$\Longrightarrow$ VORLESUNG 18 QM

H-ATOM $\quad R(r) = \dfrac{u(r)}{r}$ $\quad \rho = \kappa r$

$$E = -\frac{\hbar^2}{2m}\kappa^2$$

$$u(\rho) = \rho^{\ell+1} e^{-\rho} v(\rho)$$

$$v(\rho) = \sum_{j=0}^{j_{max}} c_j \rho^j$$

$$\boxed{c_{j+1} = \frac{\left[2(j+\ell+1) - \rho_0\right]}{(j+1)(j+2\ell+2)} c_j}$$

- $j_{max} \Longrightarrow c_{j_{max}+1} = 0$

$$2(\underbrace{j_{max}+\ell+1}_{n}) = \rho_0$$

$$n = 1, 2, \ldots$$

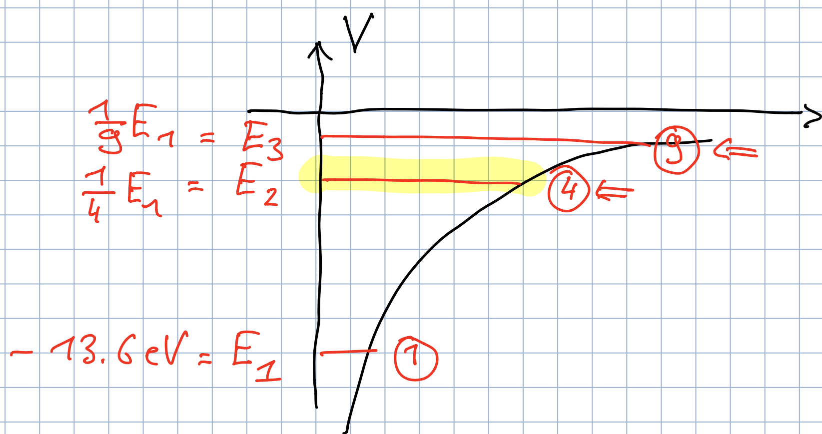
$$\kappa = \frac{1}{an}$$

BOHR RADIUS $\quad a = \dfrac{1}{m\alpha}\dfrac{\hbar c}{c^2} \simeq 0.5\, 10^{-10}\, m$

$$E_n = -\frac{E_1}{n^2} \qquad n = 1, 2, \dots$$

$$E_1 = \left(\frac{1}{2} m c^2\right) \alpha^2 \qquad \alpha = \frac{1}{137}$$

$$\approx 13.6 \quad eV$$



- **GRUNDZUSTAND** $\quad n = 1$

$$n = j_{max} + l + 1$$

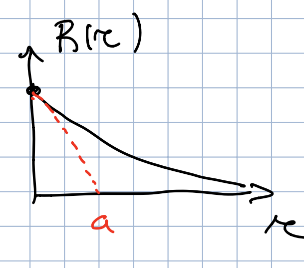
$$j_{max} = 0 \;, \quad l = 0$$

$$v(\rho) = c_0$$

$$u(\rho) = c_0\, \rho^1 e^{-\rho}$$

$$\rho = Kr = \frac{1}{an} r = \frac{r}{a}$$

$$R(r) = \frac{1}{\cancel{r}} C_0 \left(\frac{\cancel{r}}{a}\right) e^{-r/a}$$



$$R(r) = \frac{C_0}{a} e^{-r/a}$$

NORMIERUNG

$$\int_0^\infty dr\, r^2 \, |R(r)|^2 = 1$$

$$1 = \frac{C_0^2}{a^2} \int_0^\infty dr\, r^2 \, e^{-2r/a}$$

$x = 2r/a$

$$= \frac{C_0^2}{a^2} \frac{a^3}{8} \underbrace{\int_0^\infty dx\, x^2 \, e^{-x}}_{2}$$

$$= C_0^2 \frac{a}{4}$$

$$C_0 = \frac{2}{\sqrt{a}} \longrightarrow R_{10}(r) = \frac{2}{a\sqrt{a}} e^{-r/a}$$

$$\psi_{n\ell m}(\bar{r}) = R_{n\ell}(r)\, Y_{\ell m}(\theta, \phi)$$

$n = 1\,,\ \ell = 0,\ m = 0$

$$= R_{10}(r)\, \underbrace{Y_{00}(\theta, \phi)}_{\frac{1}{\sqrt{4\pi}}}$$

$$\boxed{\psi_{100}(\bar{r}) = \frac{1}{\sqrt{4\pi}}\, \frac{2}{a\sqrt{a}}\, e^{-r/a}}$$

