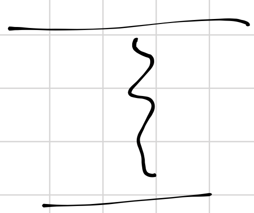
Lecture 23

We computed tree-level amplitudes for $2 \to 2$ scattering processes

Interaction at tree-level = exchange o 1 part.



Properties of exchanged part. define the interaction

($m_\gamma = 0 \longrightarrow \infty$ range Coulomb force

$m \neq 0 \longrightarrow$ finite range Yukawa)

1 to 1 correspondence between the cross section and the coupling

However we have seen that whenever more particles can be produced this relation may change

ϕ^3

$i\mathcal{M} =$ 

$-i\lambda^2 \quad \sim -i\lambda^4 \ln\frac{s}{m^2} \quad \sim -i\lambda^6 \ln^2\frac{s}{m^2} \quad \ldots$

$$|\mathcal{M}|^2 \sim \lambda^4 \xrightarrow[\text{intermediate states}]{\text{sum over } 2+n} \underline{\lambda^4 \left(\frac{s}{m^2}\right)^\alpha}$$

The ladders was a fortunate example of finite loop diagrams

But most of the time loops $= \infty$

So, how can we make sense of ∞?

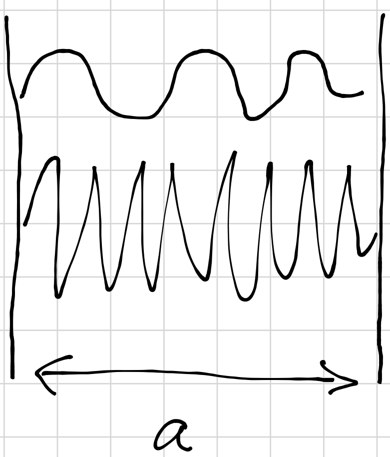
Let's take a look at ∞ that we have already seen: zero mode energy

$$H = \int \frac{d^3\vec{k}}{(2\pi)^3}\, \omega_k \left[a_k^\dagger a_k + \frac{1}{2}\right]$$

$\omega_k = |\vec{k}|$

$$E = \langle 0| \int \frac{d^3k}{(2\pi)^3} \frac{\omega_k}{2} |0\rangle = \infty$$

Consider a box of side "a" in the vacuum



For simplicity $\rightarrow$ 1D

Also, assume spin-0 γ

$E(a) \longrightarrow F(a) = -\frac{dE}{da}$

"Standing waves" $\omega_n = \frac{\pi}{a} n$

$$E(a) = \frac{1}{2}\sum_n \omega_n = \frac{\pi}{2a}\sum_n n = \infty$$

Ramanujan summation

$$\sum_n n = -1/12$$

$$A = 1 - 1 + 1 - 1 + \underset{\infty}{\ldots} = 1/2$$

$$1 - A = 1 - (1 - 1 + 1 - 1 \ldots) = A \implies A = 1/2$$

$$B = 1 - 2 + 3 - 4 + 5 - 6 + \ldots = 1/4$$

$$A - B = \begin{matrix} 1 - 1 + 1 - 1 + \ldots \\ -(1 - 2 + 3 - 4 + \ldots) \end{matrix}$$

$$= 1 - 2 + 3 - 4 + \ldots = B \implies B = \tfrac{1}{2}A = \tfrac{1}{4}$$

$C = 1+2+3+4+5+\dots = -1/12$

$$C - B = \cancel{1}+2+\cancel{3}+4+\cancel{5}+6+\dots - (\cancel{1}-2+\cancel{3}-4+\cancel{5}-6+\dots)$$

