⇒ VORLESUNG 24 QM

↳ FEINSTRUKTUR DES H-ATOM

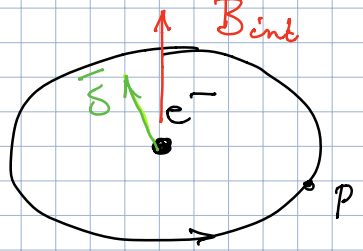
1) REL. KINETISCHE ENERGIE

$$H^1_{rel} = -\frac{\hat{p}^4}{8m^3c^2}$$

$$E^1_{rel} = -\frac{E_n^2}{2mc^2}\left\{-3 + \frac{4n}{\ell + 1/2}\right\}$$

$$E_n = \frac{E_1}{n^2} \qquad E_1 = -\frac{1}{2}mc^2\alpha^2 \simeq -13.6\,eV$$

2) SPIN-BAHN KOPPLUNG



$$\vec{B}_{int} \sim \vec{L}$$

$$H^1_{SO} \sim \vec{L}\cdot\vec{S}$$

$\vec{L}\cdot\vec{S}$

$\vec{J} = \vec{L} + \vec{S} \rightsquigarrow \vec{L}\cdot\vec{S} = \frac{1}{2}\left(J^2 - L^2 - S^2\right)$

$H^1_{SO}, H^0, J^2, J_z, L^2, S^2 \ldots$

$n, j, m_j, \ell, s = \frac{1}{2}$

$$E^1_{SO} = \frac{E_n^2}{mc^2}\, n\, \frac{j(j+1) - \ell(\ell+1) - \frac{3}{4}}{\ell(\ell+1)\left(\ell+\frac{1}{2}\right)}$$

GESAMMT REL. KORREKTUR

$$E^1_{FS} = E^1_{rel} + E^1_{SO} \qquad (\text{DIRAC})$$

$$= \frac{E_n^2}{2mc^2}\left\{ 3 - \frac{\cancel{4}^{\,6}\, n}{\ell+\frac{1}{2}} + 2n\, \frac{j(j+1) - \frac{3}{4}}{\ell(\ell+1)\left(\ell+\frac{1}{2}\right)} - \cancel{\frac{2n}{\ell+\frac{1}{2}}} \right\}$$

$$\longrightarrow \underbrace{n, \ell, j}$$

$$j = \ell \pm \frac{1}{2}$$

- $j = \ell + \frac{1}{2} \quad \longrightarrow \quad \ell = j - \frac{1}{2}$

$$\ell(\ell+1)\left(\ell+\frac{1}{2}\right) = \left(j-\frac{1}{2}\right)\left(j+\frac{1}{2}\right) j$$

$$j(j+1) - \frac{3}{4} = \left(j-\frac{1}{2}\right)\left(j+\frac{3}{2}\right)$$

$$E_{FS}^{1} = \frac{E_n^2}{2mc^2}\left\{3 - \frac{6n}{j} + 2n\,\frac{(j-\frac{1}{2})(j+\frac{3}{2})}{j(j+\frac{1}{2})(j-\frac{1}{2})}\right\}$$

$$= \frac{E_n^2}{2mc^2}\left\{3 - \frac{2n}{j(j+\frac{1}{2})}\underbrace{\left[3(j+\tfrac{1}{2}) - (j+\tfrac{3}{2})\right]}_{2j}\right\}$$

$$E_{FS}^{1} = \frac{E_n^2}{2mc^2}\left\{3 - \frac{4n}{j+\frac{1}{2}}\right\}$$

- $j = \ell - \frac{1}{2} \rightarrow \ell = j + \frac{1}{2}$

$$\ell(\ell+1)\left(\ell+\tfrac{1}{2}\right) = \left(j+\tfrac{1}{2}\right)\left(j+\tfrac{3}{2}\right)(j+1)$$

$$E_{FS}^{1} = \frac{E_n^2}{2mc^2}\left\{3 - \frac{6n}{j+1} + 2n\,\frac{(j-\frac{1}{2})(j+\frac{3}{2})}{(j+\frac{1}{2})(j+\frac{3}{2})(j+1)}\right\}$$

$$= \frac{E_n^2}{2mc^2}\left\{3 - \frac{2n}{(j+\frac{1}{2})(j+1)}\underbrace{\left[3(j+\tfrac{1}{2}) - (j-\tfrac{1}{2})\right]}_{2j+2 = 2(j+1)}\right\}$$

