$\Rightarrow$ VORLESUNG 25 QM

# ZEIT-ABHÄNGIGE STÖRUNGSTHEORIE

2 ZUSTAND SYSTEM $\nearrow |\psi_a\rangle$, $\searrow |\psi_b\rangle$

$$\hookrightarrow \quad H^0 |\psi_a\rangle = E_a |\psi_a\rangle$$

$$H^0 |\psi_b\rangle = E_b |\psi_b\rangle$$

$$\langle \psi_a | \psi_b \rangle = \delta_{ab}$$

$$|\Psi(t=0)\rangle = c_a |\psi_a\rangle + c_b |\psi_b\rangle$$

$$H^0 \downarrow \qquad |c_a|^2 + |c_b|^2 = 1$$

$$|\Psi(t)\rangle_{H^0} = c_a e^{-\frac{i}{\hbar}E_a t} |\psi_a\rangle + c_b e^{-\frac{i}{\hbar}E_b t} |\psi_b\rangle$$

$$\hookrightarrow \quad H^1(t)$$

$$H^0 + H^1(t)$$

$$\{ |\psi_a\rangle, |\psi_b\rangle \}$$ COMPLETENESS

$$\rightsquigarrow \quad |\Psi(t)\rangle_{H^0+H^1} = c_a(t)\, e^{-\frac{i}{\hbar}E_a t}\,|\psi_a\rangle + c_b(t)\, e^{-\frac{i}{\hbar}E_b t}\,|\psi_b\rangle$$

$H^1$ : ÜBERGÄNGE ZWISCHEN a & b

$$\begin{matrix} c_a(t=0) = 1 & & c_a(t_1) = 0 \\ & \overset{H^1}{\Longrightarrow} & \\ c_b(t=0) = 0 & & c_b(t_1) = 1 \end{matrix}$$

$c_a(t)$ , $c_b(t)$ ?

$$\left(H^0 + H^1(t)\right)|\Psi(t)\rangle = i\hbar \frac{\partial}{\partial t}|\Psi(t)\rangle$$

$$H^0\left(\cancel{c_a(t)\, e^{-\frac{i}{\hbar}E_a t}}\,|\psi_a\rangle + \cancel{c_b(t)\, e^{-\frac{i}{\hbar}E_b t}\,|\psi_b\rangle}\right)$$

$$+\, H^1\left(c_a(t)\, e^{-\frac{i}{\hbar}E_a t}\,|\psi_a\rangle + c_b(t)\, e^{-\frac{i}{\hbar}E_b t}\,|\psi_b\rangle\right)$$

$$= \cancel{E_a\, c_a(t)\, e^{-\frac{i}{\hbar}E_a t}}\,|\psi_a\rangle + E_b\, \cancel{c_b(t)\, e^{-\frac{i}{\hbar}E_b t}\,|\psi_b\rangle}\Big)$$

$$+\, i\hbar\, \dot{c}_a\, e^{-\frac{i}{\hbar}E_a t}\,|\psi_a\rangle + i\hbar\, \dot{c}_b\, e^{-\frac{i}{\hbar}E_b t}\,|\psi_b\rangle$$

$$H^1\left(c_a(t)e^{-\frac{i}{\hbar}E_a t}|\psi_a\rangle + c_b(t)e^{-\frac{i}{\hbar}E_b t}|\psi_b\rangle\right)$$

$$= +i\hbar\dot{c}_a\, e^{-\frac{i}{\hbar}E_a t}|\psi_a\rangle + i\hbar\,\dot{c}_b\, e^{-\frac{i}{\hbar}E_b t}|\psi_b\rangle$$

$$\Downarrow$$

$$c_a(t),\quad c_b(t)$$

$$\langle\psi_a| \quad , \quad \langle\psi_b|$$

$$\langle\psi_a|H^1|\psi_b\rangle \equiv H^1_{ab}$$

$$\langle\psi_a|$$

$$c_a(t)e^{-\frac{i}{\hbar}E_a t}H^1_{aa} + c_b(t)e^{-\frac{i}{\hbar}E_b t}H^1_{ab}$$

