle O_{\text{had}}(y) O_{\text{had}}^\dagger(x) \right\rangle = \sum_n w_n(p) e^{-E_n(p)(y_0-x_0)}
\]

\[
(y_0-x_0) \to \infty \quad w_1(p) e^{-E_1(p)(y_0-x_0)}
\]

- \(O_{\text{had}}(x)\): interpolating operator
  - projects on all states with the same quantum numbers

  Nucleon: \( O_N = \epsilon_{abc} (u^a C \gamma_5 d^b) u^c \)

- Ground state dominates at large Euclidean times: \( y_0 - x_0 \to \infty \)

- Excited states are sub-leading contributions
The $H$ Dibaryon in Lattice QCD

Flavour structure

- $H$ dibaryon lies in the 1-dimensional irrep. of $SU(3)_{\text{flavour}}$
- Flavour structure of two octet baryons:

$$8 \otimes 8 = (1 \oplus 8 \oplus 27)_S \oplus (8 \oplus 10 \oplus \bar{10})_A$$

- Upon SU(3)-symmetry breaking, 8 and 27 mix with singlet
- Singlet, octet and 27plet interpolators constructed from linear combinations of $\Lambda\Lambda$, $\Sigma\Sigma$ and $N\Xi$ operators

Other interesting dibaryons

- Dineutron lies in 27 irrep.
- Deuteron lies in $\bar{10}$ irrep. with $J^P = 1^+$
Interpolating operators

Hexaquark operators (inspired by Jaffe’s original bag model calculation):

\[
[rstuvw] = \epsilon_{ijk}\epsilon_{lmn}(s^a C\gamma_5 P_+ t^b)(v^l C\gamma_5 P_+ w^m)(r^k C\gamma_5 P_+ u^n)
\]

\[
H^{(1)} = \frac{1}{48} ([sudsud] - [udusds] - [dudsus])
\]

\[
H^{(27)} = \frac{1}{48 \sqrt{3}} (3[sudsud] + [udusds] - [dudsus])
\]

Momentum-projected two-baryon operators:

\[
B_\alpha \equiv [rst]_\alpha = \epsilon_{ijk}(s^i C\gamma_5 P_+ t^j) r^k_\alpha
\]

\[
(BB)(P; t) = \sum_x e^{-ip_1 \cdot x} B_1(x, t) (C\gamma_5 P_+) \sum_y e^{-ip_2 \cdot y} B_2(y, t), \quad P = p_1 + p_2
\]

→ project onto \((BB)^{(1)}, (BB)^{(8)}, (BB)^{(27)}\)
Correlation matrices

* Consider set of $N_{\text{op}}$ interpolating operators for a given hadron:

Correlation matrix: $C_{ij}(P, \tau) = \left\langle O_i(P, t) O_j(P, t')^\dagger \right\rangle$, $\tau = t - t'$

* Variational method: solve Generalised Eigenvalue Problem (GEVP):

$$C(t_1) \nu_n(t_1, t_0) = \lambda_n(t_1, t_0) C(t_0) \nu_n(t_1, t_0)$$

$$w_n^\dagger(t_1, t_0) C(t_1) = \lambda_n(t_1, t_0) w_n^\dagger(t_1, t_0) C(t_0), \quad n = 1, \ldots, N_{\text{op}}$$

* Project on approximately diagonal correlator:

$$\Lambda_{mn}(t) = w_n^\dagger C(t) \nu_m$$

* Compute the effective $n^{\text{th}}$ energy level:

$$E_n^{\text{eff}}(t) = \frac{1}{\Delta t} \ln \frac{\Lambda_{nn}(t)}{\Lambda_{nn}(t + \Delta t)}$$
HAL QCD Method

* Obtain baryon-baryon potential from Nambu-Bethe-Salpeter amplitude computed on the lattice

\[ G_4(r, t - t_0) = \left< 0 \left| (BB)^{(\alpha)}(r, t) (\overline{BB})^{(\alpha)}(r, t_0) \right| 0 \right> = \phi(r, t) e^{-2M(t-t_0)} \]

\((BB)^{(\alpha)}(r, t)\) : 2-baryon interpolating operator; flavour irrep. \(\alpha\)

\(\phi(r, t)\) : NBS wave function

\(M\) : single baryon mass

* Determine potential via

\[ V(r) = \frac{[-H_0 - (\partial/\partial t)] \phi(r, t)}{\phi(r, t)} \]

* Solve Schrödinger equation

→ determine binding energies and scattering phase shifts
HAL QCD Method

Details of the calculation:

\[ N_f = 3, \ \text{i.e. mass-degenerate} \ u, \ d, \ s \ \text{quarks} \]

Single lattice spacing: \[ a = 0.121(2) \text{ fm} \]

5 pion masses in the range: \[ m_\pi = 469 - 1171 \text{ MeV} \]

HAL QCD Method

Details of the calculation:

$N_f = 2 + 1, \ O(a)$ improved Wilson fermions

Single lattice spacing: $a = 0.0846 \text{ fm}$; Volume: $L \approx 8.1 \text{ fm}$