$$= \frac{E_n^2}{2mc^2}\left\{3 - \frac{4n}{j+\frac{1}{2}}\right\}$$

$$E^1_{FS} = \frac{E_n^2}{2mc^2}\left\{3 - \frac{4n}{j+\frac{1}{2}}\right\}$$

HÄNGT NUR AB VON n, j

NICHT REL $E_n = \frac{E_1}{n^2} = -\frac{1}{2}mc^2\alpha^2\frac{1}{n^2}$

DIRAC $E_{nj} = E_n + \frac{E_n^2}{2mc^2}\left\{3 - \frac{4n}{j+\frac{1}{2}}\right\}$.

$$= E_n\left[1 - \frac{\alpha^2}{4n^2}\left(3 - \frac{4n}{j+\frac{1}{2}}\right)\right]$$

$$E_{nj} = E_n\left[1 + \frac{\alpha^2}{n^2}\left(\frac{n}{j+\frac{1}{2}} - \frac{3}{4}\right)\right]$$

$O(\alpha^2) \sim 10^{-4}$

- $n = 1 \quad \ell = 0 \rightsquigarrow j = \frac{1}{2}$

$$\frac{n}{j+\frac{1}{2}} - \frac{3}{4} = 1 - \frac{3}{4} = \frac{1}{4}$$

- $n = 2. \quad \ell = 0 \rightsquigarrow j = \frac{1}{2}$

$\ell = 1 \rightsquigarrow j = \frac{1}{2}$, $j = \frac{3}{2}$

$\frac{n}{j+\frac{1}{2}} - \frac{3}{4} = \frac{5}{4}$

$\frac{n}{j+\frac{1}{2}} - \frac{3}{4} = \frac{1}{4}$

H^0

$H^0 + H^1_{FS}$

E_3 — $\ell=0$ — (18) $\ell=1$ — $\ell=2$

E_2 — $\ell=0$ — $\ell=1$

$\ell=1$: $4\ j=3/2$, $2\ j=1/2$ — $\ell=2$: $6\ j=5/2$, $4\ j=3/2$ — 9 — 1

2 — $j=1/2, \ell=0$ — $4\ j=3/2$

2 — $j=\frac{1}{2}$, $\ell=0$ — 2 — $j=\frac{1}{2}$, $\ell=1$

E_1 — $\ell=0$, $n=1$

$=$

$-13.6\,eV$

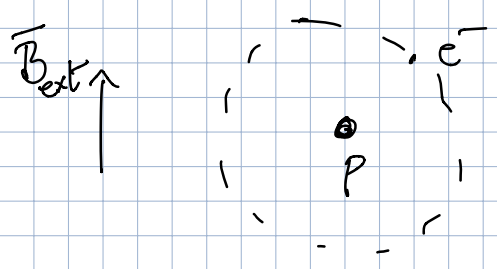
2 — $j=\frac{1}{2}$ ↕ E^1_{FS}

ENTARTUNG $2n^2$

$\Rightarrow$ SPLITTING : LAMB SHIFT

QUANTEN FELDTHEORIE

ZEEMAN EFFEKT



$e^- \nearrow \vec{L} \rightarrow \vec{\mu}_e = \gamma_e \vec{L}, \gamma_e = -\frac{e}{2m}$

$\searrow \vec{S} \rightarrow \vec{\mu}_s = \gamma_s \vec{S}, \gamma_s = -\frac{e}{2m} 2$

