# Photon propagator

$$\frac{-i}{q^2+i\varepsilon}\left[g^{\mu\nu}-(1-\xi)\,\frac{q^\mu q^\nu}{q^2}\right]$$

$\xi \longrightarrow$ gauge fixing parameter

Gauge invariance $\longleftrightarrow$ $\xi$-independence

$\longrightarrow$ Ward identity $\leftrightarrow$ Lorentz invariance

$$A = \varepsilon_\mu(q) M^\mu \qquad \left\{ \underline{q_\mu \cdot M^\mu = 0} \right.$$

## Scalar QED

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\varphi)^*(D^\mu\varphi) - m^2\varphi^*\varphi$$

$$D^\mu\varphi = (\partial^\mu - \underline{ieQA^\mu})\varphi \qquad (D^\mu\varphi)^* = (\partial^\mu + \underline{ieQA^\mu})\varphi^*$$

$Q \to$ charge in units of $e^+$ charge

For $e^-$ $\quad Q = -\underline{1}$

Gauge inv. $\quad A^\mu \longrightarrow A^\mu + \partial^\mu\alpha$

$$\varphi \to e^{-i\alpha(x)}\varphi$$

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$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (\partial_\mu\varphi)^*(\partial^\mu\varphi) - m^2\varphi^*\varphi$$

$$\underbrace{+\, ieQA^\mu\left[\varphi^*\partial_\mu\varphi - (\partial_\mu\varphi^*)\varphi\right] + e^2Q^2A_\mu A^\mu|\varphi|^2}_{\mathcal{L}_{int}}$$

Recall: no way to couple $A^\mu$ to real scalar!

We need charge

Charge arises from a global phase sym.

$$\varphi \to e^{-i\alpha}\varphi \qquad \alpha = \text{const.}$$

$$(\varphi_1, \varphi_2) \to \quad \cdot\ \varphi = \frac{1}{\sqrt{2}}(\varphi_1 + i\varphi_2)$$
$$\cdot\ \varphi^* = \frac{1}{\sqrt{2}}(\varphi_1 - i\varphi_2)$$

$$\varphi(x) = \int \frac{d^3\vec{p}}{(2\pi)^3} \frac{1}{\sqrt{2\omega_p}} \left[ a_p e^{-ipx} + b_p^+ e^{ipx} \right]$$

$$\varphi^*(x) = \int \frac{d^3\vec{p}}{(2\pi)^3} \frac{1}{\sqrt{2\omega_p}} \left[ b_p e^{-ipx} + a_p^+ e^{ipx} \right]$$

$b_p$ destroys a particle of the same mass and opposite charge to that destroyed by $a_p$

$$(\Box + m^2)\varphi(x) = 0 \qquad \Box e^{-ipx} = -p^2 e^{-ipx}$$

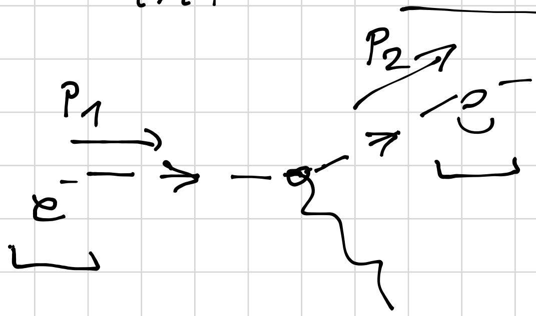
$$m^2 = p^2 \qquad \omega_p = \sqrt{\vec{p}^{\,2} + m^2}$$

Global symmetry w.r.t. phase rotation $\Rightarrow$ charge $\Rightarrow$ complex fields $\Rightarrow$

$\Rightarrow$ antiparticles $(a_p, a_p^+;\ b_p, b_p^+)$

Feinman rules for scalar QED

$$\mathcal{L}_{int}^{(1)} = +ieQA^\mu\left[\varphi^*(\partial_\mu\varphi) - (\partial_\mu\varphi^*)\varphi\right]$$

1. 

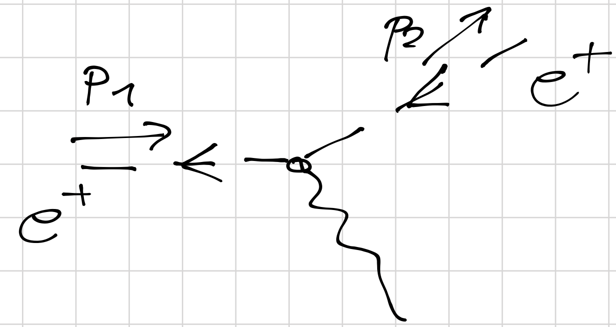

$$Q(e^-) = -1$$

$$a_p e^{-ip_1x} \qquad \partial_\mu \to -ip_{1\mu}$$

$$a_p^+ e^{ip_2x} \qquad \partial_\mu \to +ip_{2\mu}$$

$$-ie(p_1+p_2)^\mu$$

2.



