$\Rightarrow$ # VORLESUNG 20 QM

EPR PARADOX & BELL UNGLEICHUNGEN

KL $\qquad x, p$

QM $\qquad$ WAHRSCHEINLICHKEITEN

$$\rightsquigarrow \quad |\psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle + |\downarrow\rangle\right)$$

$$P_{\Uparrow} = \frac{1}{2}$$

$$P_{\Downarrow} = \frac{1}{2}$$

$\Rightarrow$ EINSTEIN, PODOLSKY, ROSEN (1935)

QUANTUM UNBESTIMMTHEIT

D. BOHM (1952)

$$\pi^0 \longrightarrow e^- e^+ \qquad m_\pi \simeq 135 \text{ MeV}$$

SPIN 0 $\qquad \uparrow \; \uparrow$ SPIN 1/2 $\qquad m_{e^-} = m_{e^+} \simeq 0.5$ MeV

$$e^- \longleftarrow \underset{\pi^0}{\bullet} \longrightarrow e^+$$

$$\frac{1}{2} \oplus \frac{1}{2} \begin{array}{l} \nearrow \; 0 \;\Longleftarrow \\ \searrow \; \underline{1} \end{array}$$

$$|e^- e^+ \rangle = \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) \Longleftarrow$$

VERSCHRÄNKTER ZUSTAND

(ENTANGLED)

$$|\psi_{12}\rangle \neq \underset{e^-}{\underset{\uparrow}{|\psi_1\rangle}} \; \underset{e^+}{\underset{\uparrow}{|\psi_2\rangle}}$$

$$|\psi_1\rangle = a_1 |\uparrow\rangle + b_1 |\downarrow\rangle$$

$$|\psi_2\rangle = a_2 |\uparrow\rangle + b_2 |\downarrow\rangle$$

$$|\psi_1\rangle |\psi_2\rangle = a_1 a_2 |\uparrow\uparrow\rangle + b_1 b_2 |\downarrow\downarrow\rangle$$
$$+ a_1 b_2 |\uparrow\downarrow\rangle + b_1 a_2 |\downarrow\uparrow\rangle$$

$$a_1 a_2 = b_1 b_2 = 0$$

$$a_1 b_2 = -b_1 a_2 = \frac{1}{\sqrt{2}}$$

$$a_1 = 0 \longrightarrow a_1 b_2 \neq \frac{1}{\sqrt{2}}$$

$$a_2 = 0 \longrightarrow b_1 a_2 \neq -\frac{1}{\sqrt{2}}$$

$$|\psi_{12}\rangle \neq |\psi_1\rangle|\psi_2\rangle$$

ALICE $e^- \longleftarrow \underset{\pi^0}{\bullet} \longrightarrow e^+$ BOB

$$P_{\Uparrow} = \frac{1}{2} \qquad\qquad P_{\Uparrow} = \frac{1}{2}$$

$$P_{\Downarrow} = \frac{1}{2} \qquad\qquad P_{\Downarrow} = \frac{1}{2}$$

A $\quad e^- \uparrow \qquad\qquad e^+ \downarrow$

$\quad e^- \downarrow \qquad\qquad e^+ \uparrow$

EINSTEIN : LOKALITÄT

QM : (SPUKHAFTE FERNWIRKUNGEN)

$\downarrow$

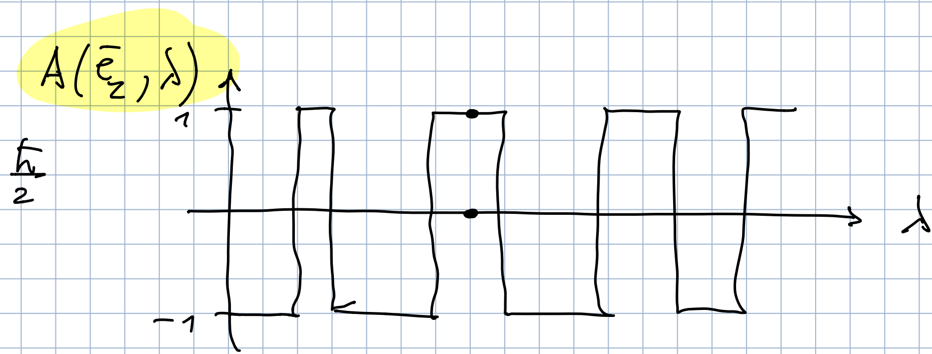
QM IST NICHT VOLLSTÄNDIG

(INCOMPLETE)