$$H_Z^1 = -(\vec{\mu}_e + \vec{\mu}_s) \cdot \vec{B}_{ext}$$

$$= \frac{e}{2m} (\vec{L} + 2\vec{S}) \cdot \vec{B}_{ext}$$

1) $H_Z^1 \ll H_{FS}^1$

$B_{ext} \ll \vec{B}_{int} \sim \frac{1}{m r^3} \vec{L}$

2) $H_{FS}^1 \ll H_Z^1$ $\quad r \sim a$

3) $H_{FS}^1 \sim H_Z^1$

1) SCHWACHE FELD ZEEMAN EFFEKT

$$H^0 \rightarrow |n\, j\, m_j\, l\, s\rangle$$

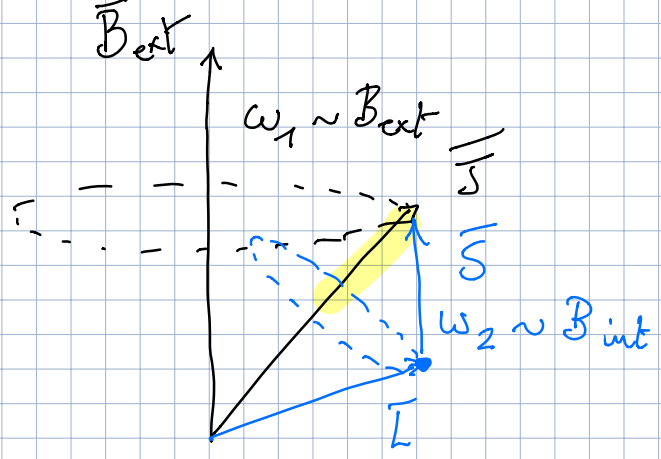
$$E_z^1 = \langle n j m_j l s | H_z^1 | n j m_j l s \rangle$$

$$\downarrow \qquad \vec{B}_{ext} = B_{ext}\vec{e}_z$$

$$= \frac{e}{2m} B_{ext} \langle n j m_j l s | L_z + 2S_z | \ldots \rangle$$

$$\vec{J} = \vec{L} + \vec{S}$$

$$\vec{L} + 2\vec{S} = \vec{J} + \vec{S}$$



$$B_{ext} << B_{int} \Rightarrow \omega_1 << \omega_2$$

$$\omega_2 \gg \omega_1$$

$$\vec{S} \Rightarrow \vec{S}_{av} = \frac{(\vec{S}\cdot\vec{J})}{J^2}\vec{J}$$

$$\vec{S}_{av}\cdot\vec{J} = \vec{S}\cdot\vec{J}$$

$$\vec{L} + 2\vec{S} = \vec{J} + \vec{S} \Rightarrow \vec{J} + \vec{S}_{av}$$

$$= \left(1 + \frac{\vec{S}\cdot\vec{J}}{J^2}\right)\vec{J}$$

$$\vec{E}_Z = \frac{e}{2m} B_{ext}$$

$$\cdot \langle n j m_j l s | \left(1 + \frac{\vec{S}\cdot\vec{J}}{J^2}\right) J_z | n j m_j l s \rangle$$

$$\vec{J} = \vec{L} + \vec{S}$$

$$(\vec{J} - \vec{S})^2 = L^2$$

$$J^2 + S^2 - 2\vec{J}\cdot\vec{S} = L^2$$

$$2\vec{S}\cdot\vec{J} = J^2 - L^2 + S^2$$

$$E_Z^1 \simeq \frac{e}{2m} B_{ext} (\hbar m_j) \cdot \left(1 + \frac{j(j+1) - l(l+1) + \frac{3}{4}}{2 j(j+1)} \right)$$

$$\mu_B \equiv \frac{e\hbar}{2m} \qquad \text{BOHR MAGNETON}$$

$$E_Z^1 = \mu_B B_{ext} \cdot m_j \cdot g_j$$

$$\rightsquigarrow g_j \equiv 1 + \frac{j(j+1) - l(l+1) + \frac{3}{4}}{2 j(j+1)}$$

