$\Longrightarrow$ VORLESUNG 13 QM

QUASI-KLASSISCHE ZUSTÄNDE

H.O. $$a\,|\alpha\rangle = \alpha\,|\alpha\rangle$$

$$\Downarrow$$

$$\hat{H}\,|n\rangle = \hbar\omega\left(n+\frac{1}{2}\right)|n\rangle$$

$$\Longrightarrow \quad |\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}}\,|n\rangle$$

$$\langle \hat{H} \rangle = \hbar\omega\left(|\alpha|^2 + \frac{1}{2}\right)$$

$$\langle X \rangle = \sqrt{2}\,\mathrm{Re}\,\alpha$$

$$\langle P \rangle = \sqrt{2}\,\mathrm{Im}\,\alpha$$

$$\sigma_x \cdot \sigma_p = \frac{\hbar}{2} \qquad \alpha = \rho\, e^{i\phi}$$

$$\rho \gg 1$$

$$\sigma_X = \frac{1}{\sqrt{2}} = \sigma_P$$

$$\frac{\sigma_X}{|\langle X\rangle|} \sim \frac{1}{2\,|\mathrm{Re}\,\alpha|} \ll 1$$

$$\rho \ll 1$$

ZEIT ENTWICKLUNG

$$t=0 \quad |\psi(0)\rangle = |\alpha_0\rangle$$

$$\Big\downarrow \quad \alpha_0 = \rho\, e^{i\phi}$$

$$|\psi(t)\rangle = e^{-\frac{|\alpha_0|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha_0^n}{\sqrt{n!}}\, e^{-\frac{i}{\hbar}\underbrace{E_n}_{\hbar\omega\left(n+\frac{1}{2}\right)} t}\, |n\rangle$$

$$= e^{-i\omega t/2}\, |\alpha(t)\rangle$$

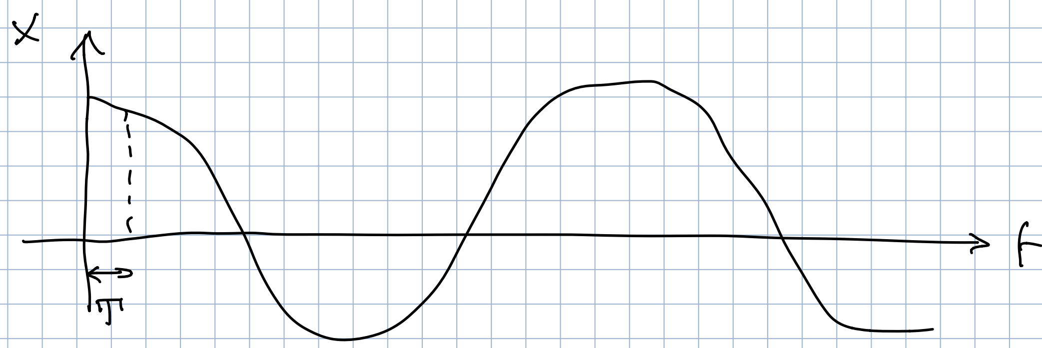
$$\alpha(t) = \alpha_0\, e^{-i\omega t}$$

$$= \rho\, e^{-i(\omega t - \phi)}$$

$$\begin{cases} \langle x \rangle = x_0 \cos(\omega t - \phi) \\ \langle p \rangle = -p_0 \sin(\omega t - \phi) \end{cases}$$

↳ 'SCHRÖDINGER CAT' ZUSTAND

$$|\psi(t=0)\rangle = |\alpha\rangle$$



$$t \in \underbrace{[0, T]}_{\text{KOPPLUNG}} \qquad \omega T \ll 1$$

$$\hat{W} = \hbar g \, (a^{\dagger} a)^2$$

$$\omega \ll g$$

$$\hat{W} \, |n\rangle = \hbar g \, n^2 \, |n\rangle$$

$$|\psi(T)\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} \, e^{-\frac{i}{\hbar} \hbar g n^2 T} \, |n\rangle$$

