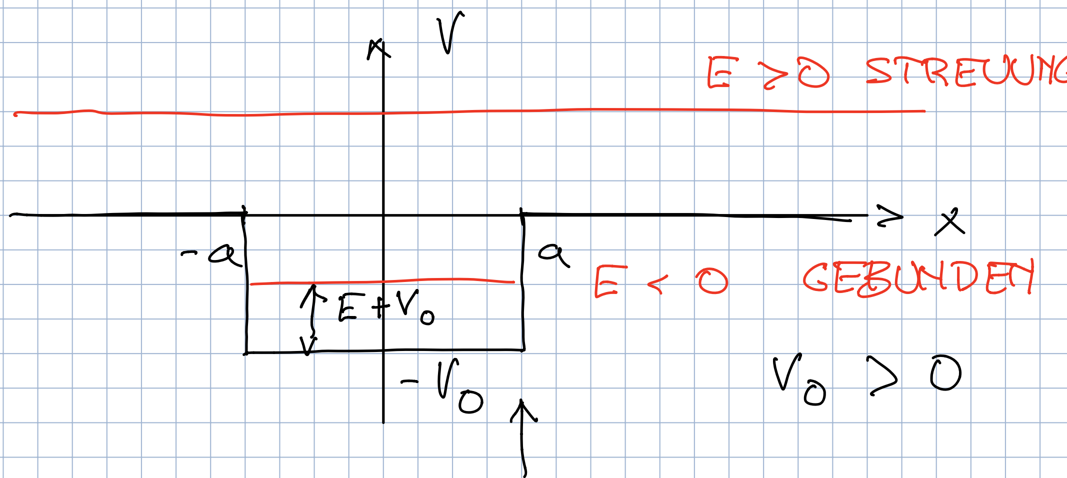
$\Longrightarrow$ VORLESUNG 7 QM

2.6 ENDLICHER POTENTIALTOPF



$\Longrightarrow$ $\underline{E < 0}$ GEBUNDEN

$\underset{\sim\sim\sim}{x < -a}$ & $\underset{\sim\sim\sim}{x > a}$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi$$

$$E = -\frac{\hbar^2}{2m}K^2$$

$$\frac{d^2\psi}{dx^2} = -K^2\psi \qquad (K > 0)$$

$$\psi(x) = A\,e^{Kx} + B\,e^{-Kx}$$

$x < -a$ $\quad \psi(-\infty) = 0.$

$$\psi(x) = A e^{Kx}$$

$x > a$ $\quad \psi(+\infty) = 0$

$$\psi(x) = B e^{-Kx}$$

$-a < x < a$

$$-\frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} - V_0 \psi = E \psi$$

$$-\frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} = (E + V_0) \psi$$

$E < 0$

$E + V_0 > 0$

$$E + V_0 = \frac{\hbar^2}{2m} \ell^2$$

$$\frac{d^2\psi}{dx^2} = -\ell^2 \psi$$

$$\psi(x) = C \sin(\ell x) + D \cos(\ell x)$$

- $\underline{\psi(-x) = \psi(x)}$ SYMMETRISCH GERADE

$$\psi(x) = D \cos(\ell x) \qquad \underline{x < a}$$

$$(1) \quad \psi(a-) = \psi(a+)$$

$$\frac{d\psi}{dx}(a-) = \frac{d\psi}{dx}(a+)$$

$$(1) \quad D \cos(\ell a) = B e^{-Ka}$$

$$(2) \quad -D\ell \sin(\ell a) = -BK e^{-Ka}$$

$$\frac{(2)}{(1)} \Rightarrow \ell a \tan(\ell a) = Ka$$

$$z \equiv \ell a$$

$$z \tan z = \underbrace{Ka}$$

$$(*) \quad z^2 = \ell^2 a^2 = \frac{2m}{\hbar^2} a^2 (E + V_0)$$

$$K^2 = -\frac{2m}{\hbar^2} E$$

$$\rightsquigarrow \quad z_0^2 = \frac{2m}{\hbar^2} a^2 V_0 \qquad \text{GEKANNT}$$