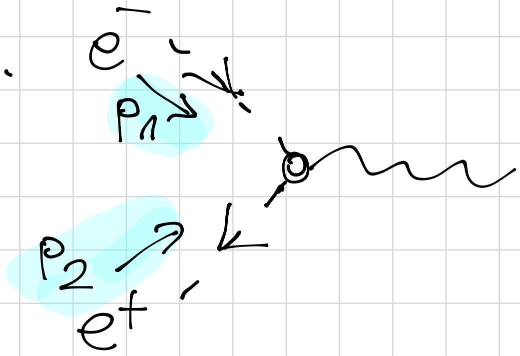

$b_p e^{-ip_1 x}$ $\qquad \partial_\mu \to -ip_{1\mu}$

$b_p^+ e^{ip_2 x}$ $\qquad \partial_\mu \to +ip_{2\mu}$

$Q = +1$

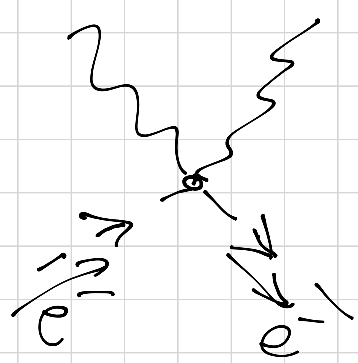
$+ie(p_1+p_2)^\mu$

3.




$$\left.\begin{matrix} a_p \\ b_p \end{matrix}\right) \longrightarrow ie(-p_1+p_2)_\mu$$

4. $\mathcal{L}_{int}^{(2)} = e^2 Q^2 A_\mu A^\mu |\varphi|^2$




$\exp(i\mathcal{L}_{int})$

$2ie^2Q^2 g^{\mu\nu}$

2 identical photons

Two kinds of arrows:

1. momentum flow (arrows over part. lines)
   convention: left $\longrightarrow$ right
2. charge flow (arrows on particle lines)
   for particles along momentum arrow
   antiparticle $\longrightarrow$ against

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What should the loose Lorentz indices be contracted with?

$\mathcal{L}_{int} \sim \underline{A^\mu}\ \underline{J_\mu}$

$$A^\mu(x) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2\omega_k}} \sum_i \left[ \varepsilon_i^\mu(k) e^{-ikx} + \varepsilon_i^{*\mu}(k) e^{ikx} \right]$$

Contract $M^\mu$ or $M^{\mu\nu}$ with

$\varepsilon^\mu$ for incoming photon

$\varepsilon^{*\mu}$ for outgoing photon

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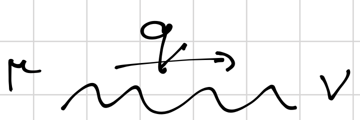
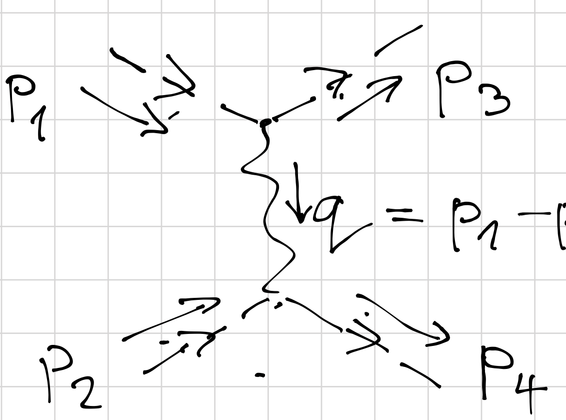



$$\frac{-i}{q^2+i\varepsilon}\left[g^{\mu\nu} - (1-\xi)\frac{q^\mu q^\nu}{q^2}\right]$$

We can start computing Feynman diags.

1. Elastic "$e^-$"–"$e^-$" scattering (scalar Møller scattering)

leading order in $e$




$q = p_1 - p_3 = p_4 - p_2$

$$i(2\pi)^4\delta^4(p_1+p_2-p_3-p_4)$$

$$iA = -ie(p_1+p_3)_\mu(-ie)(p_2+p_4)_\nu \frac{-i}{q^2+i\varepsilon}\left[g^{\mu\nu} - (1-\xi)\frac{q^\mu q^\nu}{q^2}\right]$$