WAHL $\quad gT = \frac{\pi}{2}$

$$e^{-igTn^2} = e^{-i\frac{\pi}{2}n^2}$$

$$= \frac{1}{\sqrt{2}}\left\{ e^{-i\frac{\pi}{4}} + e^{+i\frac{\pi}{4}}(-1)^n \right\}$$

z.B. $\quad n$ GERADE

$$\frac{1}{\sqrt{2}}\, 2\cos\left(\frac{\pi}{4}\right) \Rightarrow 1$$

n UNGERADE

$$-\frac{1}{\sqrt{2}}\, 2i\sin\left(\frac{\pi}{4}\right) \Rightarrow -i$$

$$|\psi(T)\rangle = \frac{1}{\sqrt{2}}\left\{ e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}}\, e^{-i\frac{\pi}{4}} |n\rangle \right.$$

$$\left. + e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{(-\alpha)^n}{\sqrt{n!}}\, e^{+i\frac{\pi}{4}} |n\rangle \right\}$$

$$\Rightarrow |\psi(T)\rangle = \frac{1}{\sqrt{2}}\left\{ e^{-i\frac{\pi}{4}} |\alpha\rangle + e^{i\frac{\pi}{4}} |-\alpha\rangle \right\}$$

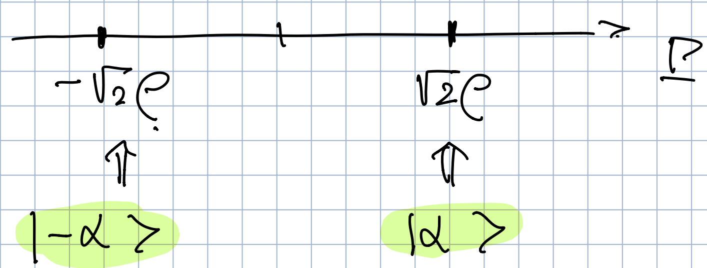
$$\alpha = \rho\, e^{i\phi} \qquad \phi = \frac{\pi}{2}$$

$$= i\rho$$

$$\langle x \rangle_{\pm\alpha} = 0$$

$$\langle p \rangle_{\pm\alpha} = \pm\sqrt{2}\,\rho \qquad \text{FÜR } |\pm\alpha\rangle$$

$$\sigma_p = \frac{1}{\sqrt{2}}$$



$\Rightarrow$ VERTEILUNG IN X

$$P(X) = |\langle X | \psi(T)\rangle|^2$$

$$= \frac{1}{2}\left| e^{-i\frac{\pi}{4}}\, \psi_\alpha(x) + e^{i\frac{\pi}{4}}\, \psi_{-\alpha}(x)\right|^2$$

$$\psi_\alpha(x) = C\, e^{-\frac{1}{2}\left(x - \sqrt{2}\, i \rho\right)^2}$$

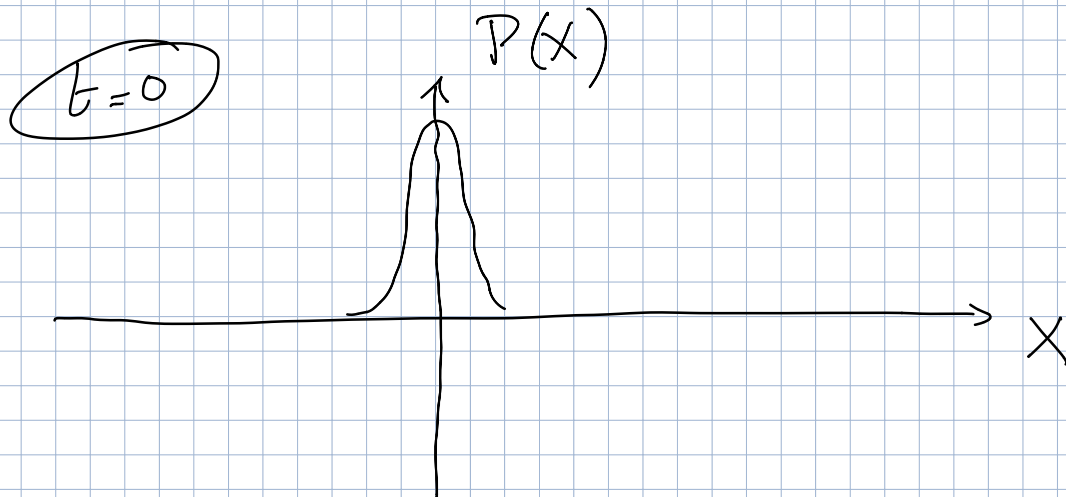
$$\psi_{-\alpha}(x) = C\, e^{-\frac{1}{2}\left(x + \sqrt{2}\, i \rho\right)^2}$$

