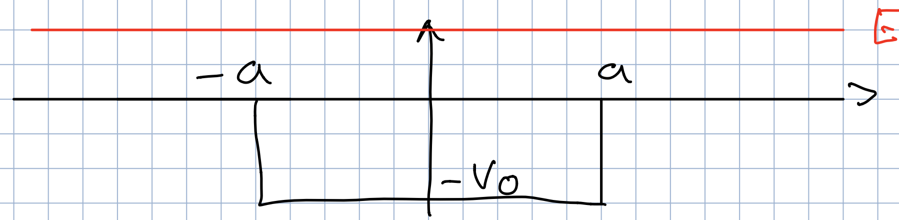
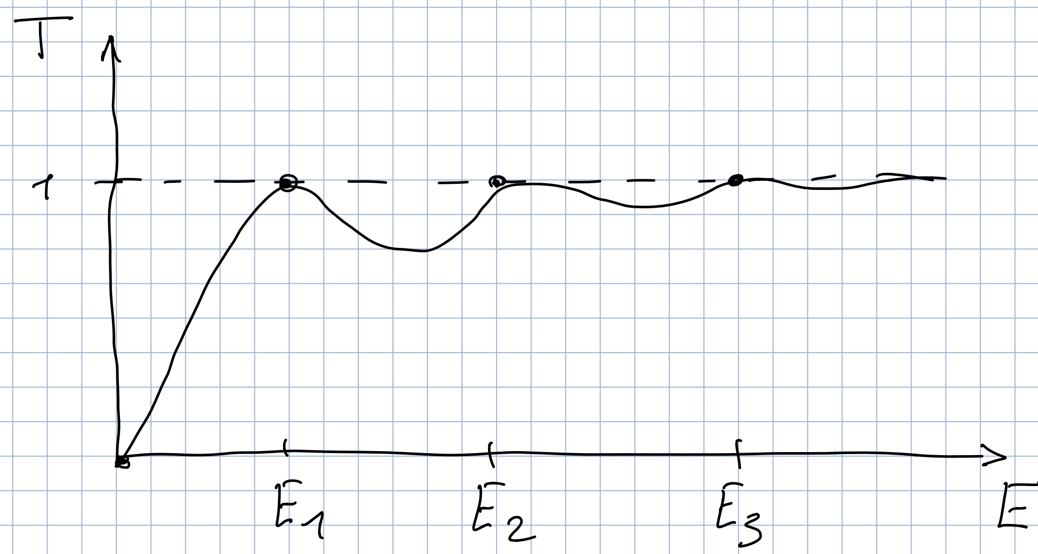
$\Longrightarrow$ VORLESUNG 8 QM



TRANSMISSION

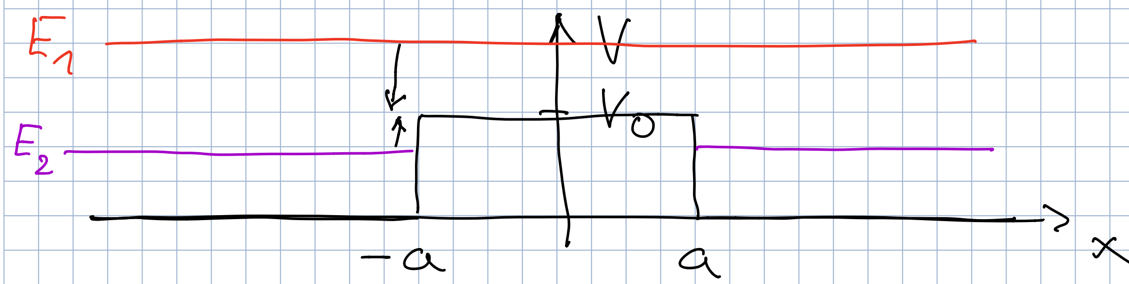
$$T^{-1} = 1 + \frac{V_0^2}{4E\,(E+V_0)} \sin^2\left(\frac{2a}{\hbar}\sqrt{2m(E+V_0)}\right)$$