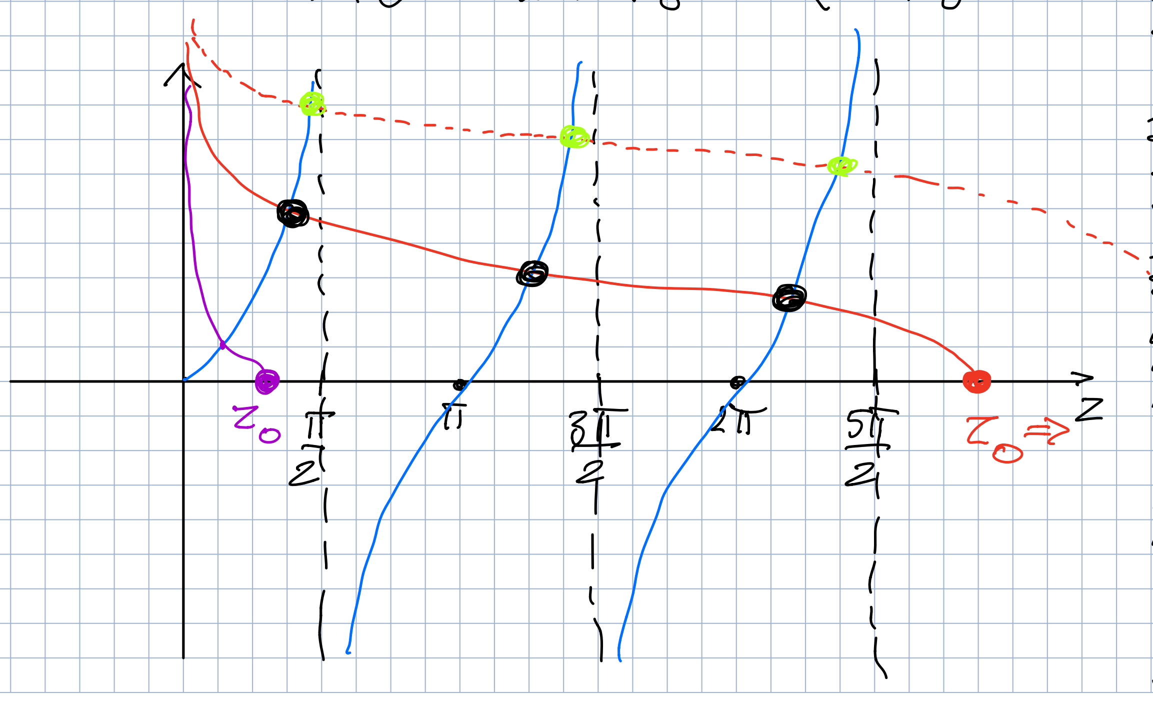
$$(*) \quad z^2 - z_0^2 = \frac{2m}{\hbar^2} a^2 E = - a^2 K^2$$

$$z \tan z = \sqrt{z_0^2 - z^2}$$

$$\tan z = \sqrt{\left(\frac{z_0}{z}\right)^2 - 1}$$

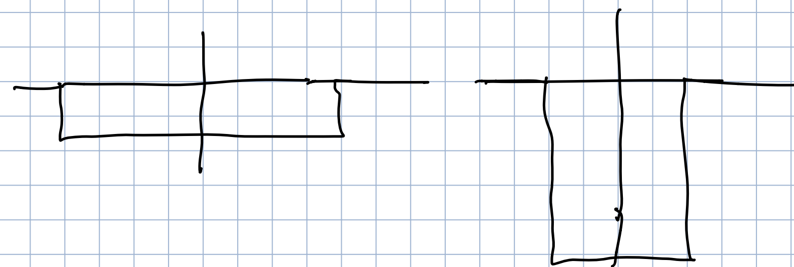
$$z_0 > z$$

$$z^2 \sim E + V_0 < V_0$$



SPEZIAL FÄLLE

- $z_0 \gg$ $\qquad a \gg \quad$ ODER $\quad V_0 \gg$



$$z \longrightarrow z_n = n\frac{\pi}{2} \qquad (n = 1, 3, 5, \dots)$$

$$z_n^2 = \frac{2m}{\hbar^2} a^2 (E_n + V_0) = n^2 \left(\frac{\pi}{2}\right)^2$$

$$E_n = -V_0 + \frac{\hbar^2}{2m} n^2 \left(\frac{\pi}{2a}\right)^2$$

$n = 1, 3, 5, \dots$

SYMMETRISCHE LÖSUNG

- $z_0 \ll$ $\qquad z_0 < \frac{\pi}{2}$

$$\Downarrow$$

IMMER 1 LÖSUNG

• $\underline{\psi(-x) = -\psi(x)}$ UNGERADE

$$
\begin{cases}
\psi(x) = C\sin(\ell x) & -a < x < a \\
\psi(x) = Be^{-Kx} & x > a
\end{cases}
$$