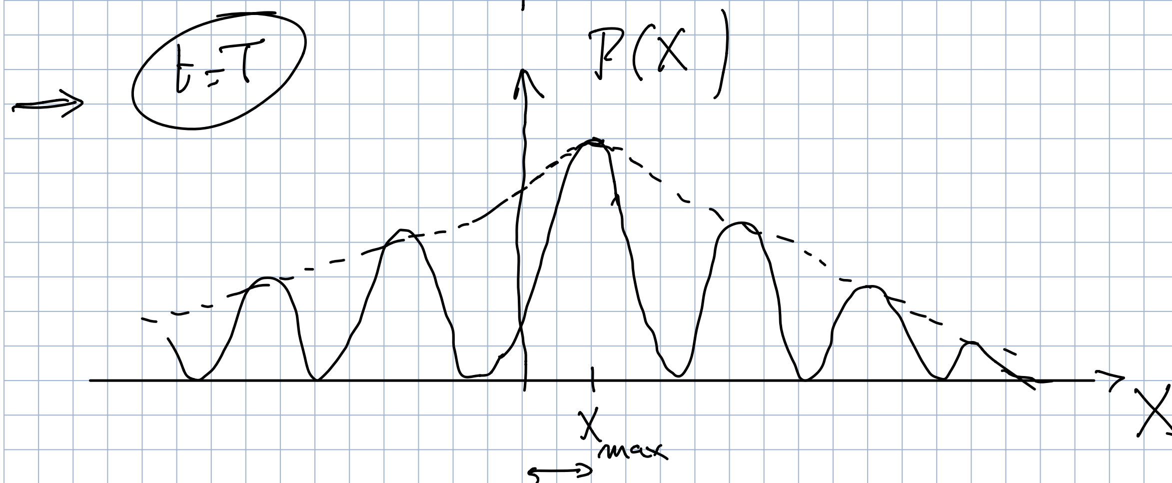
$$P(x) = \frac{1}{2} |C|^2 \left| e^{-i\frac{\pi}{4}}\, e^{-\frac{1}{2}\left(x - \sqrt{2}\, i \rho\right)^2} + e^{+i\frac{\pi}{4}}\, e^{-\frac{1}{2}\left(x + \sqrt{2}\, i \rho\right)^2} \right|^2$$

$$e^{-\frac{1}{2}\left(x \mp \sqrt{2}\, i \rho\right)^2} = e^{-\frac{1}{2}x^2}\, e^{+\rho^2}\, e^{\pm x \sqrt{2}\, i \rho}$$

$$= \frac{1}{2} |C|^2\, e^{-x^2}\, e^{2\rho^2} \Big| \underbrace{e^{-\frac{i\pi}{4}} \cdot e^{+i x \sqrt{2} \rho} + e^{+i\frac{\pi}{4}}\, e^{-i x \sqrt{2} \rho}}_{2\cos\left(x\sqrt{2}\rho - \frac{\pi}{4}\right)} \Big|^2$$

$$P(x) \sim e^{-x^2} \cos^2\left(x \sqrt{2} \rho - \frac{\pi}{4}\right)$$





MAX $\quad X_{max}\sqrt{2}\,\rho - \frac{\pi}{4} = 0$

$$X_{max} = \frac{\pi}{4}\,\frac{1}{\sqrt{2}}\quad\frac{1}{\rho}$$

$\Rightarrow$ VERTEILUNG IN $\underline{P}$

$$P(\underline{P}) = \left| \langle \underline{P} \,|\, \psi(T) \rangle \right|^2$$

