Relativistic QFT (Theo 6a): Exercise Sheet 6
Total: 100 points

11/12/2020

Ladder diagram resummation in ϕ^3-theory (60 points)

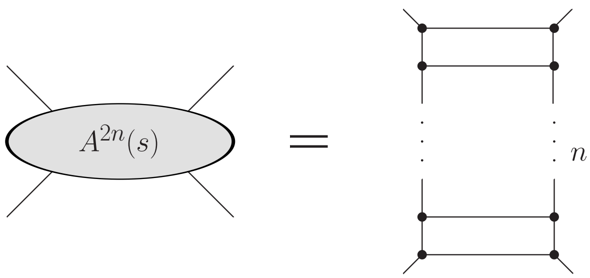

Figure 1: Ladder diagram with n loops in ϕ^3-theory.

Prove by the method of mathematical induction that the imaginary and real parts of a ladder diagram, depicted in Fig.1 are given by

$$\frac{1}{\pi}\mathrm{Im}\, A^{(2n)}(s\to\infty) = \frac{g^2}{s}\left(\frac{g}{4\pi m}\right)^2 \frac{1}{(n-2)!}\left[\left(\frac{g}{4\pi m}\right)^2 \log\frac{s}{m^2}\right]^{n-2}\theta(s-(n\,m)^2), \quad n\geq 2 \tag{1}$$

and

$$\mathrm{Re}\, A^{(2n)}(s\to\infty) = -\frac{g^2}{s}\frac{1}{(n-1)!}\left[\left(\frac{g}{4\pi m}\right)^2 \log\frac{s}{m^2}\right]^{n-1}, \tag{2}$$

respectively. (Observe that the above formulas hold for the special cases of $A^{(4)}$ and $A^{(6)}$ considered in the lectures.) To do that, assume that Eq.(1) holds true at the order n and compute the imaginary part of the $(n+1)$-loop diagram, Fig.2 using the cutting rules. Demonstrate that it can be cast in the form analogous to Eq.(1) with $n = n+1$. Work in the approximation $(nm)^2 \ll s$, thus $\ln\frac{s}{n^2m^2} \approx \ln\frac{s}{m^2}$. Use the fact that $\int\frac{ds}{s}\ln^p(s) = 1/(p+1)\ln^{p+1}(s)$.

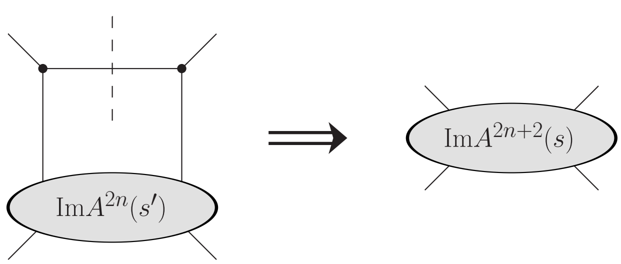

Figure 2: The last step of mathematical induction for imaginary part.

Consequently, insert Eq.(1) in the dispersion relation

$$\mathrm{Re}A^{(2n+2)}(s) = \frac{1}{\pi} \int\limits_{(n+1)^2m^2}^{\infty} \frac{ds'}{s'-s} \mathrm{Im}A^{(2n+2)}(s'), \tag{3}$$

change the integration variable to $x = (n+1)^2m^2/s'$ and perform the integral using the formula

$$\mathcal{P}\int_0^1 \frac{\log^n \frac{1}{x}}{x-y} = -n!\,\mathrm{Li}_{n+1}(y). \tag{4}$$

Above, $\mathrm{Li}_n(y)$ stands for the polylogarithm function with the following behavior at large y

$$\lim_{y\to\infty} \mathrm{Li}_n(y) = -\frac{1}{n!}\log^n(y). \tag{5}$$

$2 \to 2$ kinematics in the different frames (40 points)

Consider the elastic scattering process of two particles with masses m and M (see Fig.3), $M > m$.

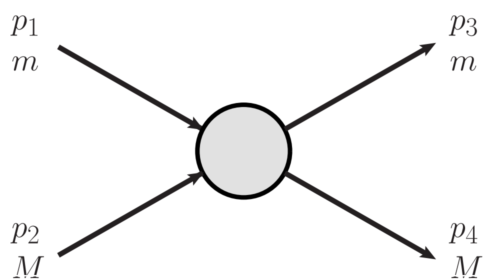

Figure 3: Elastic scattering of two particles with masses m and M

Compute the incoming and outgoing energies and momenta of particles in 3 different frames:
a) center-of-momentum frame defined by $\vec{p}_1 + \vec{p}_2 = 0$;
b) laboratory frame defined by $\vec{p}_2 = 0$;
c) Breit (or brick-wall) frame $\vec{p}_2 + \vec{p}_4 = 0$.