$\underbrace{E_n}$ $\qquad k = \frac{2\pi}{\lambda}$

$\hookrightarrow \quad 2a = n\left(\frac{\lambda}{2}\right) \qquad n = 1, 2, \ldots$

POTENTIAL BARRIÈRE



$E_1 > V_0$

$0 < E_2 < V_0$

① $\underline{E_1 > V_0}$

$V_0 \to -V_0$

$E + V_0 \to E - V_0 > 0$

$$\Rightarrow T^{-1} = 1 + \frac{V_0^2}{4E\,(E - V_0)} \sin^2\left(\frac{2a}{\hbar}\sqrt{2m(E-V_0)}\right)$$

② $\underline{E_2 < V_0}$ TUNNEL EFFEKT

$E - V_0 < 0$

$\to -\underbrace{(V_0 - E)}_{>0}$

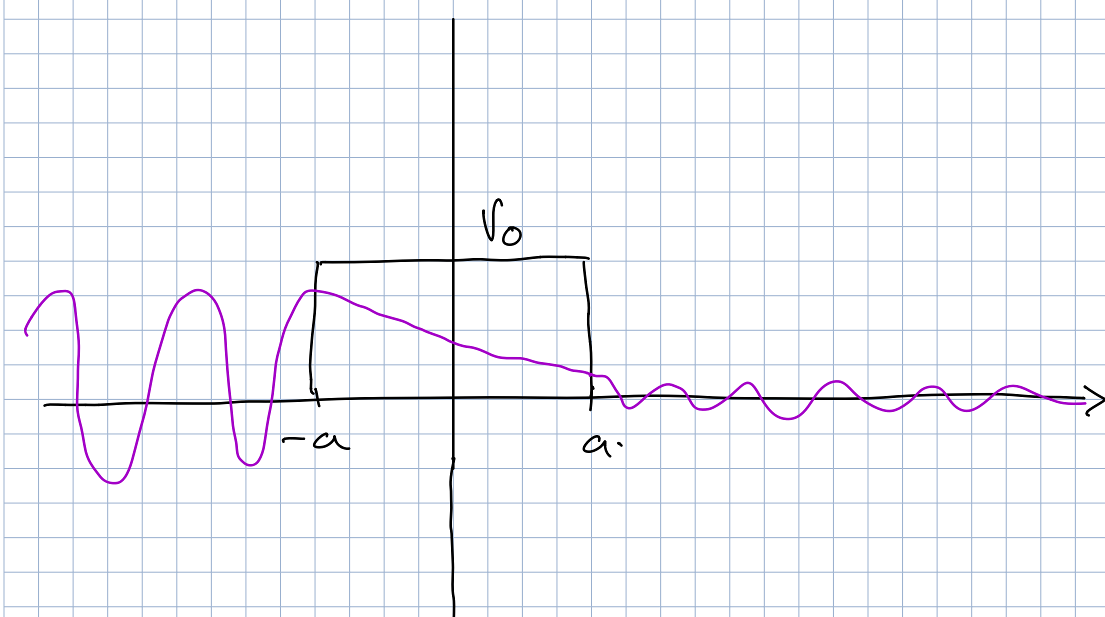
$$\sqrt{(E-V_0)} = i\sqrt{(V_0-E)}$$

$$\sin(ix) = i\sinh(x)$$

$$\sin^2(ix) = -\sinh^2(x)$$

$$\frac{1}{E-V_0} = \frac{-1}{V_0-E}$$

$$T^{-1} = 1 + \frac{V_0^2}{4E(V_0-E)}\sinh^2\left(\frac{2a}{\hbar}\sqrt{2m(V_0-E)}\right)$$