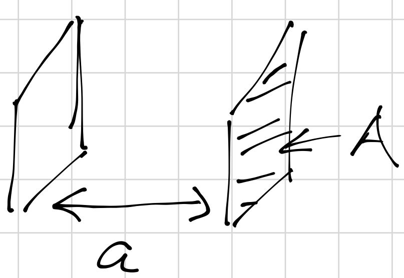
$$= 4 + 8 + 12 + \dots = 4(1+2+3+\dots) = 4C$$

$$C = -\frac{1}{3}B = -\frac{1}{12}$$

$$E(a) = -\frac{\pi}{24a} \longrightarrow F(a) = -\frac{\pi}{24a^2}$$

3D + 2 pol. of the γ

$$F(a) = -\frac{\pi \hbar c}{240 a^4} A$$



$$E(a) = \frac{1}{2}\sum_n \omega_n(a)$$

Add a regulator $f(n)$: suppress higher modes

$$\omega_n = \frac{\pi}{a} n$$

$$E(a) = \frac{1}{2}\sum_n \left(\frac{\omega_n}{\mu}\right)^{-s} \cdot \omega_n = \frac{1}{2}\left(\frac{\pi}{a}\right)^{1-s} \mu^s \underbrace{\sum_n \frac{1}{n^{s-1}}}_{\text{Riemann zeta-fn. } \zeta(s-1)}$$

If $s=0$ $\longrightarrow$ we recover the original sum $\sum_n n$

However, $\zeta(-1) = -\frac{1}{12}$

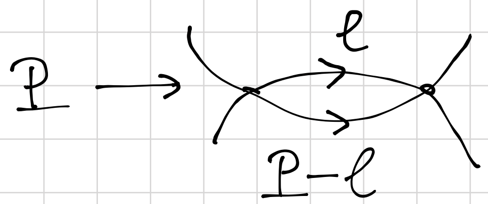
↪ take limit $s \to 0$ $\quad E(a) = -\frac{\pi}{24a}$

Chapter 15 of Schwartz "Casimir effect"

For any reg. fn. $f(n)$ the result for the Casimir force is the same!

We got a finite number by subtracting ∞.

1-loop in ϕ^4



$$i\mathcal{M}^{(2)} = \frac{\lambda^2}{2}\int \frac{d^4\ell}{(2\pi)^4}\frac{1}{\ell^2-m^2}\frac{1}{(P-\ell)^2-m^2}$$

$$2\,\mathrm{Im}\,\mathcal{M}^{(2)} = \frac{\lambda^2}{2}\int\frac{d^4\ell}{(2\pi)^4}\,2\pi\,\delta(\ell^2-m^2)\,2\pi\,\delta((P-\ell)^2-m^2)\,\theta(\ell^0)\,\theta(P^0-\ell^0)$$

$P^2 = s$

$P^0 = \sqrt{s}$

$\vec{P} = 0$

$$\Rightarrow \mathrm{Im}\,\mathcal{M}^{(2)} = \frac{\lambda^2}{32\pi}\sqrt{1-\frac{4m^2}{s}}\,\theta(s-4m^2)$$

DR

$$\mathcal{M}^{(2)} = \frac{1}{\pi}\int_{4m^2}^{\infty}\frac{ds'}{s'-s-i\varepsilon}\,\mathrm{Im}\,\mathcal{M}^{(2)}(s')$$

$$\mathcal{M}^{(2)}(\Lambda) = \frac{1}{\pi}\int_{4m^2}^{\Lambda^2}\frac{ds'}{s'-s-i\varepsilon}\,\mathrm{Im}\,\mathcal{M}^{(2)}(s')$$

$$= +\frac{\lambda^2}{32\pi^2}\ln\left(\frac{\Lambda^2}{s}\right)$$

$\Lambda\to\infty \implies \mathcal{M}^{(2)}(\Lambda)$ diverges log.