- 1° ANREGUNG $\quad n = 2$

$$n = j_{max} + \ell + 1 = 2$$

$\nearrow \quad \ell = 0 \qquad j_{max} = 1$

$\searrow \quad \ell = 1 \qquad j_{max} = 0$

$\rightsquigarrow \ell = 0$

$$v(\rho) = c_0 + c_1 \rho$$

$$c_{j+1} = \frac{\left[2(j+\ell+1) - 4\right]}{(j+1)\,(j+2\ell+2)}\, c_j$$

$$\rho_0 = 2n$$

$$c_1 = \frac{2(1) - 4}{1\cdot(2)}\, c_0 = -c_0$$

$$v(\rho) = c_0\,(1-\rho)$$

$$\Big\downarrow \qquad \rho = K r = \frac{1}{an}\, r$$

$$\rightarrow \quad v(\rho) = c_0\left(1 - \frac{r}{2a}\right), \quad \rho = \underset{\sim\sim\sim}{\left(\frac{r}{2a}\right)}$$

$$u(\rho) = \rho\, e^{-\rho}\, v(\rho)$$

$$= \left(\frac{r}{2a}\right) e^{-r/2a}\, c_0\left(1 - \frac{r}{2a}\right)$$

$$R_{\underset{\uparrow\, n}{2}\,\underset{\uparrow\, \ell}{0}}(r) = \frac{c_0}{2a}\left(1 - \frac{r}{2a}\right) e^{-\frac{r}{2a}}$$

$$C_0 = \frac{2}{\sqrt{2}\sqrt{a}}$$

$$R_{20}(0) = \frac{C_0}{2a}$$

$\ell = 0$

$$\leadsto \ell = 1 \qquad j_{max} = 0$$

$$v(\rho) = c_0$$

$$u(\rho) = \rho^2 \, e^{-\rho} \, c_0$$

$$\rho = kr = \left(\frac{1}{an}\right) r = \frac{r}{2a}$$

$$R_{21}(r) = \frac{1}{r}\left(\frac{r}{2a}\right)^2 e^{-r/2a} \, c_0$$

n ℓ

$$R_{21}(r) = \frac{C_0}{4a^2} \, r \, e^{-r/2a}$$

$$R_{n\ell}(r) \xrightarrow[r \ll]{} r^{\ell}$$

- BELIEBIGES n

$$n = j_{max} + \ell + 1$$

$$\rightsquigarrow \quad \ell = n - 1 - j_{max}$$

$$\ell = 0, \dots, n-1$$

$n = 1 \qquad \ell = 0$

$n = 2 \qquad \ell = 0, 1$

$n = 3 \qquad \ell = 0, 1, 2$

$$\forall \ell \qquad m = -\ell, \dots, +\ell$$

$$\underbrace{2\ell + 1}$$

$$\psi_{n\ell m} \rightsquigarrow E_n$$

ENTARTUNG

FÜR $n \quad \sum_{\ell=0}^{n-1} (2\ell+1) = n^2$

$n = 1 \qquad \ell = 0 \qquad m = 0 \qquad\qquad 1$

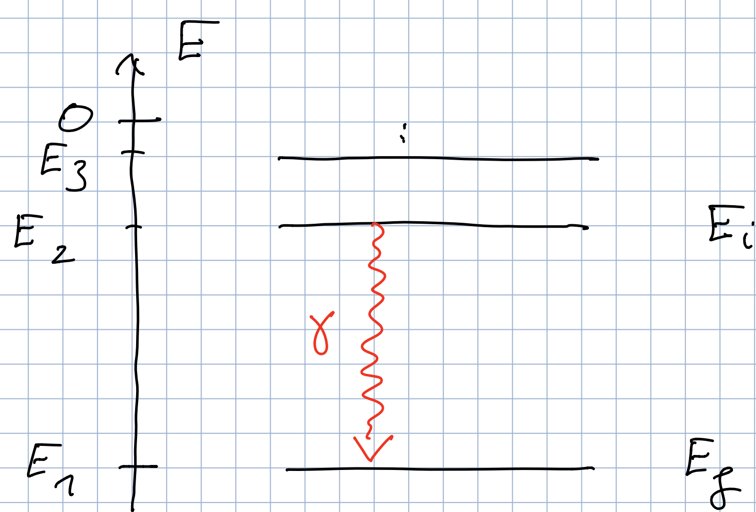
$n = 2 \qquad \left.\begin{array}{lll} \ell = 0 & m = 0 & 1 \\ \ell = 1 & m = -1, 0, 1 & 3 \end{array}\right\} \quad 4$

$n = 3 \qquad \left.\begin{array}{ll} \ell = 0 & 1 \\ \ell = 1 & 3 \\ \ell = 2 & 5 \end{array}\right\} \quad 9$

$\vdots$

$v(\rho) \Rightarrow$ ASSOZIERTE LAGUERRE POLY.