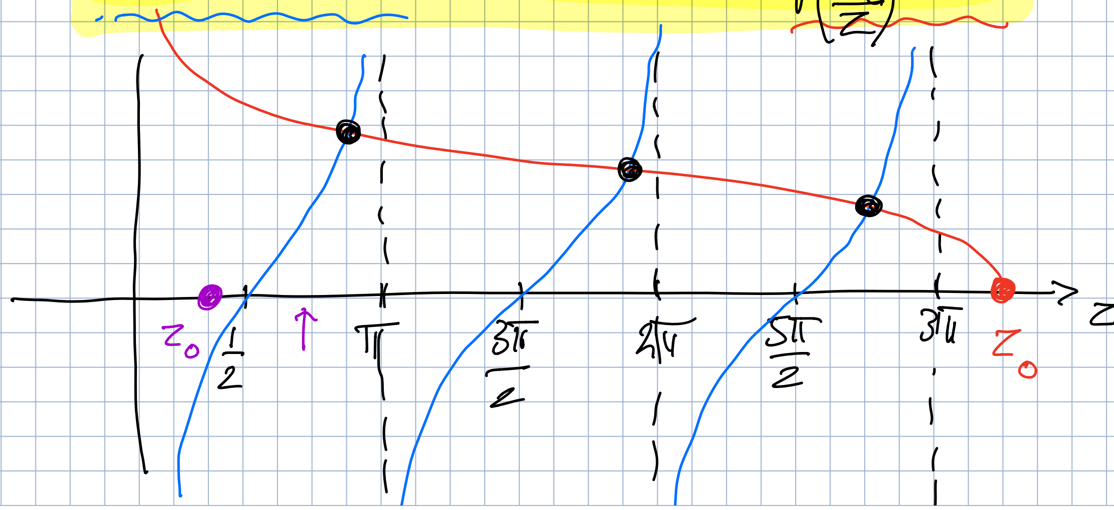
(1) $\psi(a) \Rightarrow Be^{-Ka} = C\sin(\ell a)$

(2) $\frac{d\psi}{dx}(a) \Rightarrow -BKe^{-Ka} = C\ell\cos(\ell a)$

$$\frac{(2)}{(1)} \Rightarrow -K = \ell\cot(\ell a)$$

$$\ell a = z$$

$$\boxed{-\cot z = Ka = \sqrt{\left(\frac{z_0}{z}\right)^2 - 1}}$$



$z_0 >>$

$$z \longrightarrow z_n = \tilde{n}\pi \qquad \tilde{n} = 1, 2, 3 \ldots$$

$$n = 2\tilde{n} \qquad = (2\tilde{n})\left(\frac{\pi}{2}\right) \qquad 2\tilde{n} = 2, 4, 6$$

$$\boxed{E_n = -V_0 + \frac{\hbar^2}{2m}\, n^2 \left(\frac{\pi}{2a}\right)^2}$$

$$n = 2, 4, 6$$

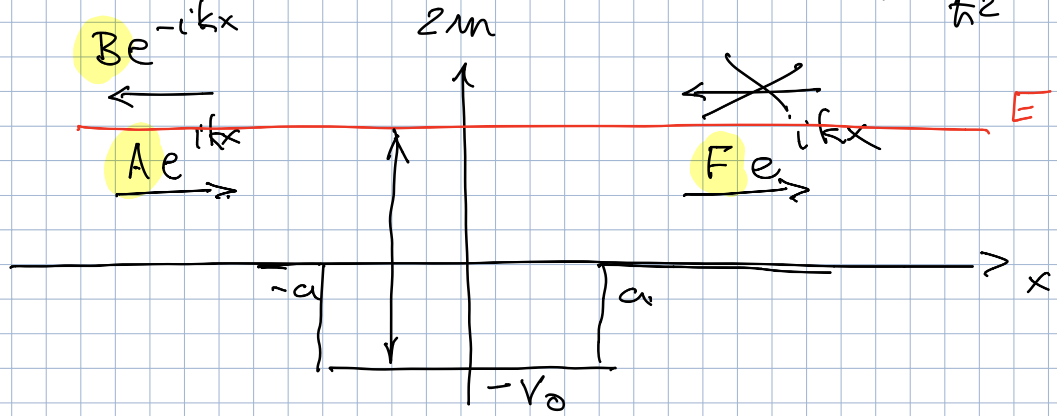
$z_0 < \frac{\pi}{2}$ KEINE LÖSUNG

(UNGERADE)