$$\bar{\mu} = - \frac{e}{2m} g_j \bar{J}$$

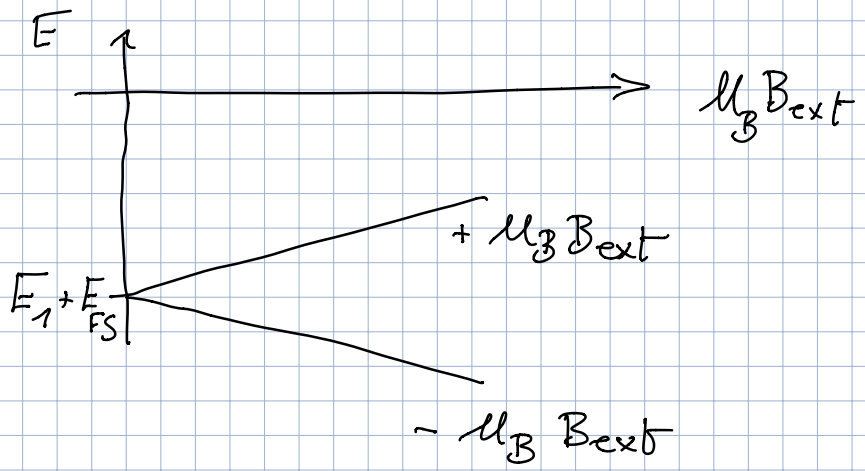
$$H_Z = - \bar{\mu} \cdot \bar{B}$$

- $n = 1 \qquad \ell = 0 \qquad j = \frac{1}{2} \qquad m_j = \pm \frac{1}{2}$

$$g_{\frac{1}{2}} = 1 + \frac{\frac{3}{4} - 0 + \frac{3}{4}}{2 \cdot \frac{3}{4}}$$

$$= 2$$

$$E^{\uparrow}_{z,\, j=\frac{1}{2}} = \mu_B B_{ext} \left(+\frac{1}{2}\right) \cdot 2$$



- $n = 2 \qquad \ell = 1 \;\Big/\; j = \frac{3}{2} \qquad m_j = -\frac{3}{2}, \ldots, +\frac{3}{2}$

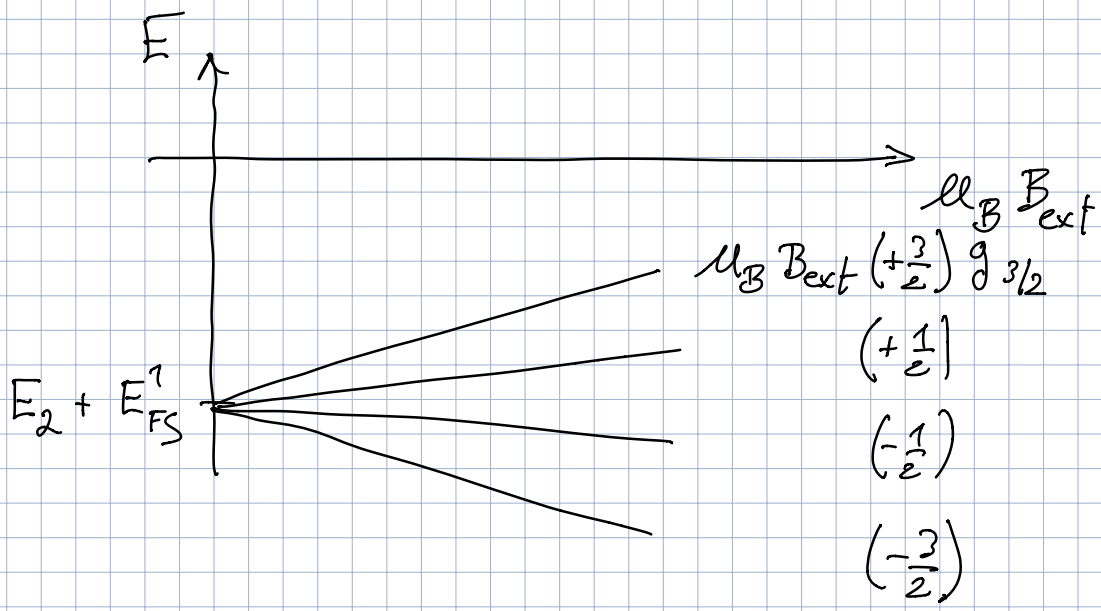
$\ell = 0 \;\Big/\; j = \frac{1}{2} \qquad m_j = \pm \frac{1}{2}$

$j = \frac{3}{2}, \quad \ell = 1$

$$g_{\frac{3}{2}} = 1 + \frac{\frac{3}{2}\cdot\frac{5}{2} - 2 + \frac{3}{4}}{2\,\frac{3}{2}\cdot\frac{5}{2}}$$

$$= 1 + \frac{\frac{9}{2} - 2}{\frac{15}{2}} = 1 + \frac{5}{15}$$

$$g_{\frac{3}{2}} = \frac{4}{3}$$



ALLGEMEIN $H^1_{FS} + H^1_Z$

$$|\psi^0\rangle = |n\, j\, m_j\, \ell\, m_\ell\rangle$$

$n = 2 \longrightarrow 8 \times 8$ MATRIX

SIEHE LECTURE NOTES