$$= \frac{1}{2} \left| e^{-i\frac{\pi}{4}} \varphi_{\alpha}(\underline{P}) + e^{+i\frac{\pi}{4}} \varphi_{-\alpha}(\underline{P}) \right|^2$$

$$\varphi_{\alpha}(\underline{P}) = C' e^{-\frac{1}{2}(\underline{P} + i\sqrt{2}\alpha)^2}$$

$$\alpha = i\rho$$

$$\varphi_{\pm\alpha}(\underline{P}) = C' e^{-\frac{1}{2}(\underline{P} \mp \sqrt{2}\rho)^2}$$

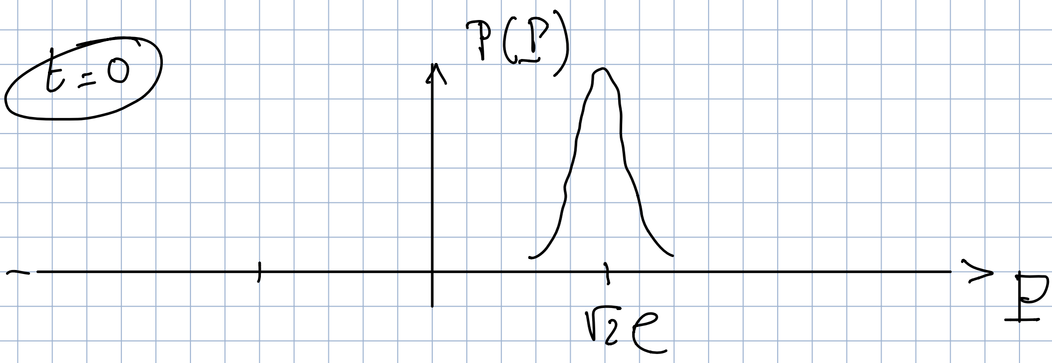
$$P(\underline{P}) \approx \left| e^{-i\frac{\pi}{4}} e^{-\frac{1}{2}(\underline{P} - \sqrt{2}\rho)^2} + e^{+i\frac{\pi}{4}} e^{-\frac{1}{2}(\underline{P} + \sqrt{2}\rho)^2} \right|^2$$

$$= e^{-(\underline{P} - \sqrt{2}\rho)^2} + e^{-(\underline{P} + \sqrt{2}\rho)^2}$$

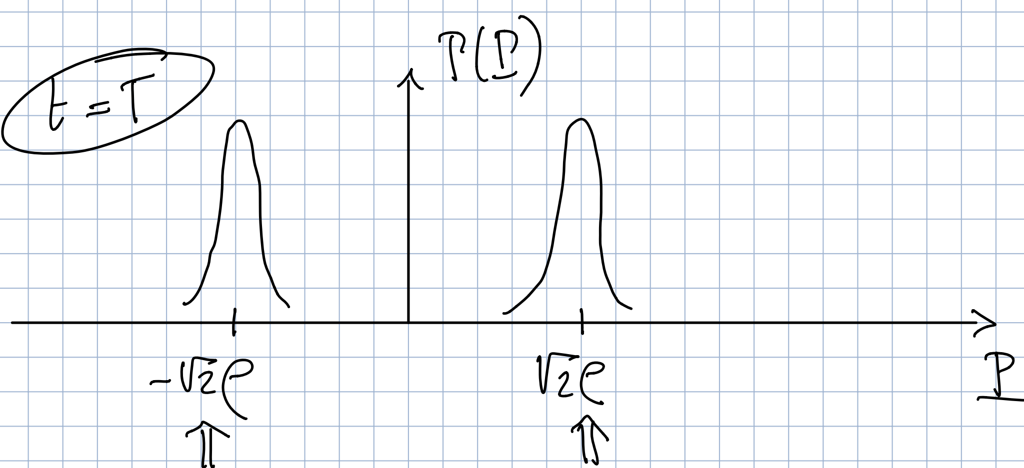
$$+ 2 \,\mathrm{Re} \left(\cancel{e^{-\frac{1}{2}(\underline{P} - \sqrt{2}\rho)} e^{-\frac{1}{2}(\underline{P} + \sqrt{2}\rho)}} \underbrace{e^{-i\frac{\pi}{2}}}_{(-i)} \right)$$

$$P(\underline{P}) \sim e^{-(\underline{P}-\sqrt{2}\rho)^2} + e^{-(\underline{P}+\sqrt{2}\rho)^2}$$