$$\mathcal{M}^{(2)}(s_1) - \mathcal{M}^{(2)}(s_2) = \frac{\lambda^2}{32\pi^2}\ln\left(\frac{s_2}{s_1}\right)$$

$$\frac{d\sigma}{d\Omega} \sim |\mathcal{M}^{(n+2)}|^2 \sim \underline{f(\lambda)}$$

$$A\left(\frac{\lambda}{f_a}\right) \qquad |A| \leq A^{\text{exp.}}$$

$$\hookrightarrow \left|\frac{\lambda}{f_a}\right| \leq \dots$$

We measure

$$i\mathcal{M} \qquad -i\lambda \qquad -\frac{i\lambda^2}{32\pi^2} \ln s + c_\infty$$

$$c_\infty = \frac{i\lambda^2}{32\pi^2} \ln \Lambda^2$$

$$\downarrow$$

Define renormalized coupling λ_R

$$\underline{\lambda_R} \equiv -\mathcal{M}(s_0) = \lambda + \frac{\lambda^2}{32\pi^2} \ln \frac{s_0}{\Lambda^2} + \dots$$

$$\lambda = \lambda_R + \underset{\sim}{a}\, \lambda_R^2 + \dots$$

$$\lambda_R = (\lambda_R + a\lambda_R^2 + \dots) + \frac{(\lambda_R + a\lambda_R^2 + \dots)^2}{32\pi^2} \ln \frac{s_0}{\Lambda^2} + \dots$$

$$\rightarrow a = -\frac{1}{32\pi^2} \ln \frac{s_0}{\Lambda^2}$$

$$\lambda = \lambda_R - \frac{\lambda_R^2}{32\pi^2} \ln \frac{s_0}{\Lambda^2}$$

$$\Downarrow$$

$$\mathcal{M}(s) = -\lambda - \frac{\lambda^2}{32\pi^2} \frac{s}{\Lambda^2} + \dots$$