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

$\nearrow$ <u>LOKAL</u>

$\searrow$ VERBORGENE VARIABLEN (λ)



$\Longrightarrow$ BELL'S UNGLEICHUNGEN

J. BELL (1964)

- <u>EPR EXPERIMENT</u>

$$\langle S=0 \mid \sigma_z^{(1)} \, \sigma_z^{(2)} \mid S=0 \rangle$$

$$|S=0\rangle = \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$

$$= \frac{1}{2} \langle \uparrow\downarrow - \downarrow\uparrow \mid \sigma_z^{(1)} \, \sigma_z^{(2)} \mid \uparrow\downarrow - \downarrow\uparrow \rangle$$

$$= \frac{1}{2} \underbrace{\langle \uparrow\downarrow \mid \sigma_z^{(1)} \, \sigma_z^{(2)} \mid \uparrow\downarrow \rangle}_{-1}$$

$$+\frac{1}{2}\underbrace{\langle\downarrow\uparrow|\;\ldots\;|\downarrow\uparrow\rangle}_{-1}$$

$$-\frac{1}{2}\underbrace{\langle\uparrow\downarrow|\;\ldots\;|\downarrow\uparrow\rangle}_{0}$$

$$-\frac{1}{2}\underbrace{\langle\downarrow\uparrow|\;\ldots\;|\uparrow\downarrow\rangle}_{0}$$

$$\langle\uparrow|\sigma_z^{(1)}|\uparrow\rangle$$

$$=(1\ \ 0)\begin{pmatrix}1&0\\0&-1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}=1$$

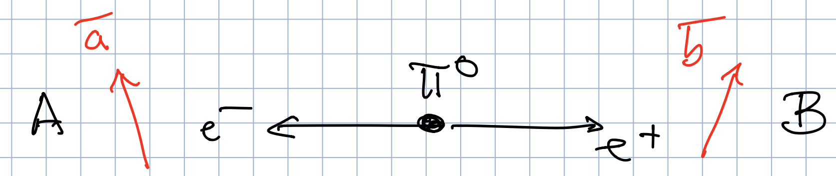
$$\langle\downarrow|\sigma_z^{(1)}|\downarrow\rangle=-1$$

$$\langle\uparrow|\sigma_z^{(1)}|\downarrow\rangle=0$$

$$(1\ \ 0)\begin{pmatrix}1&\\&-1\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}=0$$

$$\langle S=0|\sigma_z^{(1)}\sigma_z^{(2)}|S=0\rangle=-1$$

• VERALLGEMEINERUNG EPR EXP.



$$P(\vec{a},\vec{b}) = \langle S=0 | (\vec{\sigma}^{(1)} \cdot \vec{a})(\vec{\sigma}^{(2)} \cdot \vec{b}) | S=0 \rangle$$

VORHER $\vec{a} = \vec{e}_z$

$\vec{b} = \vec{e}_z$

$$= \frac{1}{2} \langle \uparrow\downarrow | \vec{\sigma}^{(1)} \cdot \vec{a} \quad \vec{\sigma}^{(2)} \cdot b | \uparrow\downarrow \rangle$$

$$+ \frac{1}{2} \langle \downarrow\uparrow | \quad \ldots \quad | \downarrow\uparrow \rangle$$

$$- \frac{1}{2} \langle \uparrow\downarrow | \quad \ldots \quad | \downarrow\uparrow \rangle$$

$$- \frac{1}{2} \langle \downarrow\uparrow | \quad \ldots \quad | \uparrow\downarrow \rangle$$

$$\vec{\sigma}^{(1)} \cdot a \quad \vec{\sigma}^{(2)} \cdot b$$

$$= \left(a_x \sigma_x^{(1)} + a_y \sigma_y^{(1)} + a_z \sigma_z^{(1)} \right)$$

$$\cdot \left(b_x \sigma_x^{(2)} + b_y \sigma_y^{(2)} + b_z \sigma_z^{(2)} \right)$$

