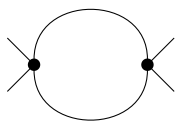
Relativistic QFT (Theo 6a): Exercise Sheet 5
Total: 100 points

04/12/2020

1. One loop diagram asymptotics in ϕ^4-theory (50 points)

Consider the one-loop diagram at the second order in the expansion of the S-matrix for a $2 \to 2$ scattering process in the ϕ^4-theory:



$$= \frac{\lambda^2}{2} \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m^2)[(p_1 + p_2 - k)^2 - m^2]}. \tag{1}$$

Here, p_1 and p_2 are the incoming momenta of the initial particles.

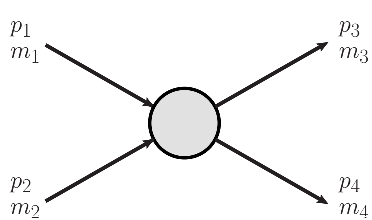
(a) Represent the momentum $k^\mu = \alpha n^\mu + \beta \bar{n}^\mu + k_\perp^\mu$ using the Sudakov's vectors $n^\mu = \sqrt{s}/2\{1, 0, 0, 1\}$ and $\bar{n}^\mu = \sqrt{s}/2\{1, 0, 0, -1\}$, and calculate the loop diagram (1) at high s by the method of regions from Lecture 9. For the final integration over $k_\perp$ assume that $|k_\perp| \leq \Lambda$, where Λ is some real number. How does the result behave with Λ? (**25 points**)

(b) Calculate the imaginary part of (1) now applying the Cutkosky's cutting rules. (**15 points**)

(c) Using Cauchy theorem, try to evaluate the real part of (1) from the imaginary part. Assume that the value of (1) is known at $s = s_0$, and make the subtraction at this point to get the convergent integral.(**10 points**)

2. Identity for the Mandelstam variables (10 points)

Show that for a $2 \to 2$ scattering process,



in the general case when incoming and outgoing particles have different masses, i.e. $p_i^2 = m_i^2$, $i = 1, \ldots, 4$ with $m_1 \neq m_2 \neq m_3 \neq m_4$, the Mandelstam variables $s = (p_1 + p_2)^2$, $t = (p_1 - p_3)^2$ and $u = (p_1 - p_4)^2$,obey the identity:

$$s + t + u = \sum_{i=1}^{4} m_i^2. \tag{2}$$

3. Kinematical Boundaries for Meson-Nucleon Scattering (40 points)

Consider a theory with the interaction given by two Yukawa terms,

$$\mathcal{L}_{\text{int}} = -g_1\phi_1\psi^*\psi - g_2\phi_2\psi^*\psi. \tag{3}$$

Here the real scalar meson fields ϕ_1 and ϕ_2 have different masses m_1 and m_2, and the complex "nucleon" field ψ with mass M. For the process of non-diagonal meson-nucleon scattering $\phi_1 N \to \phi_2 N$

(a) draw the tree-level Feynman diagrams (second order in couplings) and obtain the corresponding invariant transition amplitude in terms of the Mandelstam invariants (**10 points**).

(b) derive the kinematical boundaries for the Mandelstam invariants s, t and u in terms of m_1, m_2 and the nucleon mass M. Assume $m_2 > m_1$ and work in the center of momentum frame of the incoming particles (**30 points**)