$$= -\lambda_R + \frac{\lambda_R^2}{32\pi^2} \ln \frac{s_0}{\Lambda^2} + \dots$$

$$- \frac{\lambda_R^2}{32\pi^2} \ln \frac{s}{\Lambda^2} + \dots$$

$$= -\lambda_R - \frac{\lambda_R^2}{32\pi^2} \ln \frac{s}{s_0} + \dots$$

$$\underline{\lambda_R \doteq -\mathcal{M}(s_0)}$$

<u>Again</u>: what we did is subtracting the ∞

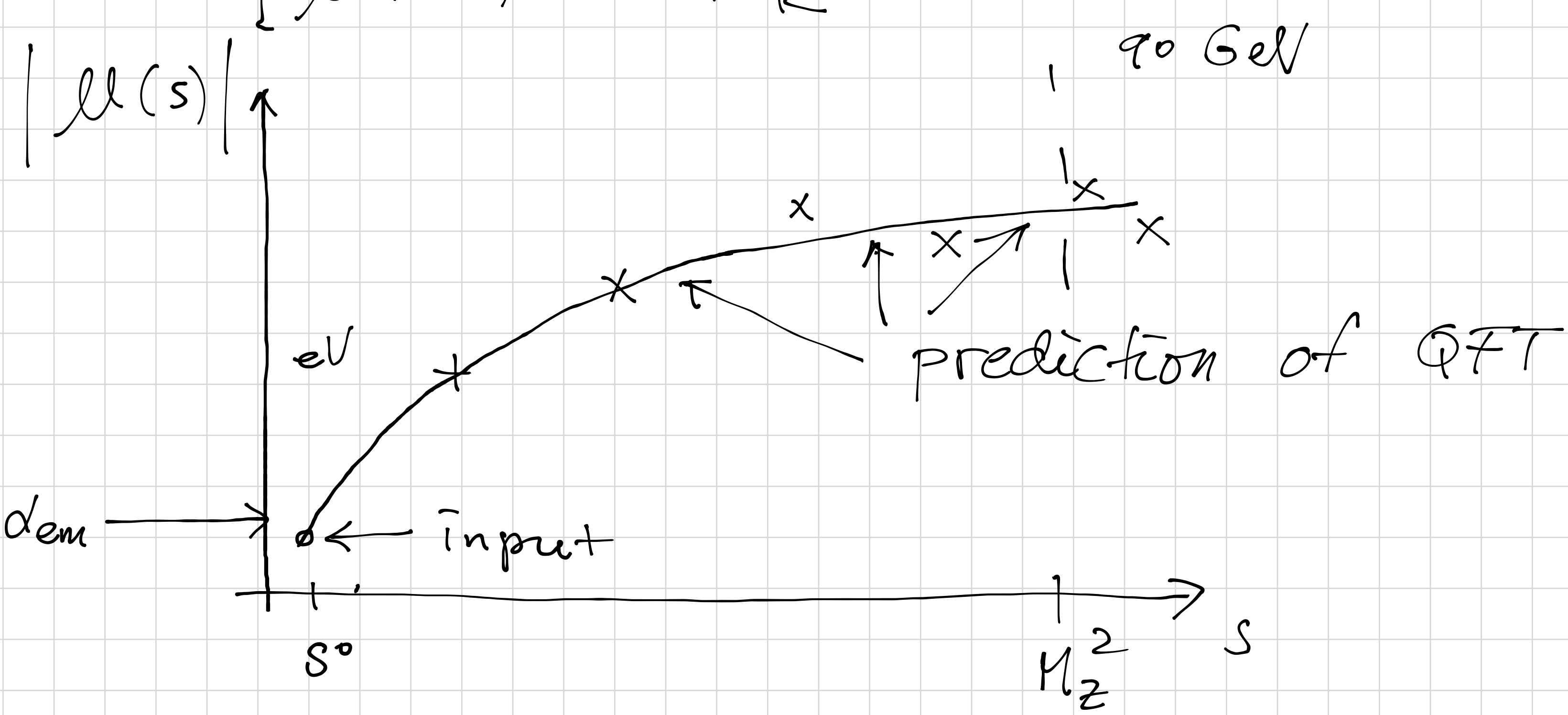
Counterterm

$$\mathcal{L} = -\frac{1}{2}\phi\Box\phi - \frac{\lambda_R}{4!}\phi^4 - \frac{\delta_\lambda}{4!}\phi^4$$

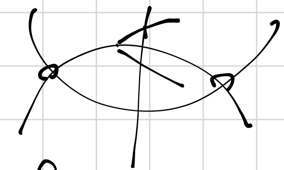
$$\mathcal{M}(s) = -\lambda_R - \delta_\lambda - \frac{\lambda_R^2}{32\pi^2} \ln \frac{s}{\Lambda^2} + \dots$$

$$\delta_\lambda = -\frac{\lambda_R^2}{32\pi^2} \ln \frac{s_0}{\Lambda^2}$$

$$\hookrightarrow \begin{cases} \mathcal{M}(s) = -\lambda_R - \frac{\lambda_R^2}{32\pi^2} \ln \frac{s}{s_0} + \dots \\ \mathcal{M}(s_0) = -\lambda_R \end{cases}$$



Measuring this curve $\longrightarrow$ substance of precision tests of Standard Model

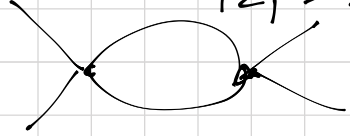
In the example 

The result depends on λ

on the spectrum

If different types of part.

$\phi_{1,2,3} \ldots$



$\phi_{1,2,3} \ldots$

If you find a deviation $\underline{\mathcal{M}^{exp}(s) \neq \mathcal{M}^{th.}(s)}$

Possibility: the spectrum $\phi_{1,2,3} \ldots$ incomplete

↳ add a new particle ϕ_{n+1}:

$$\mathcal{M}^{exp} = \mathcal{M}^{th}$$

Example: t-quark $m_t \cong 170$ GeV

Predicted from a comparison of PT to exp. corrections $\sim \ln\left(\frac{m_t^2}{s}\right)$