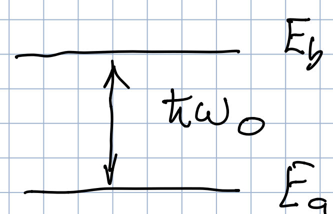
$$= i\hbar\,\dot{c}_a e^{-\frac{i}{\hbar}E_a t} + 0$$

$$\boxed{\dot{c}_a = -\frac{i}{\hbar}\left[c_a(t)\,H^1_{aa} + c_b(t)\,e^{-\frac{i}{\hbar}(E_b-E_a)t}\,H^1_{ab}\right]}$$

$$\boxed{\dot{c}_b = -\frac{i}{\hbar}\left[c_b(t)\,H^1_{bb} + c_a(t)\,e^{-\frac{i}{\hbar}(E_a-E_b)t}\,H^1_{ba}\right]}$$

SPEZIAL FALL

$$H^1_{aa} = H^1_{bb} = 0$$




$$\hbar\omega_0 \equiv E_b - E_a$$

$$\begin{cases} \dot{c}_a = -\frac{i}{\hbar}\, c_b\, e^{-i\omega_0 t}\, H^1_{ab} \\ \dot{c}_b = -\frac{i}{\hbar}\, c_a\, e^{+i\omega_0 t}\, H^1_{ba} \end{cases}$$

$\Rightarrow$ STÖRUNGS THEORIE $\quad H^1_{ab} \ll H^0$

$\hookrightarrow$ $\underline{t=0}$

$$\begin{cases} c_a(t=0) = 1 \\ c_b(t=0) = 0 \end{cases}$$

↳ ORDNUNG 0 $\quad H^1 = 0$

$$\dot{c}_a^{(0)} = 0 \quad \Longrightarrow \quad c_a^{(0)}(t) = 1$$

$$\dot{c}_b^{(0)} = 0 \quad \Longrightarrow \quad c_b^{(0)}(t) = 0$$

↳ ORDNUNG 1

$$\dot{c}_a^{(1)} = -\frac{i}{\hbar} \underbrace{c_b^{(0)}}_{0} e^{-i\omega_0 t} H^1_{ab}$$

$$= 0$$

$$\Downarrow$$

$$\Longrightarrow \quad c_a^{(1)}(t) = 1$$

$$\dot{c}_b^{(1)} = -\frac{i}{\hbar} \underbrace{c_a^{(0)}}_{1} e^{+i\omega_0 t} H^1_{ba}$$

$$= -\frac{i}{\hbar} e^{i\omega_0 t} H^1_{ba}$$

$$c_b^{(1)}(t) - \underset{\overset{\|}{0}}{c_b^{(1)}(0)} = -\frac{i}{\hbar} \int_0^t dt' \, e^{i\omega_0 t'} H^1_{ba}(t')$$

$$c_b^{(1)}(t) = -\frac{i}{\hbar} \int_0^t dt' \, e^{i\omega_0 t'} H^1_{ba}(t')$$

↳ ORDNUNG 2

$$\dot{c}_a^{(2)} = -\frac{i}{\hbar} c_b^{(1)} e^{-i\omega_0 t} H^1_{ab}$$

$$\Big\downarrow \int_0^t$$

$$c_a^{(2)}(t) - \underbrace{c_a^{(2)}(0)}_{1} = -\frac{i}{\hbar}\int_0^t dt' \, c_b^{(1)}(t') \, e^{-i\omega_0 t'} H^1_{ab}(t')$$

$$c_a^{(2)}(t) = 1 - \frac{1}{\hbar^2}\int_0^t dt' \, e^{-i\omega_0 t'} H^1_{ab}(t') \int_0^{t'} dt'' \, e^{i\omega_0 t''} H^1_{ba}(t'')$$

$$\dot{c}_b^{(2)} = -\frac{i}{\hbar} \underbrace{\cancel{c_a^{(1)}}}_{1} e^{+i\omega_0 t} H^1_{ba}$$