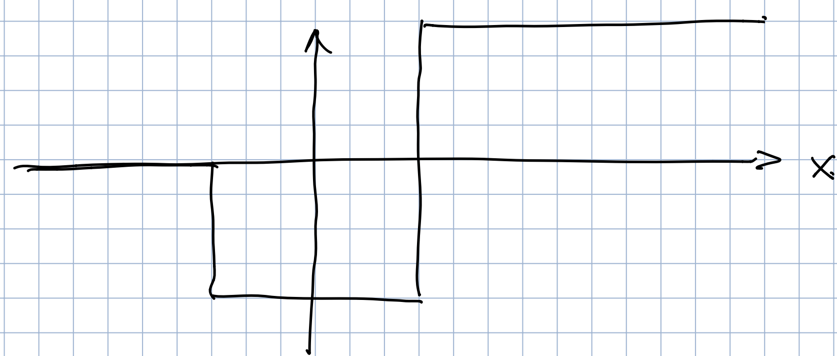
$$E = V_0$$

$$\sinh x \simeq x + \dots$$

$$\frac{e^{x} - e^{-x}}{2}$$

$$T^{-1} = 1 + \frac{V_0^{2}}{4E\cancel{(V_0 - E)}} \underbrace{\left(\frac{2a}{\hbar} \sqrt{2m(V_0 - E)} \right)^{2}}_{\left(\frac{2a}{\hbar}\right)^{2} 2m \cancel{(V_0 - E)}}$$

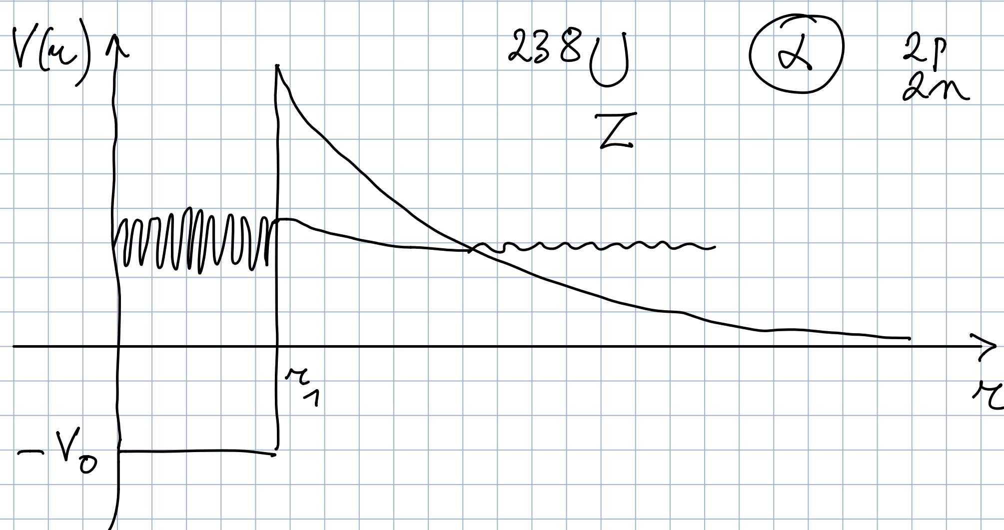
$$T^{-1} = 1 + \frac{2m}{4E} V_0^{2} \left(\frac{2a}{\hbar} \right)^{2}.$$



