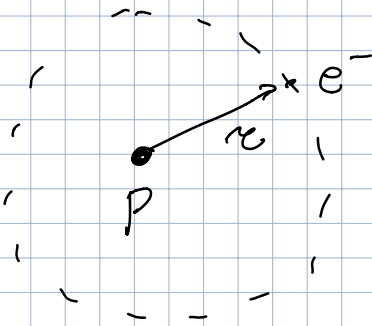


⇒ VORLESUNG 17 QM

H-ATOM



$$V(r) = - \frac{d}{r} (\hbar c)$$

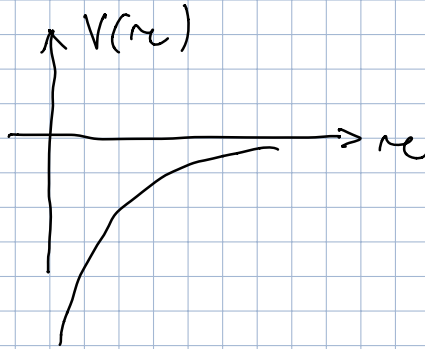
$$d \approx \frac{1}{137} \quad (\hbar c = 197 \text{ MeV fm})$$

$$\psi(r) = R(r) Y_{lm}(\theta, \phi)$$

$$R(r) = \frac{U(r)}{r}$$

$$-\frac{\hbar^2}{2m} \frac{d^2 U}{dr^2} + \left[V(r) + \frac{\hbar^2}{2m} \frac{l(l+1)}{r^2} \right] U - \frac{d}{r} \hbar c = E U$$

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GEBUNDENE ZUSTÄNDE

$$E = -\frac{\hbar^2}{2m} k^2$$

$$p \equiv \hbar k$$

$$\frac{d^2 U}{dr^2} = \left[1 - \frac{\alpha \hbar c}{2m} \frac{1}{p} + \frac{l(l+1)}{r^2} \right] U$$

$$\leadsto \frac{d^2 U}{dr^2} = \left[1 - \frac{p_0}{p} + \frac{l(l+1)}{r^2} \right] U$$

↳ ASYMPTOTISCHES VERHALTEN

$$p \rightarrow \infty$$

$$\frac{d^2 U}{dr^2} = U$$

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$$U(\rho) \xrightarrow{\rho \rightarrow \infty} \cancel{B e^{\rho}} + A e^{-\rho}$$

\hookrightarrow e-AM KERN

$$\rho \rightarrow 0$$

$$\frac{d^2 U}{d\rho^2} = \frac{l(l+1)}{\rho^2} U$$

$$U(\rho) \xrightarrow{\rho \rightarrow 0} C e^{\rho+1} + \cancel{D e^{-\rho}}$$

$$U(\rho) = \rho^a$$

$$a(a-1)\rho^{a-2} = l(l+1)\rho^{a-2}$$

$$a(a-1) = l(l+1)$$

$$\nearrow a = l+1$$

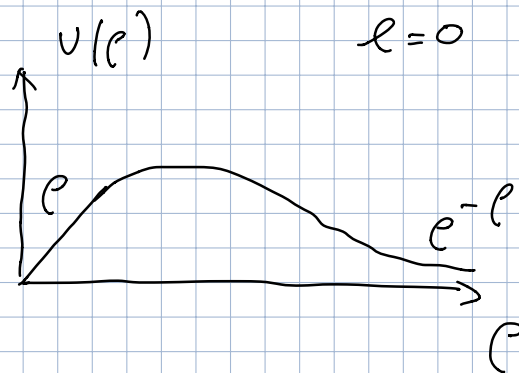
$$\searrow a = -l$$

$$R(r) = \frac{U(\rho)}{r} = K \frac{U(\rho)}{\rho}$$

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$$U(\rho) \xrightarrow{\rho \rightarrow 0} 0$$

$$l \in \mathbb{N} \quad l = 0, 1, 2, \dots$$



$$\leadsto U(\rho) = \rho^{l+1} e^{-\rho} v(\rho)$$

$$v(\rho) = \sum_{j=0}^{\infty} c_j \rho^j$$

$$\begin{aligned} \frac{dU}{d\rho} &= e^{-\rho} \left[(l+1) \rho^l v - \rho^{l+1} v + \rho^{l+1} \frac{dv}{d\rho} \right] \\ &= \rho^l e^{-\rho} \left[(l+1 - \rho) v + \rho \frac{dv}{d\rho} \right] \end{aligned}$$

$$\begin{aligned} \frac{d^2U}{d\rho^2} &= e^{-\rho} \left[-(l+1-\rho) \rho^l v - \rho^{l+1} \frac{dv}{d\rho} \right. \\ &\quad \left. + l(l+1) \rho^{l-1} v \right. \\ &\quad \left. - (l+1) \rho^l v \right] \end{aligned}$$