Near physical point: $m_\pi = 146 \text{ MeV}, \ m_K = 525 \text{ MeV}$

[Sasaki et al., arXiv:1912.08630]

- $\Lambda\Lambda$ interaction weakly attractive

$\Rightarrow$ No bound or resonant dihyperon near $\Lambda\Lambda$ threshold at the physical point

Needs verification using a different methodology
The Mainz Dibaryon Project

Collaborators:

A. Francis, J.R. Green, A. Hanlon, P. Junnarkar, Ch. Miao, T.D. Rae, H.W.

Gauge ensembles provided by the CLS effort:

* $N_f = 2$ flavours of $O(a)$ improved Wilson fermions; quenched strange quark
  Pion masses: $m_\pi = 450 - 1000$ MeV (to compare with earlier studies)
  [Francis, Green, Junnarkar, Miao, Rae, HW, Phys Rev D99 (2019) 074505]

* $N_f = 2 + 1$ flavours of $O(a)$ improved Wilson fermions
  Pion masses: $m_\pi = 200 - 420$ MeV
  [Hanlon, Francis, Green, Junnarkar, HW, arXiv:1810.13282]

* SU(3)-symmetric and SU(3)-broken situations

* Three different lattice spacings to investigate lattice artefacts
Finite-volume spectrum

- Ensemble E1: $N_f = 2$, SU(3)-symmetric, $m_\pi \approx 960$ MeV
- Point-to-all propagators: hexaquark operators at source, two-baryon or hexaquark operators at the sink

Hexaquark operators: noisier, slower convergence towards ground state
Finite-volume spectrum

* Compute timeslice-to-all propagators
  → “distillation” — Laplace-Heaviside (LapH) smearing
  [Peardon et al., PRD 80 (2009) 054506; Morningstar et al., PRD 83 (2011) 114505]

* Quark propagator with smearing matrix at source and sink:

\[ S D^{-1} S, \quad S^{(t)}(x, y) = \sum_{k=1}^{N_{\text{LapH}}} V^{(k)}(x, t) \otimes V^{(k)}(y, t)^\dagger \]

\[ V^{(k)} : k^{\text{th}} \text{ eigenvector of Laplacian } \Delta; \text{ has support on entire timeslice} \]
Finite-volume spectrum

* Compute timeslice-to-all propagators
  → “distillation” — Laplace-Heaviside (LapH) smearing

[Peardon et al., PRD 80 (2009) 054506; Morningstar et al., PRD 83 (2011) 114505]

* Much better statistical signal
Finite-volume spectrum

* Compute timeslice-to-all propagators
  $\rightarrow \text{“distillation”} \quad \text{— Laplace-Heaviside (LapH) smearing}$

[Peardon et al., PRD 80 (2009) 054506; Morningstar et al., PRD 83 (2011) 114505]

\[
\langle BB(t) BB^+(0) \rangle
\]

* Much better statistical signal

* Ensemble E5: broken SU(3)-flavour symmetry ($m_\pi = 450 \text{ MeV}$)
Scattering phase shifts — Lüscher method

* Scattering momentum: \[ p^2 = \frac{1}{4}(E^2 - P \cdot P) - m^2_\Lambda \]

* Scattering phase shifts: \[ p \cot \delta(p) = \frac{2}{\gamma L \sqrt{\pi}} Z_{00}(1, q^2), \quad q = \frac{pL}{2\pi} \]

\[ Z_{00}(1, q^2) = \frac{1}{\sqrt{4\pi}} \left\{ \sum_{q^2 \neq n^2} \frac{1}{q^2 - n^2} - 4\pi \Lambda_n \right\} \]

[Lüscher 1990/91; Rummukainen & Gottlieb 1995]
Scattering phase shifts — Lüscher method

* Scattering momentum:  
  \[ p^2 = \frac{1}{4}(E^2 - P \cdot P) - m^2_\Lambda \]

* Scattering phase shifts:  
  \[ p \cot \delta(p) = \frac{2}{\gamma L \sqrt{\pi}} Z_{00}(1, q^2), \quad q = \frac{pL}{2\pi} \]

* Pole of the scattering amplitude:  
  \[ \mathcal{A} \propto \frac{1}{p \cot \delta(p) - ip} \]

* Fit to effective range expansion:  
  \[ p \cot \delta_0(p) = A + Bp^2 + \ldots \overset{!}{=} -\sqrt{-p^2} \]

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Preliminary

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**E1 ground states, uncorrelated fit, \( \Delta E = 16.3 \pm 4.2 \text{ MeV} \)**
Comparison with other calculations

* **NPLQCD Collaboration**: point-to-all propagators

* **HAL QCD Collaboration**: energy levels from lattice calculation of NBS wavefunction \((N_f = 3)\)

![Graph showing binding energy comparison](image)

**Binding energy from FV analysis (SU(3)-symmetric case)**

\[
B_{\Lambda\Lambda} = 19 \pm 10 \text{ MeV}
\]

\((m_\pi = 960 \text{ MeV})\)

[Green: SU(3)-symmetric; blue: SU(3)-broken]