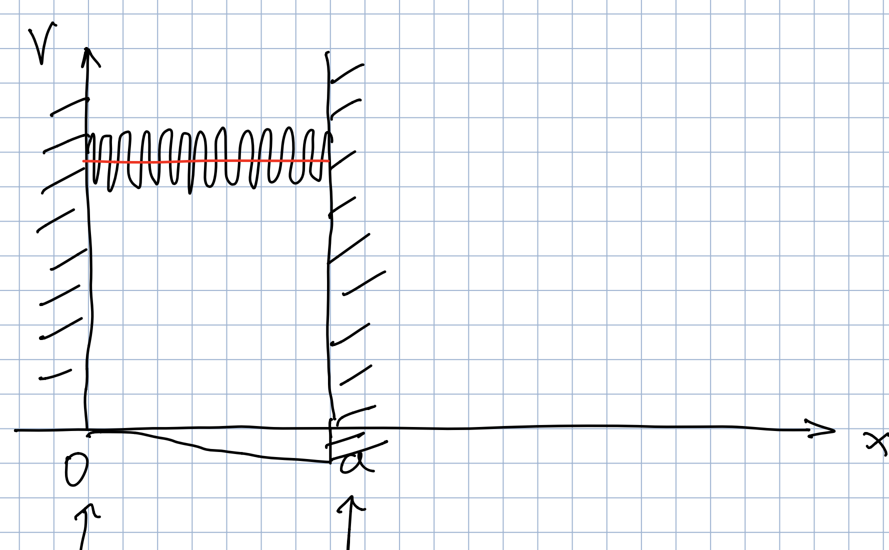
⟹ 2.7 WKB NÄHERUNG

WENTZEL, KRAMERS, BRILLIOIN

SEMI-KLASSISCHE METHODE



VARIATION V(x) LANGSAM
IM VERGLEICH ZUR λ



$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\psi = E\psi$$

$$\frac{d^2\psi}{dx^2} = \underbrace{-\frac{2m}{\hbar^2}\big(E - V(x)\big)}_{-\,k^2(x)}\psi$$

$$k(x) \equiv \frac{1}{\hbar}\sqrt{2m\big(E - V(x)\big)}$$

$$\longrightarrow \quad \frac{d^2\psi}{dx^2} = -k^2(x)\,\psi \qquad E > V(x)$$

$$\psi(x) = A(x)\, e^{i\phi(x)}$$

A & ϕ REAL

$$\psi' = A' e^{i\phi} + iA\phi' e^{i\phi}$$

$$= \left(A' + iA\phi'\right) e^{i\phi}$$

$$\psi'' = \left(A'' + iA'\phi' + iA\phi''\right) e^{i\phi}$$

$$+ \left(A' + iA\phi'\right) i\phi' e^{i\phi}$$

$$= \left(A'' + 2iA'\phi' + iA\phi'' - A(\phi')^2\right) e^{i\phi}$$

$$A'' + 2iA'\phi' + iA\phi'' - A(\phi')^2 = -k^2(x)\, A$$

REAL TEIL $\quad A'' - A(\phi')^2 = -k^2(x)\, A$

IM TEIL $\quad 2A'\phi' + A\phi'' = 0$

$$\Downarrow$$

$$\underbrace{\frac{d}{dx}\left(A^2\phi'\right)}_{2AA'\phi' + A^2\phi''} = 0$$

$$= A\left(2A'\phi' + \phi''\right)$$

$$A^2\phi' = C$$

$$A(x) = \frac{C}{\sqrt{\phi'(x)}}$$

$$\rightsquigarrow \phi(x)$$

$$A'' - A(\phi')^2 = -k^2(x)\, A$$

$$A'' \simeq 0 \qquad (\text{WKB})$$

$$\phi'^2 = k^2(x)$$

$$= \frac{2m}{\hbar^2}\left(E - V(x)\right)$$

$$\downarrow$$

$$\frac{d\phi}{dx} = \pm k(x)$$

$$\phi(x) = \pm \int^x dx\; k(x)$$

$$\boxed{\psi(x) \simeq \frac{C}{\sqrt{k(x)}}\, e^{\pm i \int dx\, k(x)}}$$

<u>BEISPIEL</u> UNENDLICHER POT. TOPF MIT LANGSAM VARIERENDER BODEN

$$\psi(x) = \frac{1}{\sqrt{k(x)}} \left\{ C_+ e^{i\phi(x)} + C_- e^{-i\phi(x)} \right\}$$

$$\phi(x) = \frac{\sqrt{2m}}{\hbar} \int_0^x dx \sqrt{E - V(x)}$$

$$\psi(x) = \frac{1}{\sqrt{k(x)}} \left\{ C_1 \sin \phi(x) + C_2 \cos \phi(x) \right\}$$