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_\pm = \sigma_x \pm i\,\sigma_y \quad \Rightarrow \quad \sigma_x = \frac{1}{2}\left(\sigma_+ + \sigma_-\right)$$

$$\sigma_y = \frac{1}{2i}\left(\sigma_+ - \sigma_-\right)$$

$$\sigma_+ = \begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix}$$

$$\sigma_- = \begin{pmatrix} 0 & 0 \\ 2 & 0 \end{pmatrix}$$

$$\vec{\sigma}^{(1)} \cdot \vec{a} \qquad \vec{\sigma}^{(1)} \cdot \vec{b}$$

$$= \left[\frac{1}{2}\left(a_x - i a_y\right) \sigma_+^{(1)} + \frac{1}{2}\left(a_x + i a_y\right)\sigma_-^{(1)} + a_z \sigma_z^{(1)}\right]$$

$$\cdot \left[\frac{1}{2}\left(b_x - i b_y\right) \sigma_+^{(2)} + \frac{1}{2}\left(b_x + i b_y\right)\sigma_-^{(2)} + b_z \sigma_z^{(2)}\right]$$

$$\left\langle \substack{\uparrow \\ \downarrow} \right| \sigma_+^{(1)} \left| \substack{\uparrow \\ \downarrow} \right\rangle = 0$$

$$\langle S=0 \,|\, \vec{\sigma}^{(1)}\cdot \vec{a} \quad \vec{\sigma}^{(2)}\cdot \vec{b} \,|\, S=0 \rangle$$

$$= \frac{1}{2}\Big\{ a_z b_z (-1) + a_z b_z (-1)$$

$$\rightsquigarrow \quad - \frac{1}{\cancel{2}}(a_x - i a_y)(\cancel{2})\, \frac{1}{\cancel{2}}(b_x + i b_y)\, \cancel{2}$$

$$- \frac{1}{\cancel{2}}(a_x + i a_y)(\cancel{2})\, \frac{1}{\cancel{2}}(b_x - i b_y)\, \cancel{2} \Big\}$$

$$\langle \downarrow | \sigma_+^{(1)} | \uparrow \rangle = 0$$

$$\begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = 2$$

$$\langle \uparrow | \sigma_+^{(1)} | \downarrow \rangle = 2$$

$$\langle \uparrow | \sigma_-^{(1)} | \downarrow \rangle = 0$$

$$\langle \downarrow | \sigma_-^{(1)} | \uparrow \rangle = 2$$

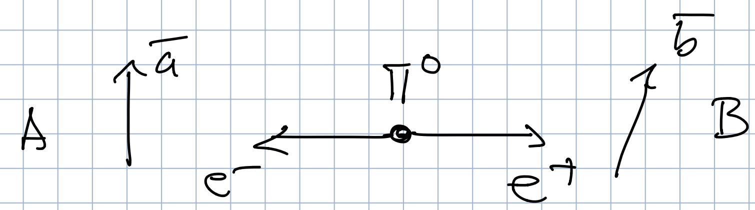
$$\therefore \quad \langle S=0 \,|\, \vec{\sigma}^{(1)}\cdot \vec{a} \quad \vec{\sigma}^{(2)}\cdot \vec{b} \,|\, S=0 \rangle$$

$$= \frac{1}{2}\left(-2 a_z b_z - 2 a_x b_x - 2 a_y b_y \right)$$

$$= - \vec{a}\cdot\vec{b} \qquad (\mathrm{QM})$$

$$\bar{a} = \bar{b} = \bar{e}_z \curvearrowright -1$$

- LOKALEN VERBORGENE VARIABLEN THEORIE



$$A(\bar{a}, \lambda) = \pm 1$$

$$B(\bar{b}, \lambda) = \pm 1$$

$$A(\bar{a}, \cancel{\bar{b}}, \lambda)$$

$$B(\cancel{\bar{a}}, \bar{b}, \lambda)$$

LOKALITÄT !