Example 2: <u>Higgs boson</u> $m_H \cong 125$ GeV

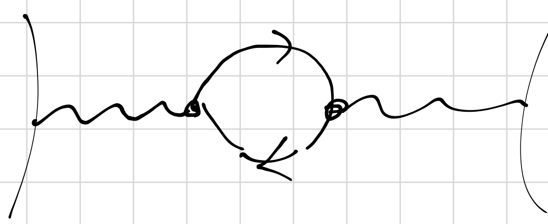
LHC 2012

↳ corrections tell you how obs. depend on masses

↳ global fits ⟶ determine parameters

Higgs mass <u>had</u> to be in a rel. narrow window around 125 GeV

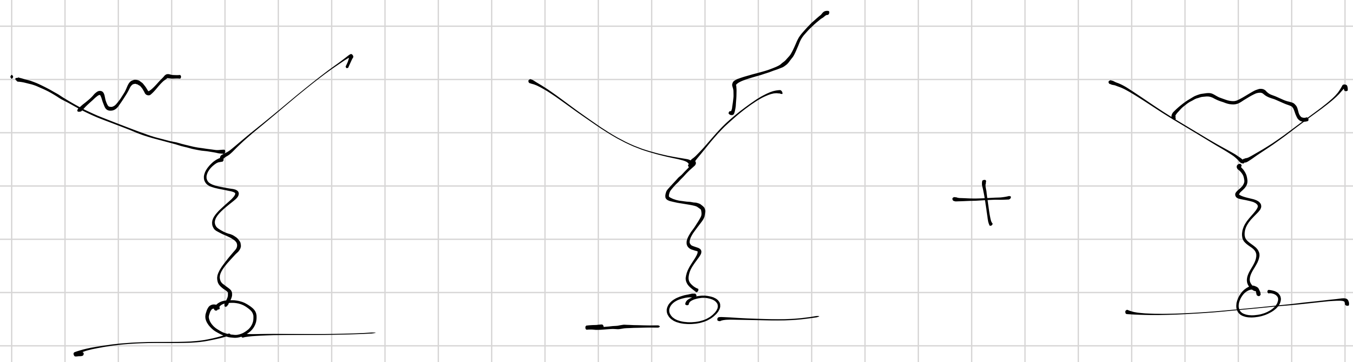
$\longrightarrow$ Vacuum polarization

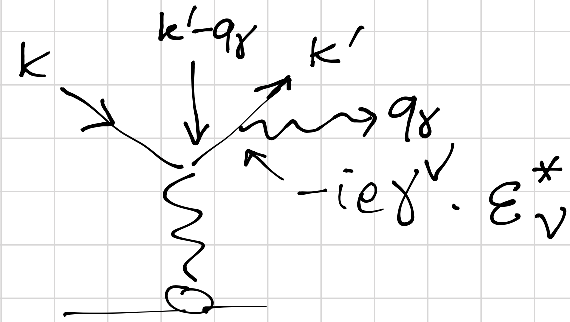


$\longrightarrow \alpha_{em}(Q^2)$

UV div. (high modes)

$\longrightarrow$ IR div. $\longrightarrow$ γ's are massless





$\mathcal{M}^{(0)}(e+p \longrightarrow e+p)$

$$= -\frac{e^2}{t}\,\bar{u}(k')\gamma^\mu u(k)\cdot J_\mu$$

Consider $\mathcal{M}^{(1)}(e+p \longrightarrow e+p+\gamma)$

$\hookrightarrow$ carrées away energy

$$\mathcal{M}^{(1)} = -\frac{e^2}{t}\,\bar{u}(k')\,\frac{\not{\varepsilon}^*(q_\gamma)(\not{k}'-\not{q}_\gamma+m_e)\gamma^\mu}{(k'-q_\gamma)^2-m_e^2}\,u(k)\cdot J_\mu$$

Study the soft limit: $q_\gamma \to 0$

$$\underbrace{(k'-q_\gamma)^2-m_e^2} = -2k'q_\gamma \sim \underset{\sim}{\omega_\gamma}$$

✓