$$\psi(0) = 0 \longrightarrow \phi(0) = 0$$

$$\Downarrow$$

$$\psi(a) = 0. \qquad C_2 = 0.$$

$$\psi(x) = \frac{C_1}{\sqrt{k(x)}} \sin \phi(x)$$

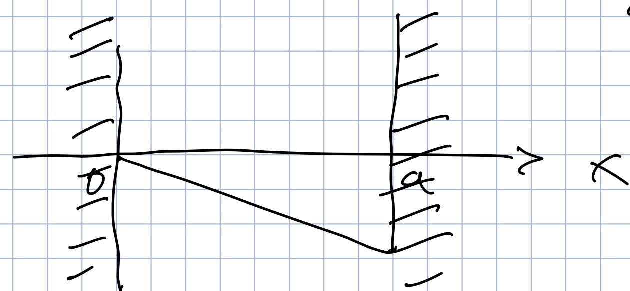
$$\Phi(a) = n\pi \qquad n = 1, 2, 3, \ldots$$

$$\frac{\sqrt{2m}}{\hbar}\int_0^a dx \, \sqrt{E_n - V(x)} = n\pi$$

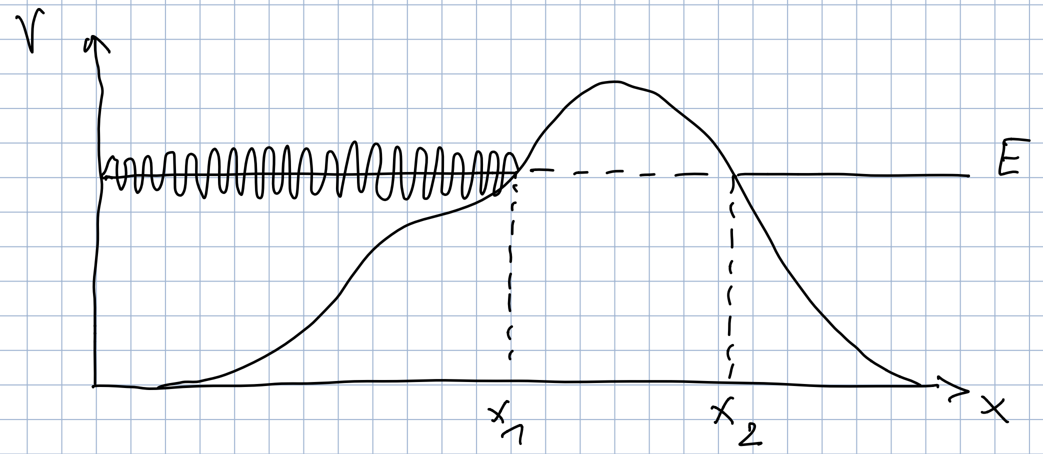
$\underline{\underline{V = 0}}$ $\qquad E_n = \frac{\hbar^2}{2m} k_n^2$

$$k_n \cdot a = n\pi$$

$$k_n = n\frac{\pi}{a}$$



↳ 'TUNNELING' DURCH EINE BARRIERE



$$\lambda \ll (x_2 - x_1)$$

$$x_1 < x < x_2$$

$$\frac{d^2\psi}{dx^2} = \frac{1}{\hbar^2}\, 2m \underbrace{\left[V(x) - E\right]}_{>0} \psi$$