$\Rightarrow$ SPEKTRUM DES H-ATOM



$$E_\gamma = E_i - E_f$$

$$= (-13.6)\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right)$$

$$E_\gamma = h\nu = \hbar\omega$$

$$\frac{1}{\lambda} = \frac{\nu}{c} = \frac{h\nu}{hc} = \frac{E_\gamma}{2\pi\hbar c}$$

$$\frac{1}{\lambda} = \underbrace{\frac{(-E_1)}{2\pi\hbar c}}_{R} \left(\frac{1}{n_i^2} - \frac{1}{n_f^2} \right)$$

(RYDBERG)

$$-E_1 = \left(\frac{1}{2} m c^2\right) \alpha^2$$

$$R \simeq 1.1 \quad 10^7 \quad m^{-1}$$

$\Longrightarrow$ SPIN

- ORBITAL $\bar{L}$

EIGENZUSTÄNDE $\begin{array}{cc} L^2, & L_z \\ \downarrow & \downarrow \\ \ell & m \end{array}$

$$\langle \bar{r} \,|\, \ell m \rangle = Y_{\ell m}(\hat{r})$$

$$\begin{cases} L^2 \,|\ell m\rangle = \hbar^2 \ell(\ell+1) \,|\ell m\rangle \\ L_z \,|\ell m\rangle = \hbar m \,|\ell m\rangle \end{cases}$$

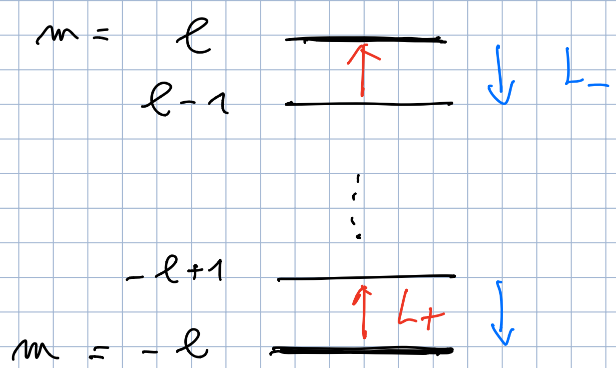
$$\ell = 0, 1, 2, \ldots$$

$$m = -\ell, \ldots, +\ell$$

$$L_\pm = L_x \pm i L_y$$

$$L_\pm \,|\ell m\rangle = C_\pm \,|\ell\; m \pm 1\rangle$$

$$C_\pm = \hbar \sqrt{\ell(\ell+1) - m(m \pm 1)}$$



$2\ell + 1$ ZUSTÄNDE

2ℓ SCHRITTE

$$2\ell \in \mathbb{N}$$

$$\ell = 0, \ \frac{1}{2}, \ 1, \ \frac{3}{2}, \ldots$$

INTRINSISCHER SPIN

SICHTBARE MATERIE

SPIN $\frac{1}{2}$

LEPTONEN $\quad e^-, \mu^-, \tau^-$

ν_e, ν_μ, ν_τ

QUARKS $\quad u, d, s, c, b, t$

p, n

$p : uud$

$n : ddu$

- SPIN 1/2

$$S = \frac{1}{2} \qquad S_z = \pm \frac{\hbar}{2}$$

$$S_z = +\frac{\hbar}{2}$$

$$S_z = -\frac{\hbar}{2}$$

$$|S\,S_z\rangle \rightarrow \left|\frac{1}{2}, +\frac{1}{2}\right\rangle \; , \; \left|\frac{1}{2}, -\frac{1}{2}\right\rangle$$

$$\begin{cases} S^2 \left|\frac{1}{2} \pm \frac{1}{2}\right\rangle = \hbar^2 \frac{3}{4} \left|\frac{1}{2} \pm \frac{1}{2}\right\rangle \\ S_z \left|\frac{1}{2} \pm \frac{1}{2}\right\rangle = \pm \frac{\hbar}{2} \left|\frac{1}{2} \pm \frac{1}{2}\right\rangle \end{cases}$$

$$\left|\frac{1}{2} + \frac{1}{2}\right\rangle \longleftrightarrow \chi_\uparrow = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$ SPIN UP

$$\left|\frac{1}{2} - \frac{1}{2}\right\rangle \longleftrightarrow \chi_\downarrow = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$ SPIN DOWN

$$\chi^\dagger \chi = \underline{1}$$

S^2, S_z IN MATRIX DARSTELLUNG ?