$t=0$



→ $t=T$



$$\rho \gg 1$$

- QM ÜBERLAGERUNG

$|\psi(T)\rangle$ 1 SYSTEM

N SYSTEME $N \gg$

MESSUNG x, p

$$\frac{1}{\sqrt{2}} \ll \delta p \ll \sqrt{2}\,\rho$$

AUFLÖSUNG

σ_p

$N \gg 1$

$$\sqrt{2}\,\rho \longleftarrow N/2$$

$$-\sqrt{2}\,\rho \longleftarrow N/2$$

- **STATISCHE MISCHUNG**

$N/2$	SYSTEME	IM	$\lvert\alpha\rangle$
$N/2$	"	"	$\lvert-\alpha\rangle$

p		$\langle p\rangle$
$N/2$	$\lvert\alpha\rangle$	$+\sqrt{2}\,\rho$
$N/2$	$\lvert-\alpha\rangle$	$-\sqrt{2}\,\rho$

SELBE RESULTAT !

ORTSMESSUNG

STATISTISCHE MISCHUNG

$$\langle X \rangle$$

$$|\alpha\rangle \longrightarrow 0$$

$$|-\alpha\rangle \longrightarrow 0$$

QM ÜBERLAGERUNG

$$\delta X \ll \frac{1}{\rho}$$

$$\delta x \ll \sqrt{\frac{\hbar}{m\omega}} \frac{1}{\rho}$$

$$\frac{x_0}{\rho}$$

$$\delta x \ll \frac{x_0}{\rho^2} \sim 10^{-6} \, 10^{-19} \, m$$

PENDULUM

$$\delta x \ll 10^{-25} \, m$$

$$x_0 \sim 10^{-6} \, m$$

$$\rho \sim 3 \cdot 10^{9}$$

↳ 3) ZERBRECHLICHKEIT EINER QM ÜBERLAGERUNG

'FRAGILITY'

$$\alpha(t) \Rightarrow \alpha(t)\, e^{-\gamma t}$$

γ DÄMPFUNG

$$\alpha_1(t) = \alpha(t)\, e^{-\gamma t}$$

$$\langle \hat{H} \rangle_{\alpha_1} = \hbar\omega\left(|\alpha_1|^2 + \frac{1}{2}\right)$$

$$= \hbar\omega\left(|\alpha|^2 e^{-2\gamma t} + \frac{1}{2}\right)$$

$$e^{-2\gamma t} \approx 1 - 2\gamma t + \ldots$$

$\gamma t \ll 1$

$$\langle \hat{H} \rangle_{\alpha_1} = \hbar\omega\left(|\alpha|^2 + \frac{1}{2}\right) - \hbar\omega\, 2\gamma t \cdot |\alpha|^2$$

$$|\alpha|^2 2\gamma t = |\beta|^2$$

QM ÜBERLAGERUNG

STAT. MISCHUNG

$$|\beta| \sim 1 \qquad |\alpha|^2 2\gamma t \lesssim 1$$

$$t \lesssim \frac{1}{\gamma |\alpha|^2}$$

$$\rho \sim 10^9$$

$$(2\gamma)^{-1} \sim 1 \text{ JAHR} \sim \underbrace{10^7}\,s$$

$$t \lesssim \frac{1}{\gamma |\alpha|^2} \sim \frac{10^7}{10^{18}}$$

$$\sim \underline{\underline{10^{-11}\,s}}$$

MESOSCOPISCHE SYSTEME

$|e\rangle^2 \sim 10$

NOBEL PREIS 2012

HAROCHE, WINELAND