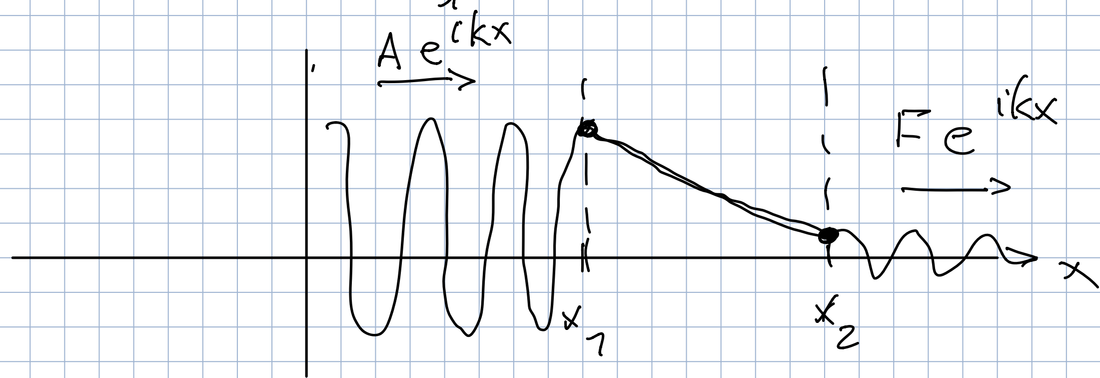
$$= \kappa^2(x)\, \psi$$

$$\kappa(x) = \frac{\sqrt{2m}}{\hbar} \sqrt{V(x) - E}$$

$$\psi(x) = \frac{C_1}{\sqrt{K(x)}}\, e^{\int dx\, K(x)} + \frac{C_2}{\sqrt{K(x)}}\, e^{-\int dx\, K(x)}$$

$$\downarrow \qquad x_2 - x_1 \gg \lambda$$

$$\int_{x_1}^{x} dx\, K(x) \sim (x - x_1)$$



$$C_1 = 0$$

$$\psi(x) \simeq \frac{C_2}{\sqrt{K(x)}}\, e^{-\int_{x_1}^{x} dx\, K(x)}$$

$$x_1 < x < x_2$$

$$\frac{|F|}{|A|} \sim e^{-\int_{x_1}^{x_2} dx\, K(x)}$$

$$T = \frac{|F|^2}{|A|^2} = e^{-2\gamma}$$

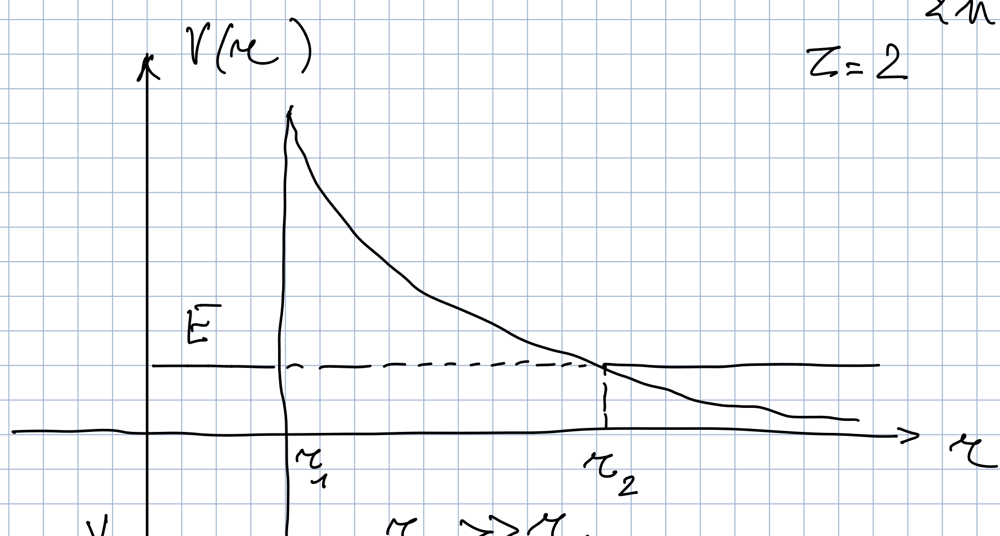
$$\rightsquigarrow \quad \gamma \equiv \int_{x_1}^{x_2} dx\, K(x)$$