$\Rightarrow$ $E > 0$ STREUZUSTÄNDE

$$\left.\begin{array}{l} x < -a \\ x > a \end{array}\right\} \quad V = 0$$

$$E = \frac{\hbar^2}{2m} k^2 \quad \Rightarrow \quad k = \sqrt{\frac{2mE}{\hbar^2}}$$



$\underline{x < -a}$ $\quad \psi(x) = A\, e^{ikx} + B\, e^{-ikx}$

$\underline{x > a}$ $\quad \psi(x) = F\, e^{ikx} + \cancel{G\, e^{-ikx}}$

$\underline{-a < x < a:}$ $\quad E + V_0 = \frac{\hbar^2}{2m} \ell^2$

$$\Rightarrow \psi(x) = C \sin(\ell x) + D \cos(\ell x)$$

RANDBEDINGUNGEN

$$\psi(-a) \Rightarrow A e^{-ika} + B e^{ika} = -C\sin(\ell a) + D\cos(\ell a)$$

$$\frac{d\psi}{dx}(-a) \Rightarrow ik\left[A e^{-ika} - B e^{ika}\right] = \ell\left[C\cos(\ell a) + D\sin \ell a\right]$$

$$\psi(a) \Rightarrow F e^{ika} = C\sin(\ell a) + D\cos(\ell a)$$

$$\frac{d\psi}{dx}(a) \Rightarrow ik F e^{ika} = \ell\left[C\cos(\ell a) - D\sin(\ell a)\right]$$

$\rightsquigarrow C$

$$\sin(\ell a)\,\ell\left(F e^{ika}\right) + ik F e^{ika} \cos(\ell a)$$

$$= C\,\ell\left(\sin^2(\ell a) + \cos^2(\ell a)\right)$$

$$= C\ell$$

$$C = F e^{ika}\left(\sin \ell a + \frac{ik}{\ell} \cos \ell a\right)$$

$\rightsquigarrow D$

$$-\cos(la)\, l\, (F e^{ika}) + ik F e^{ika} \sin(la)$$

$$= -D l$$

$$\boxed{D = F e^{ika} \left(\cos(la) - \frac{ik}{l} \sin(la) \right)}$$

$\rightsquigarrow \frac{F}{A}$?

(1) $$A e^{-ika} + B e^{ika} = -C \sin(la) + D \cos(la)$$

(2) $$ik\left[A e^{-ika} - B e^{ika}\right] = l\left[C \cos(la) + D \sin la\right]$$

(1) $$A e^{-ika} + B e^{ika} = F e^{ika} \Big\{ -\sin^2 la - \frac{ik}{l} \sin la \cos la + \cos^2 la - \frac{ik}{l} \sin la \cos la \Big\}$$

(1) $$A e^{-ika} + B e^{ika} = F e^{ika} \Big\{ \cos 2la - \frac{ik}{l} \sin 2la \Big\}$$

(2) $$ik\left[A e^{-ika} - B e^{ika}\right] = l F e^{ika} \cdot \Big\{ \sin 2la + \frac{ik}{l} \cos 2la \Big\}$$

$$\frac{F}{A} \curvearrowright B \text{ eliminieren}$$

$$ik\,(1) + (2)$$

$$2ik\, A\, e^{-ika} = F e^{ika} \Big\{ 2ik \cos 2\ell a + \left(\frac{k^2}{\ell} + \ell\right) \sin 2\ell a \Big\}$$

$$\frac{F}{A} = \frac{e^{-2ika}}{\cos(2\ell a) - \frac{i}{2k\ell}\left(k^2+\ell^2\right)\sin(2\ell a)}$$

$$T = \frac{|F|^2}{|A|^2}$$

$$T^{-1} = \cos^2(2\ell a) + \frac{\left(k^2+\ell^2\right)^2}{4k^2\ell^2}\sin^2(2\ell a)$$