[Francis et al., PRD 99 (2019) 074505; Green et al., in prep.]
Comparison with other calculations

* NPLQCD Collaboration: point-to-all propagators
* HAL QCD Collaboration: energy levels from lattice calculation of NBS wavefunction \((N_f = 3)\)

![Graph](attachment:image.png)

Binding energy from FV analysis
(SU(3)-symmetric case)

\[
B_{\Lambda\Lambda} = 16.3 \pm 4.2 \text{ MeV} \\
(m_\pi = 960 \text{ MeV})
\]

\[
B_{\Lambda\Lambda} = 4.5 \pm 3.7 \text{ MeV} \\
(m_\pi = 450 \text{ MeV})
\]

(Preliminary)

[Green: SU(3)-symmetric; blue: SU(3)-broken]

[Francis et al., PRD 99 (2019) 074505; Green et al., in prep.]
Higher spin states

* So far: focus on $J^P = 0^+$ and $S = -2$
* Extend calculation to higher spins $\rightarrow$ include additional irreps.
* Spin-1 interpolating operators:

$$B_\alpha \equiv [rst]_\alpha = \epsilon_{ijk} \left( s^i C \gamma_5 P^+_j t^k \right) r_\alpha$$

$$(BB)_i(p_1, p_2) = \sum_x e^{-ip_1 \cdot x} B_1(x, t) (C \gamma_i P_+) \sum_y e^{-ip_2 \cdot y} B_2(y, t)$$

$\Rightarrow$ Deuteron: $$(BB)^{(n)}_{i; T_1^+} = \frac{1}{N} \sum_{p; p^2=n} (BB)_i(-p, p)$$

* Study $H$ dibaryon and additional states in QCD with $N_f = 2 + 1$
* Move toward physical pion mass
Gauge ensembles with $N_f = 2+1$

* $O(a)$ improved Wilson fermions — CLS effort

* SU(3)-symmetric point: $m_\pi = m_K \approx 420$ MeV

<table>
<thead>
<tr>
<th>Label</th>
<th>$L^3 \times T$</th>
<th>$a$ [fm]</th>
<th>$m_\pi$ [MeV]</th>
<th>Symmetry</th>
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<tbody>
<tr>
<td>U103</td>
<td>$24^3 \times 128$</td>
<td>0.0865</td>
<td>420</td>
<td>SU(3)-symmetric</td>
</tr>
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<td>0.0644</td>
<td>200</td>
<td>SU(3)-broken</td>
</tr>
</tbody>
</table>

* Approach physical point along chiral trajectory defined by

\[
\text{Tr } M_q = \text{const.} \iff \frac{1}{2} m_\pi^2 + m_K^2 \approx \text{const.}
\]

* Compute spectrum using distillation and GEVP
Preliminary results for $N_f = 2+1$

* Singlet channel, spin-0, SU(3)-symmetric
* Finite-volume energy levels in $A_1$ irrep. in different frames

U103 ensemble: $m_\pi = m_K = 420$ MeV, $L = 2.08$ fm, $a = 0.0865$ fm
Preliminary results for $N_f = 2+1$

* Singlet channel, spin-0, SU(3)-symmetric
* Finite-volume energy levels in $A_1$ irrep. in different frames

H101 ensemble: $m_\pi = m_K = 420\text{ MeV}, L = 2.77\text{ fm}, a = 0.0865\text{ fm}$
Preliminary results for $N_f = 2+1$

* Singlet channel, spin-0, SU(3)-symmetric
* Finite-volume energy levels in $A_1$ irrep. in different frames

B450 ensemble: $m_\pi = m_K = 415$ MeV, $L = 2.45$ fm, $a = 0.0765$ fm
Preliminary results for $N_f = 2+1$

* Singlet channel, spin-0, SU(3)-broken
* Finite-volume energy levels in $A_1$ irrep. in different frames

U102 ensemble: $m_\pi = 350$ MeV, $m_K = 450$ MeV, $L = 2.08$ fm, $a = 0.0865$ fm
Preliminary results for $N_f = 2+1$

- Singlet channel, spin-0, SU(3)-broken
- Finite-volume energy levels in $A_1$ irrep. in different frames

Excellent resolution of excitation spectrum

Interpretation of the energy levels very involved
Preliminary results for $N_f = 2+1$

* Is the deuteron bound at $m_\pi = m_K \approx 420$ MeV?
**Distillation & GEVP:** powerful method to determine energy levels in a finite volume for a wide range of dibaryon channels

**Lüscher’s finite-volume quantisation condition:** rigorous formalism to study hadron-hadron interactions on the lattice

**H dibaryon:** Binding energies extracted from Lüscher formalism significantly lower than the naïve energy difference \( E_{\Lambda\Lambda} - 2m_\Lambda \)

**SU(3)-symmetric point:** \( B_{\Lambda\Lambda} = 5 - 20 \text{ MeV}, \quad m_\pi \geq 450 \text{ MeV} \)

⇒ significantly lower than Jaffe’s bag model estimate

**Next steps:**

* Investigate SU(3)-breaking
* Compute binding energies as the quark masses are tuned towards the physical situation