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$$\begin{aligned}
 &+ (l+1 - \rho) \rho^l \frac{d^2 v}{d\rho^2} \\
 &+ (l+1) \rho^l \frac{dv}{d\rho} \\
 &+ \rho^{l+1} \frac{d^2 v}{d\rho^2}]
 \end{aligned}$$

$$\begin{aligned}
 \frac{d^2 v}{d\rho^2} = \rho^l e^{-\rho} & \left[\left(2(l+1) + \rho + \frac{l(l+1)}{\rho} \right) v \right. \\
 & \left. + \left(2(l+1) - 2\rho \right) \frac{dv}{d\rho} \right. \\
 & \left. + \rho \frac{d^2 v}{d\rho^2} \right]
 \end{aligned}$$

$$\frac{d^2 v}{d\rho^2} = \left[1 - \frac{\rho_0}{\rho} + \frac{l(l+1)}{\rho^2} \right] v$$

$$\begin{aligned}
 &\rho \frac{d^2 v}{d\rho^2} + 2(l+1 - \rho) \frac{dv}{d\rho} \\
 &+ \left(-2(l+1) + \cancel{\rho} + \frac{l(l+1)}{\rho} \right) v \\
 &= \rho v \left[\cancel{1} - \frac{\rho_0}{\rho} + \frac{l(l+1)}{\rho^2} \right]
 \end{aligned}$$

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$$\rho \frac{d^2 v}{d\rho^2} + 2(\ell+1-\rho) \frac{dv}{d\rho}$$

$$+ [\rho_0 - 2(\ell+1)] v = 0$$

$$v(\rho) = \sum_{j=0}^{\infty} c_j \rho^j$$

$$\frac{dv}{d\rho} = \sum_{j=1}^{\infty} j c_j \rho^{j-1} = \sum_{j=0}^{\infty} (j+1) c_{j+1} \rho^j$$

$$\frac{d^2 v}{d\rho^2} = \sum_{j=1}^{\infty} j(j-1) c_j \rho^{j-2} = \sum_{j=0}^{\infty} (j+2)(j+1) c_{j+2} \rho^j$$

TERME ρ^j

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$$\sum_{j=0}^{\infty} j(j+1) c_{j+1} \rho^j$$

$$+ 2(\ell+1-\rho) \sum_{j=0}^{\infty} (j+1) c_{j+1} \rho^j$$

$$+ [\rho_0 - 2(\ell+1)] \sum_{j=0}^{\infty} c_j \rho^j = 0$$

$$\rho \sum_{j=0}^{\infty} (j+1) c_{j+1} \rho^j$$

$$= \sum_{j=1}^{\infty} j c_j \rho^j$$

$$\sum_{j=0}^{\infty} [\dots] \rho^j = 0$$

$$j(j+1) c_{j+1} + 2(\ell+1)(j+1) c_{j+1}$$

$$- 2j c_j + [\rho_0 - 2(\ell+1)] c_j = 0$$

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$$c_{j+1} = \frac{[2(j+l+1) - \rho_0]}{(j+1)(j+2l+2)} c_j$$

$$v(\rho) = e^{l+1} e^{-\rho} \sum_{j=0}^{\infty} c_j \rho^j$$

$$j \gg$$

$$c_{j+1} \approx \frac{2j}{(j+1)j} c_j = \frac{2}{j+1} c_j$$

$$c_1 = 2 c_0$$

$$c_2 = \frac{2}{2} c_1 = \frac{2^2}{2} c_0$$

$$c_3 = \frac{2}{3} c_2 = \frac{2^3}{3!} c_0$$

⋮

$$j \gg c_j = \frac{2^j}{j!} c_0$$

$$v(\rho) \xrightarrow{\rho \rightarrow \infty} \sum_{j=0}^{\infty} \frac{2^j}{j!} \rho^j = \sum_{j=0}^{\infty} \frac{(2\rho)^j}{j!}$$

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$$v(\rho) = e^{\ell+1} e^{-\rho} v(\rho) = e^{2\rho}$$

$$\xrightarrow{\rho \rightarrow \infty} e^{\ell+1} e^{-\rho} e^{2\rho} \sim \cancel{e^{\ell+1} e^{\rho}}$$