$$= \dot{c}_b^{(1)}$$

$$c_b^{(2)} = c_b^{(1)}$$

NORMIERUNG

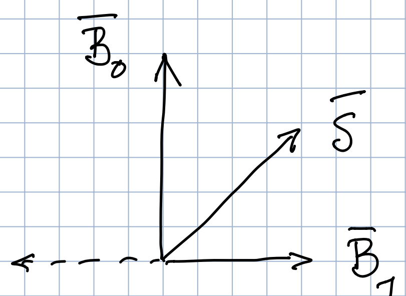
$$|c_a(t)|^2 + |c_b(t)|^2 = \underline{1}$$

z.B. ORDNUNG $\underline{1}$

$$\underbrace{|c_a^{(1)}(t)|^2}_{\underline{1}} + |c_b^{(1)}(t)|^2$$

$$\underline{1} + \cancel{O((H^1)^2)}$$

$\Rightarrow$ SINUSOIDALE STÖRUNG




$B_1 \ll B_0$

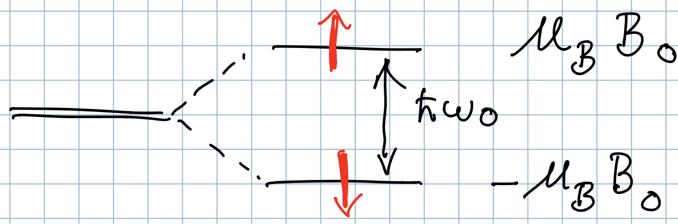
$\times$

$$H^0 = -\bar\mu \cdot \bar B_0 \qquad \bar B_0 = B_0 \bar e_z$$

$$= -B_0 \mu_z$$

$$H^0 = \mu_B B_0 \sigma_z \qquad \bar\mu = -\frac{e}{2m}\,\underset{\underset{2}{\shortparallel}}{g}\,\frac{\hbar}{2}\,\bar\sigma$$

$$= -\underbrace{\frac{e\hbar}{2m}}_{\mu_B}\bar\sigma$$




$$\hbar\omega_0 = 2\mu_B B_0$$

$$\vec{B}_1(t) = B_1 \sin\omega t \; \vec{e}_x$$

$$H^1(t) = -\vec{\mu}\cdot\vec{B}_1(t)$$

$$= \mu_B B_1 \sin\omega t \; \sigma_x$$

$$\langle \uparrow | H^1 | \uparrow \rangle = 0$$

$$\langle \uparrow | H^1 | \downarrow \rangle = \mu_B B_1 \sin\omega t$$

$$\langle \downarrow | H^1 | \downarrow \rangle = 0$$

$$\langle \downarrow | H^1 | \uparrow \rangle = \mu_B B_1 \sin\omega t$$

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$|\psi_a\rangle = |\downarrow\rangle \qquad C_\downarrow(0) = 1$$

$$|\psi_b\rangle = |\uparrow\rangle \qquad C_\uparrow(0) = 0$$

$$C_\downarrow^{(1)}(t) = 1$$

$$C_\uparrow^{(1)}(t) = -\frac{i}{\hbar}\int_0^t dt' \, e^{i\omega_0 t'} H^1_{\uparrow\downarrow}(t')$$

$$\downarrow \qquad H^1_{\uparrow\downarrow} = \underbrace{\mu_B B_1}_{V_{\uparrow\downarrow}} \sin \omega t$$

$$= -\frac{i}{\hbar} V_{\uparrow\downarrow} \int_0^t dt' \, e^{i\omega_0 t'} \frac{1}{2i}\left(e^{i\omega t'} - e^{-i\omega t'}\right)$$

$$= -\frac{V_{\uparrow\downarrow}}{2\hbar}\left\{ \frac{e^{i(\omega+\omega_0)t} - 1}{i(\omega+\omega_0)} - \frac{e^{i(\omega_0-\omega)t} - 1}{i(\omega_0-\omega)} \right\}$$

$\downarrow$ $\omega$ IN DER NÄHE $\omega_0$

$$\omega \simeq \omega_0$$

$$|\omega_0 - \omega| \ll (\omega + \omega_0)$$

$$c_{\uparrow}^{(1)}(t) \simeq \frac{V_{\uparrow\downarrow}}{2i\hbar} \; \frac{e^{i(\omega_0-\omega)t} - 1}{(\omega_0-\omega)}$$