$\Rightarrow$ GAMOW THEORIE DES α- ZERFALLS

^{238}U

α- EMISSION $\qquad \alpha$: 2p
2n

Z=2



$r_2 >> r_1$

RESTKERN : LADUNG Z

COULOMB REPULSION

$$V(r) = + \frac{2Z\ e^2}{4\pi\varepsilon_0\ r} = C\ \frac{Z}{r}$$

$$E = \frac{2Ze^2}{4\pi\varepsilon_0 r_2} = C\,\frac{Z}{r_2}$$

$$T \sim e^{-2\gamma}$$

$$\gamma = \int_{r_1}^{r_2} dr \quad K(x)$$

$$= \frac{\sqrt{2m}}{\hbar} \int_{r_1}^{r_2} dr \ \sqrt{V(r) - E}$$

$$= \frac{\sqrt{2m}}{\hbar} \int_{r_1}^{r_2} dr \ \sqrt{C\,\frac{Z}{r} - E}$$

$$= \frac{\sqrt{2m}}{\hbar} \int_{r_1}^{r_2} dr \sqrt{\underbrace{C\frac{Z}{r_2}}_{E}\frac{r_2}{r} - E}$$

$$= \frac{\sqrt{2mE}}{\hbar} \int_{r_1}^{r_2} dr \sqrt{\frac{r_2}{r} - 1}$$

$$r < r_2$$

$$r = r_2 \sin^2\theta$$

$$\frac{r_2}{r} - 1 = \frac{1}{\sin^2\theta} \cos^2\theta$$

$$\sqrt{\frac{r_2}{r} - 1} = \frac{\cos\theta}{\sin\theta}$$

$$dr = 2r_2 \sin\theta \sqrt{1 - \sin^2\theta}\, d\theta$$

$$\gamma = \frac{\sqrt{2mE}}{\hbar} 2r_2 \int_{\sin^{-1}\sqrt{\frac{r_1}{r_2}}}^{\pi/2} d\theta \, \cancel{\sin\theta} \, \frac{\cos^2\theta}{\cancel{\sin\theta}}$$

$$r = r_2 \implies \sin^2\theta = 1 \qquad \theta = \pi/2$$

$$r = r_1 \implies \sin^2\theta = \frac{r_1}{r_2}$$

$$\theta = \sin^{-1}\sqrt{\frac{r_1}{r_2}}$$

$$\gamma = \frac{\sqrt{2mE}}{\hbar}\, 2r_2 \int_{\sin^{-1}\sqrt{\frac{r_1}{r_2}}}^{\pi/2} d\theta \;\; \frac{1}{2}\left(1 + \cos 2\theta\right)$$

$$= \frac{\sqrt{2mE}}{\hbar}\, 2r_2\, \frac{1}{2}\left[\theta + \frac{1}{2}\sin 2\theta\right]_{\sin^{-1}\sqrt{\frac{r_1}{r_2}}}^{\pi/2}$$

$\sin\theta \cos\theta$

$$\gamma = \frac{\sqrt{2mE}}{\hbar}\, r_2 \left\{ \frac{\pi}{2} - \sin^{-1}\sqrt{\frac{r_1}{r_2}} - \sqrt{\frac{r_1}{r_2}}\sqrt{1 - \frac{r_1}{r_2}} \right\}$$

$$r_2 \gg r_1$$

$$\sin^{-1}\sqrt{\frac{r_1}{r_2}} \approx \sqrt{\frac{r_1}{r_2}}$$

$$\sqrt{1-\frac{r_1}{r_2}} \approx 1$$

$$\gamma = \frac{\sqrt{2mE}}{\hbar} r_2 \left\{ \frac{\pi}{2} - 2\sqrt{\frac{r_1}{r_2}} \right\}$$

$$\gamma = \frac{\sqrt{2mE}}{\hbar} \left\{ \frac{\pi}{2} r_2 - 2\sqrt{r_1 r_2} \right\}$$

$$E = C \frac{Z}{r_2}$$

$$r_2 = \frac{CZ}{E}$$

$$\gamma = \frac{\sqrt{2m}}{\hbar} \left\{ \frac{\pi}{2} \frac{CZ}{\sqrt{E}} - 2\sqrt{r_1}\sqrt{CZ} \right\}$$

$$\gamma = \underbrace{C_1}\, \frac{Z}{\sqrt{E}} - \underbrace{C_2}\, \sqrt{Z r_1}$$

FÜR ^{238}U

LEBENSDAUER $\sim 1/e^{-2\gamma}$

$\simeq 4.5 \; 10^{9}$ y