$$\hookrightarrow \quad \underbrace{S^2}_{2\times 2} \underbrace{\chi_\uparrow}_{2\times 1} = \hbar^2 \frac{3}{4} \chi_\uparrow$$

$$S^2 = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$S^2 \chi_\uparrow = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} a \\ c \end{pmatrix}$$

$$= \hbar^2 \frac{3}{4} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$c = 0 \quad , \quad a = \hbar^2 \frac{3}{4}$$

$$S^2 \chi_\downarrow = \hbar^2 \frac{3}{4} \chi_\downarrow$$

$$b = 0 \quad , \quad d = \hbar^2 \frac{3}{4}$$

$$S^2 = \hbar^2 \frac{3}{4} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$\hookrightarrow \quad S_z \chi_\uparrow = +\frac{\hbar}{2} \chi_\uparrow$

$$S_z = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$S_z \chi_\uparrow = \begin{pmatrix} a \\ c \end{pmatrix} = \frac{\hbar}{2} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$c = 0, \quad a = \frac{\hbar}{2}$$

$$S_z \chi_\downarrow = \begin{pmatrix} b \\ d \end{pmatrix} = -\frac{\hbar}{2} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$b = 0, \quad d = -\frac{\hbar}{2}$$

$$S_z = \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$\hookrightarrow \quad S_\pm = S_x \pm i S_y$

$$L_\pm |\ell m\rangle = C_\pm |\ell \; m \pm 1\rangle$$

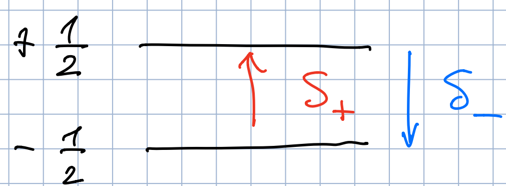
$$C_\pm = \hbar \sqrt{\ell(\ell+1) - m(m \pm 1)}$$

$$S = \frac{1}{2}$$

$$S_\pm \,|\, s \; s_z \rangle = \hbar \sqrt{\frac{3}{4} - s_z(s_z \pm 1)} \;|\, s \; s_z \pm 1 \rangle$$

$$S_+ \,|\, \tfrac{1}{2} \; +\tfrac{1}{2} \rangle = 0$$

$$S_+ \,|\, \tfrac{1}{2} \; -\tfrac{1}{2} \rangle = \hbar \;|\, \tfrac{1}{2} \; +\tfrac{1}{2} \rangle$$



$$S_- \,|\, \tfrac{1}{2} \; +\tfrac{1}{2} \rangle = \hbar \;|\, \tfrac{1}{2} \; -\tfrac{1}{2} \rangle$$

$$S_- \,|\, \tfrac{1}{2} \; -\tfrac{1}{2} \rangle = 0$$

- $$S_+ \, \chi_\uparrow = 0$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} a \\ c \end{pmatrix} \Rightarrow \underline{a = c = 0}$$

$$S_+ \chi_\downarrow = \hbar \chi_\uparrow$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} b \\ d \end{pmatrix} = \hbar \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$d = 0$

$\underset{\sim\sim\sim}{b = \hbar}$

$$S_+ = \hbar \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

- S_-

$$S_- = \hbar \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$\Rightarrow \quad S_\pm = S_x \pm i S_y$$

$$S_x = \frac{1}{2} \left(S_+ + S_- \right)$$

$$S_y = \frac{1}{2i} \left(S_+ - S_- \right)$$

$$S_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$S_y = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ +i & 0 \end{pmatrix}$$

$$S_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\overline{S}\ (S_x, S_y, S_z)$$

$$\overline{S} = \frac{\hbar}{2}\,\overline{\sigma}$$

↑

PAULI MATRIZEN

$$\begin{cases} \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\ \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \\ \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \end{cases}$$

$$[S_x, S_y] = i\hbar S_z$$

$$[S_y, S_z] = i\hbar S_x$$

$$[S_z, S_x] = i\hbar S_y$$

$$\chi_\uparrow = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \chi_\downarrow = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\chi = \begin{pmatrix} a \\ b \end{pmatrix} = a \begin{pmatrix} 1 \\ 0 \end{pmatrix} + b \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\chi = a\,\chi_\uparrow + b\,\chi_\downarrow$$

$$\chi^\dagger \chi = 1 \longrightarrow |a|^2 + |b|^2 = 1$$

$|a|^2 \Rightarrow$ WAHRSCHEINLICHKEIT $\uparrow$

$|b|^2 \Rightarrow$ " $\downarrow$