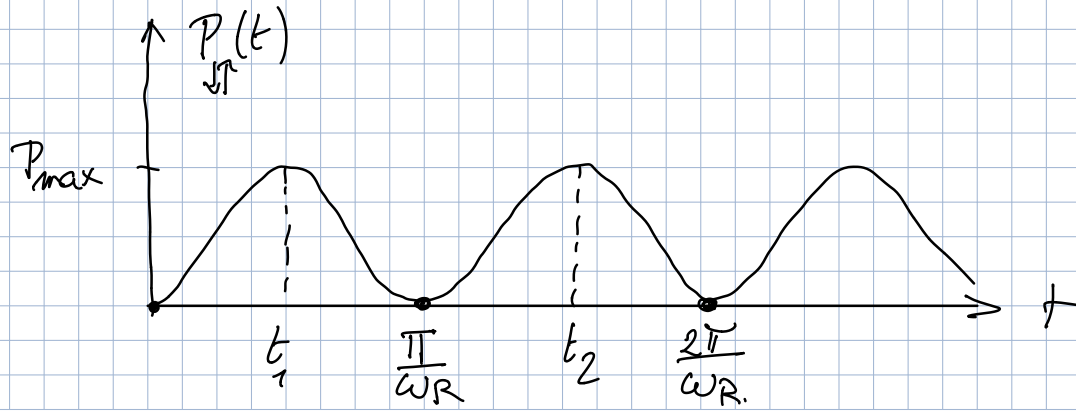
$$\underset{\omega \simeq \omega_0}{=} \frac{V_{\uparrow\downarrow}}{\hbar} \, e^{i(\omega_0-\omega)t/2} \; \frac{\sin\left((\omega_0-\omega)t/2\right)}{(\omega_0-\omega)}$$

$$P^{(1)}_{\downarrow\Rightarrow\uparrow} = \left| c_{\uparrow}^{(1)}(t) \right|^2$$

WAHRSCHEINLICHKEIT FÜR SPIN FLIP $\downarrow \Rightarrow \uparrow$

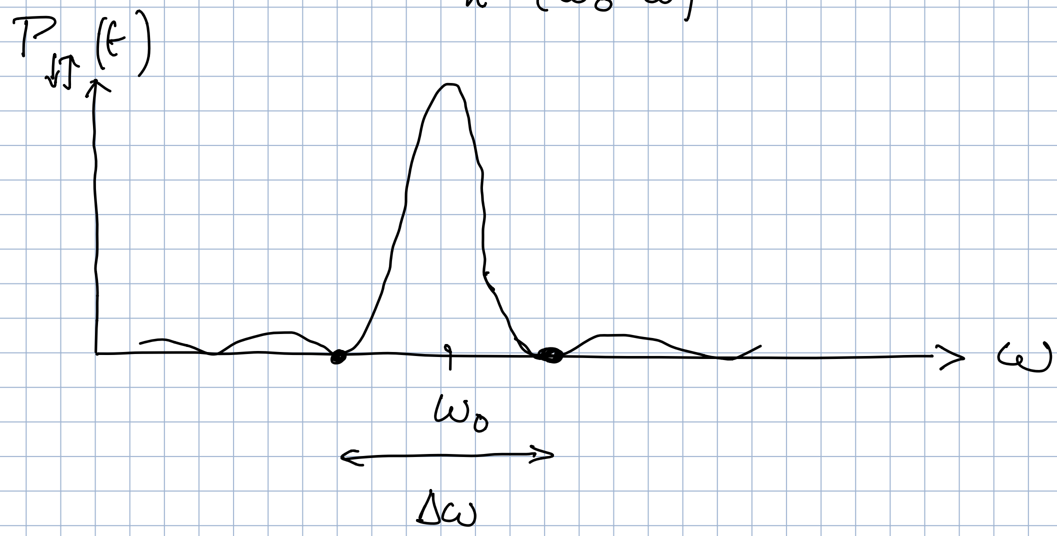
$$\boxed{P^{(1)}_{\downarrow\Rightarrow\uparrow} = \frac{|V_{\uparrow\downarrow}|^2}{\hbar^2} \; \frac{\sin^2\left((\omega_0-\omega)t/2\right)}{(\omega_0-\omega)^2}}$$