$$p_1^2 = p_2^2 = p_3^2 = p_4^2 = m^2$$

$$q^\mu(p_1+p_3)_\mu = (p_1-p_3, p_1+p_3) = p_1^2 - p_3^2 = 0$$

$\Downarrow$ No explicit $\xi$ – dependence

$$A = \frac{e^2}{q^2}(p_1+p_3, p_2+p_4) = e^2 \cdot \frac{s-u}{t}$$

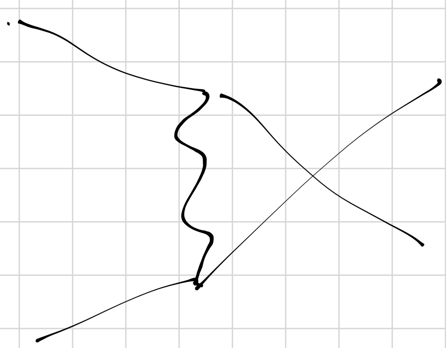
$$s = (p_1+p_2)^2 = 2m^2 + 2p_1p_2$$
$$\quad = (p_3+p_4)^2 = 2m^2 + 2p_3p_4$$

$$u = (p_1-p_4)^2 = 2m^2 - 2p_1p_4 = 2m^2 - 2p_2p_3$$
$$t = q^2$$

$$\frac{1}{q^2}\left(p_1+p_3,\, p_2+p_4\right) = \frac{1}{t}\Big(\underbrace{p_1p_2 + p_3p_4}_{= s-2m^2} + \underbrace{p_2p_3 + p_1p_4}_{= 2m^2-u}$$

$$= \frac{s-u}{t}$$

<u>Exercise</u> ! remember to account for symmetry $p_3 \leftrightarrow p_4$



To associate a number to each Feynman diagram $\longrightarrow$ have to pick a gauge (i.e. fix $\xi$)

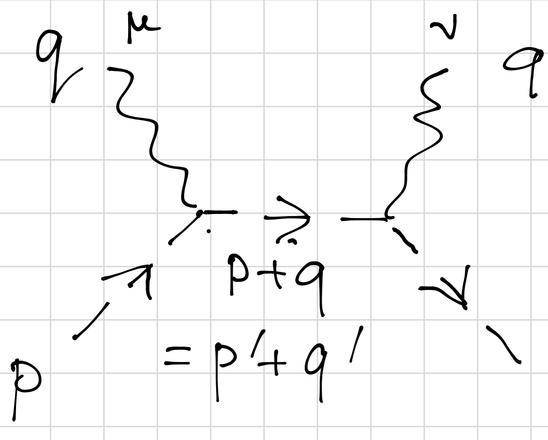
Once you account for all relevant diags. $\longrightarrow$ observe $\xi$-dependence drop.

No physical observable depends on $\xi$ !

Exercise : $e^+e^- \rightarrow e^+e^-$ (scalar Bhabha scattering)

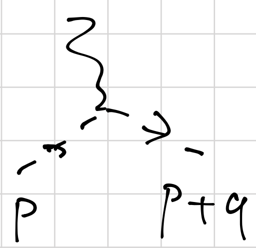
Consider Compton scattering

$\gamma e^- \to \gamma e^-$




$$iA_s = \varepsilon^\mu(q)\,\varepsilon^{*\nu}(q')\,(-ie)^2\,\frac{i}{(p+q)^2-m^2}$$

$$(2p+q)_\mu\,(2p'+q')_\nu$$




$$A_s = -e^2\,\frac{(2p+q,\varepsilon)(2p'+q',\varepsilon'^*)}{(p+q)^2-m^2}$$

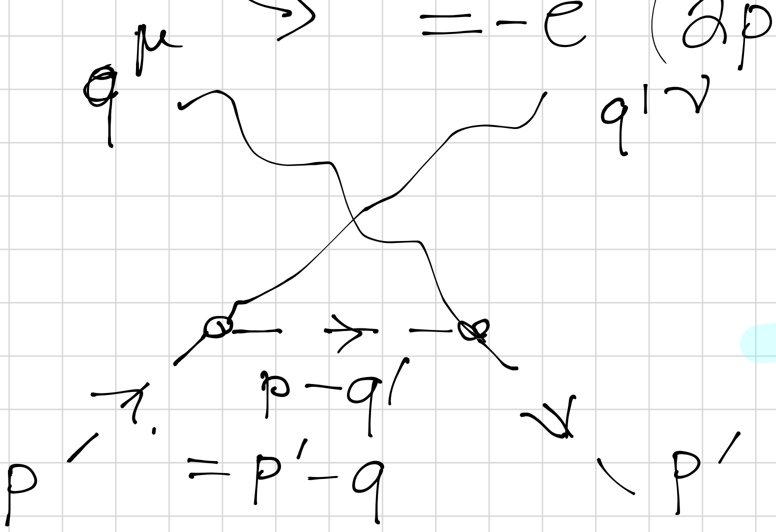
To check Ward ID: sub $\varepsilon_\mu \to q_\mu$

$$A_s = \varepsilon_\mu\,\varepsilon'^*_\nu \cdot A_s^{\mu\nu}$$

$$q_\mu\,\varepsilon'^*_\nu\,A_s^{\mu\nu} = -e^2\,\frac{(2pq+q^2)(2p'+q',\varepsilon'^*)}{(p+q)^2-m^2}$$