$$P(\lambda) > 0$$

$$\int d\lambda \, P(\lambda) = 1$$

<u>ALICE</u> $\langle S=0 | \bar{\sigma}^{(1)} \cdot \bar{a} | S=0 \rangle$ QM

$$= 0$$

$$= \int d\lambda \, P(\lambda) \; A(\bar{a}, \lambda)$$

BOB $\langle S=0 | \bar{\sigma}^{(2)} \cdot \bar{b} | S=0 \rangle$

$$= 0$$

$$= \int d\lambda \; P(\lambda) \; B(\bar{b}, \lambda)$$

$\hookrightarrow \quad \bar{b} = \bar{a}$

in $S=0 \quad A(\bar{a}, \lambda) = - B(\bar{a}, \lambda)$

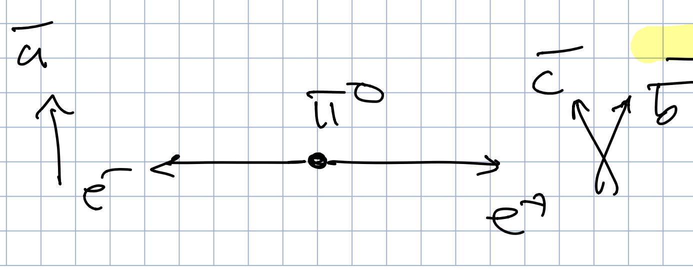
$\uparrow \downarrow$

$\downarrow \uparrow$

$$P(\bar{a}, \bar{b}) \underset{QM}{\equiv} \langle S=0 | (\bar{\sigma}^{(1)} \cdot \bar{a}) (\bar{\sigma}^{(2)} \cdot \bar{b}) | S=0 \rangle$$

$$= - \bar{a} \cdot \bar{b}$$

$$P(\bar{a}, \bar{b}) = \int d\lambda \; P(\lambda) \; A(\bar{a}, \lambda) \; \underbrace{B(\bar{b}, \lambda)}_{- A(\bar{b}, \lambda)}$$



$$P(\bar{a}, \bar{c}) = -\int d\lambda \; P(\lambda)\, A(\bar{a}, \lambda)\, A(\bar{c}, \lambda)$$

$$P(\bar{a}, \bar{b}) - P(\bar{a}, \bar{c})$$

$$= -\int d\lambda \; P(\lambda)\, A(\bar{a}, \lambda) \left[A(\bar{b}, \lambda) - A(\bar{c}, \lambda) \right]$$

$$A^2(\bar{b}, \lambda) = 1$$

$$= -\int d\lambda \; P(\lambda)\, A(\bar{a}, \lambda)\, A(\bar{b}, \lambda) \cdot \left[1 - A(\bar{b}, \lambda)\, A(\bar{c}, \lambda) \right]$$

$$-1 \leq A(\bar{a}, \lambda)\, A(\bar{b}, \lambda) \leq 1$$

$$\left| P(\bar{a}, \bar{b}) - P(\bar{a}, \bar{c}) \right|$$

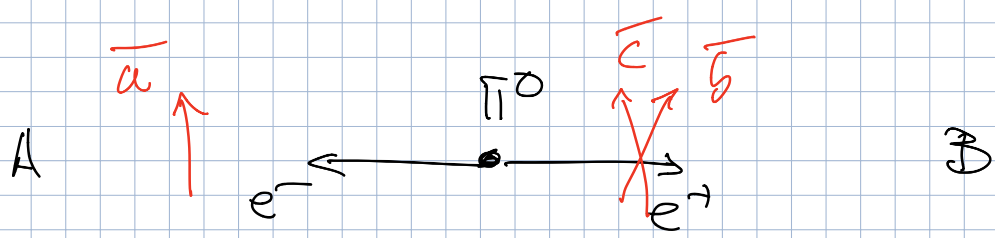
$$\leq \int d\lambda \; P(\lambda) \left[1 - A(\bar{b}, \lambda)\, A(\bar{c}, \lambda) \right]$$