$\underline{t=0}$ $\quad P_{\downarrow\Rightarrow\uparrow} = 0$



$$\rightsquigarrow \quad t_n = (2n+1)\frac{\pi}{|\omega_0-\omega|}$$

$$P_{max} = \frac{|V_{\uparrow\downarrow}|^2}{\hbar^2(\omega_0-\omega)^2}$$




$$\frac{\Delta\omega\, t}{2} = 2\pi$$

$$\Longrightarrow \quad \Delta\omega = \frac{4\pi}{t}$$

EXAKTE LÖSUNG

$$\boxed{P_{\downarrow\Rightarrow\uparrow}(t) = \frac{|V_{\downarrow\uparrow}|^2}{\hbar^2}\,\frac{\sin^2(\omega_R t)}{(2\omega_R)^2}}$$

$$(\omega_0-\omega)/2 \Rightarrow \omega_R$$

$$\omega_R = \frac{1}{2}\sqrt{(\omega_0-\omega)^2 + \frac{(V_{\downarrow\uparrow})^2}{\hbar^2}}$$

RABI FREQUENZ

STÖRUNGSTHEORIE

$$V_{\downarrow\uparrow} \ll \hbar\,|\omega_0-\omega|$$

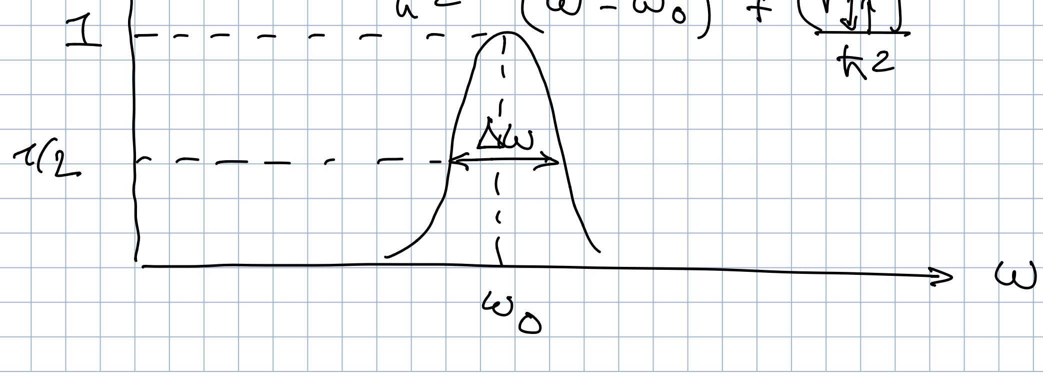
$$\Downarrow$$

$$\omega_R \simeq \frac{1}{2}\,|\omega_0-\omega|$$

$$\underbrace{\omega=\omega_0} \implies \omega_R = \frac{V_{\downarrow\uparrow}}{2\hbar}$$

$$P^{max}_{\downarrow\uparrow} = \frac{|V_{\downarrow\uparrow}|^2}{\hbar^2}\cdot\left(\frac{\hbar}{V_{\downarrow\uparrow}}\right)^2 = 1$$

$$P_{\downarrow\uparrow} = \frac{|V_{\downarrow\uparrow}|^2}{\hbar^2}\,\frac{1}{(\omega-\omega_0)^2 + \frac{(V_{\downarrow\uparrow})^2}{\hbar^2}}$$



$$\frac{\Delta\omega}{2} = \frac{|V_{\downarrow\uparrow}|}{\hbar}$$

**NMR**

$$\omega_0 = \frac{e}{2M} \underbrace{g}_{\simeq 5.6} B_0$$

$$M_p \simeq 940 \text{ MeV}$$

$$B_0 = 1T$$

$$\frac{\omega_0}{2\pi} \simeq 43 \text{ MHz}$$