$$(p+q)^2 - m^2 = \cancel{p^2} + \underline{q^2 + 2pq} \cancel{- m^2}$$

$$= -e^2(2p'+q',\varepsilon'^*) \neq 0$$




$$A_s \longrightarrow A_s\,(q \leftrightarrow -q')$$

$$A_u = -e^2\,\frac{(2p-q',\varepsilon'^*)(2p'-q,\varepsilon)}{(p'-q)^2-m^2}$$

$$A_u = \varepsilon_\mu\,\varepsilon'^*_\nu \cdot A_u^{\mu\nu}$$

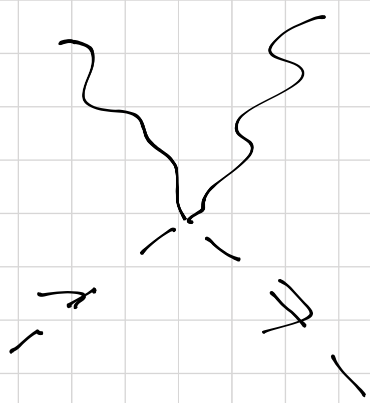
$$q_\mu\,\varepsilon'^*_\nu \cdot A_u^{\mu\nu} = -e^2\,\underbrace{\frac{(2p'-q,q)}{(p'-q)^2-m^2}}_{=-1}\,(2p-q',\varepsilon'^*)$$

$$q_\mu \varepsilon'^*_\nu (A_s^{\mu\nu} + A_u^{\mu\nu}) = -e^2(2p'+q', \varepsilon'^*) + e^2(2p-q', \varepsilon'^*)$$

$$= -e^2(2p'+2q'-2p, \varepsilon'^*)$$

$$= -2e^2(q\varepsilon'^*) \neq 0$$

$\underline{p'+q' = p+q}$



$$iA_c^{\mu\nu} = 2ie^2 g^{\mu\nu}$$

c = contact

$$q_\mu \cdot \varepsilon'^*_\nu A_c^{\mu\nu} = 2e^2(q\varepsilon'^*)$$

$$q_\mu \varepsilon'^*_\nu (A_s + A_u + A_c)^{\mu\nu} = 0$$

$$\varepsilon_\mu q'_\nu (A_s + A_u + A_c)^{\mu\nu} = 0$$

The sum of these 3 diagrams is the minimal gauge-invariant set of Feynman diagrams for Compton scatt.

$$A^{\mu\nu}_{\text{Compton}} = -e^2\left[\frac{(2p+q)^\mu(2p'+q')^\nu}{(p+q)^2-m^2} + \frac{(2p-q')^\nu(2p'-q)^\mu}{(p-q')^2-m^2} - 2g^{\mu\nu}\right]$$

<u>Exercise</u> Klein-Nishina formula

Compton cross section for spin-0

$$\frac{d\sigma}{d\Omega} \sim \left| \varepsilon_\mu \varepsilon'^*_\nu A^{\mu\nu} \right|^2$$

$$= \left( \varepsilon_\mu \varepsilon'^*_\nu A^{\mu\nu} \right) \cdot \left( \varepsilon_\alpha \varepsilon'^*_\beta A^{\alpha\beta} \right)^*$$

$$= \varepsilon_\mu \varepsilon^*_\alpha \cdot \varepsilon'^*_\nu \varepsilon'_\beta \cdot \underline{A^{\mu\nu}} A^{*\alpha\beta}$$

$$\sum_\lambda \varepsilon_\mu \varepsilon^*_\alpha = -g_{\mu\alpha} + \cancel{(1-\xi) \frac{q_\mu q_\alpha}{q^2}}$$

$$\sum_{\lambda'} \varepsilon'_\beta \varepsilon'^*_\nu = -g_{\nu\beta} + \cancel{(1-\xi) \frac{q'_\beta q'_\nu}{q'^2}}$$

$$A^{\mu\nu}: \quad q_\mu A^{\mu\nu} = q'_\nu A^{\mu\nu} = 0$$

$$\frac{d\sigma}{d\Omega} \sim A^{\mu\nu} \cdot A^*_{\mu\nu}$$

$$\varepsilon = \varepsilon_\lambda(q) \qquad \lambda = \pm$$

$$\varepsilon_\pm = \frac{1}{\sqrt{2}} (0, 1, \pm i, 0)$$

$$\Pi^{\mu\nu} = \frac{i}{q^2 + i\varepsilon} \underbrace{\sum_\lambda \varepsilon^\mu_\lambda(q)\, \varepsilon^{*\nu}_\lambda(q)}$$

Another idea for exercise:

Low-energy limit for $A$

$$q = (\omega, 0, 0, \omega) \qquad \omega \to 0$$