$$\int d\lambda \; P(\lambda) = 1$$

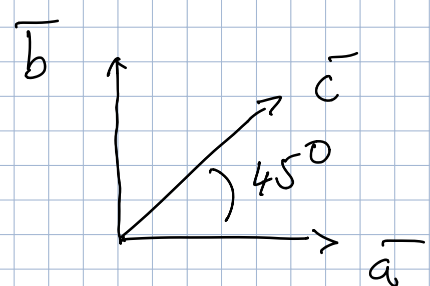
$$= 1 - \int d\lambda \, P(\lambda) \, A(\bar{b}, \lambda) \, A(\bar{c}, \lambda)$$

$$= 1 + P(\bar{b}, \bar{c})$$

$$\rightarrow \boxed{\left| P(\bar{a}, \bar{b}) - P(\bar{a}, \bar{c}) \right| \leq 1 + P(\bar{b}, \bar{c})} \qquad \text{(BELL)}$$



$\underline{\text{QM}}$ $\qquad P(\bar{a}, \bar{b}) = -\bar{a} \cdot \bar{b}$



$$P(\bar{a}, \bar{b}) = 0$$

$$P(\bar{a}, \bar{c}) = -\frac{1}{\sqrt{2}}$$

$$P(\bar{b}, \bar{c}) = -\frac{1}{\sqrt{2}}$$

$$\left| 0 - \frac{1}{\sqrt{2}} \right| \leq 1 - \frac{1}{\sqrt{2}}$$

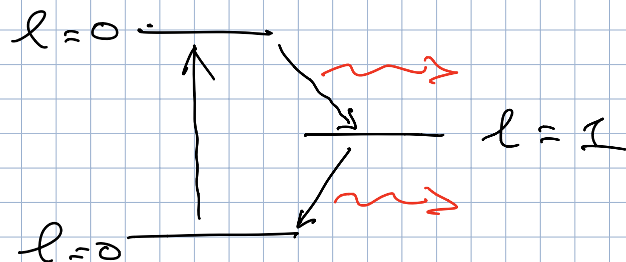
$$0.707 \not\leq 0.293$$

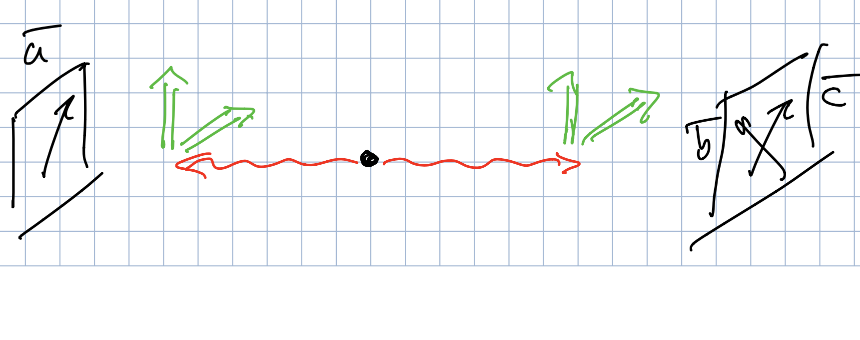
QM

EXPERIMENT

1982 → ASPECT
2015

Ca





$$|2\gamma\rangle = \frac{1}{\sqrt{2}}\left\{ |\Uparrow\Uparrow\rangle + |\Rightarrow\Rightarrow\rangle \right\}$$

~~BELL UNGL~~

QM !

LOKALE VERBORGENE VARIABLEN THEORIE

IS **NICHT** KOMPATIBEL MIT EXP.

QM **IST** KOMPATIBEL MIT EXP !