REIHE MUSS ABBRECHEN

$$v(\rho) = \sum_{j=0}^{j_{\max}} c_j \rho^j$$

$$c_{j_{\max}+1} = 0$$

$$2(j_{\max} + \ell + 1) = \rho_0$$

$$j_{\max} \geq 0$$

$$\ell \geq 0$$

$$n \equiv j_{\max} + \ell + 1$$

$$n = 1, 2, \dots$$

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HAUPT QUANTEN ZAHL

$$p_0 = \frac{\alpha \hbar c 2m}{\hbar^2 k} = \underline{2m} \quad E = - \frac{\hbar^2 k^2}{2m}$$

$$k = \frac{2m}{\hbar^2} \frac{\alpha \hbar c}{2m}$$

$$E = - \left(\frac{2m}{\hbar^2} \right) \frac{\alpha^2 (\hbar c)^2}{4 m^2}$$

$$E = - \left(\frac{1}{2} m \alpha^2 c^2 \right) \frac{1}{n^2}$$

$$E_n = - \frac{E_1}{n^2} \quad \text{BOHR FORMEL}$$

$$n = 1, 2, \dots$$

GRUNDZUSTAND

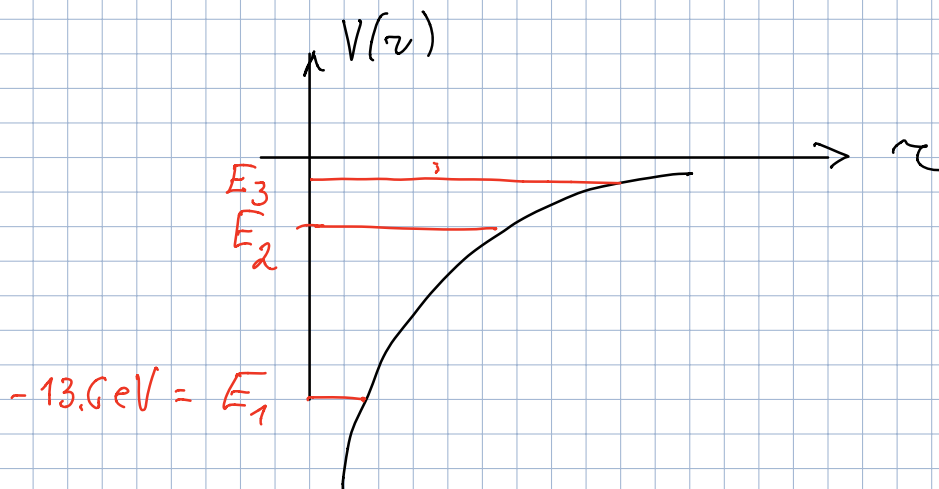
$$E_1 \equiv \left(\frac{1}{2} m c^2 \right) \alpha^2$$

$$\approx \frac{1}{2} 0.5 \text{ MeV} \left(\frac{1}{137} \right)^2$$

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$$= \frac{0.25 \cdot 10^6}{(1.37)^2 \cdot 10^4} \text{ eV}$$

$$= \frac{25}{(1.37)^2} \text{ eV} \approx \underline{\underline{13.6 \text{ eV}}}$$



$$\psi(r) \sim e^{-\rho} = e^{-Kr}$$

$$K = \frac{2m}{\hbar^2} \frac{\alpha \hbar c}{2m} \equiv \frac{1}{a} \frac{1}{m}$$

↑
LÄNGE

BOHR RADIUS \Rightarrow

$$a = \frac{1}{\alpha m c^2} \hbar c$$

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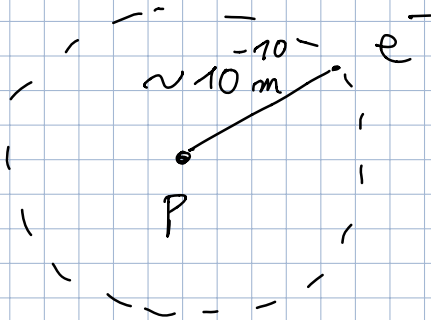
$$a = \frac{(137) (197 \text{ MeV fm})}{(0.5 \text{ MeV})}$$

$$\approx 1.37 \cdot 10^2 \cdot 4 \cdot 10^2 \cdot 10^{-15} \text{ m}$$

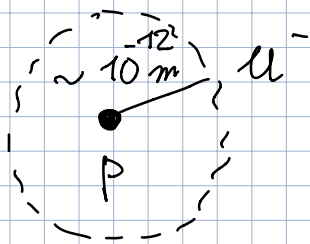
$$a = 0.53 \cdot 10^{-10} \text{ m}$$

$$U(e) \rightarrow e^{-\frac{\mu}{a} r} \sim e^{-\frac{\mu}{a} r}$$

e^- $n=1$



MUONISCHE ATOME



$$m_{\mu} = 105 \text{ MeV}$$

$$\approx 200 m_e$$

→ 200 x KLEINER

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