$$= 1 + \left(\frac{\left(k^2+\ell^2\right)^2}{4k^2\ell^2} - 1\right)\sin^2(2\ell a)$$

$$T^{-1} = 1 + \frac{(k^2 - \ell^2)^2}{4k^2\ell^2} \sin^2(2\ell a)$$

$T < 1$ ‽

$T^{-1} > 1$

$$k^2 = \frac{2m}{\hbar^2} E$$

$$\ell^2 = \frac{2m}{\hbar^2} (E + V_0)$$

$$k^2 - \ell^2 = -\left(\frac{2m}{\hbar^2}\right) V_0$$

$$T^{-1} = 1 + \frac{V_0^2}{4E\,(E + V_0)} \sin^2\left(\frac{2a}{\hbar}\sqrt{2m(E+V_0)}\right)$$

$T \leq 1$ ‽

$\underline{T = 1}$ PERFEKTE TRANSMISSION

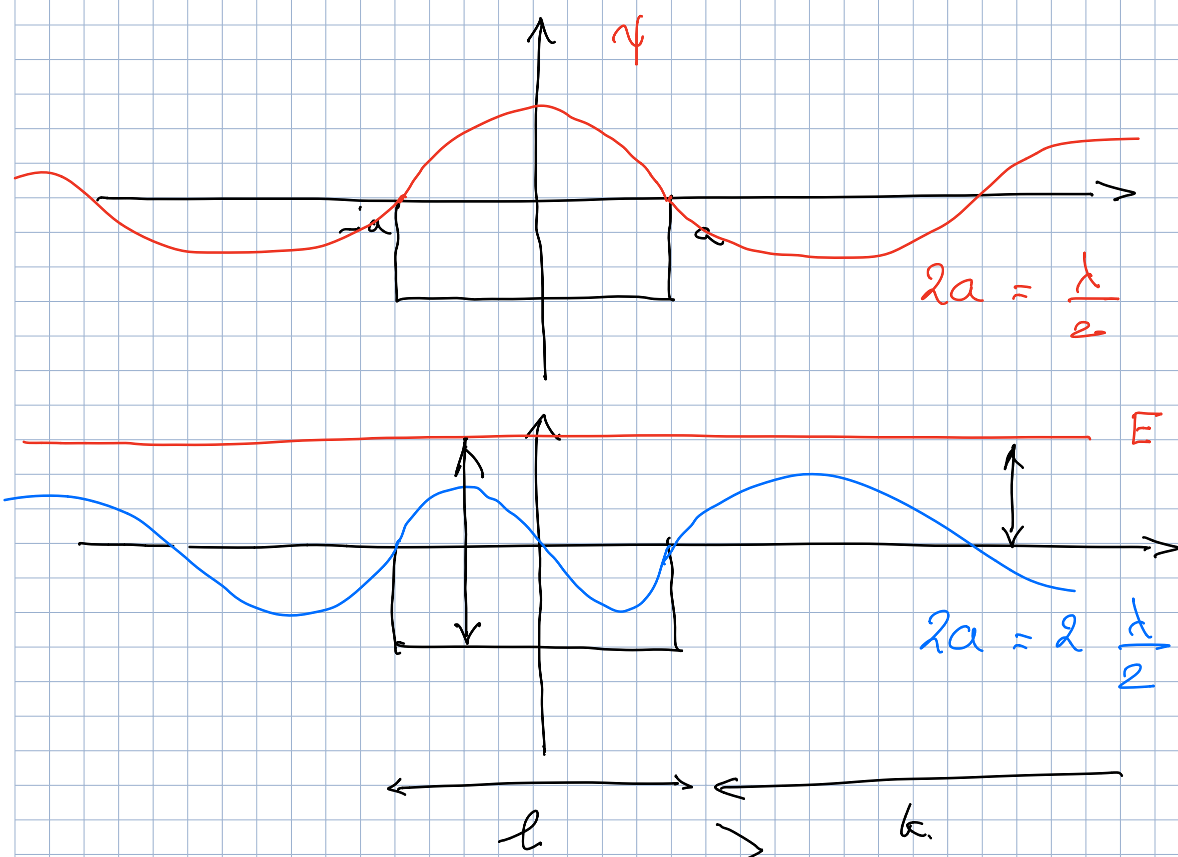
$$\frac{2a}{\hbar}\sqrt{2m(E+V_0)} = n\pi \qquad n = 1, 2, 3, \ldots$$

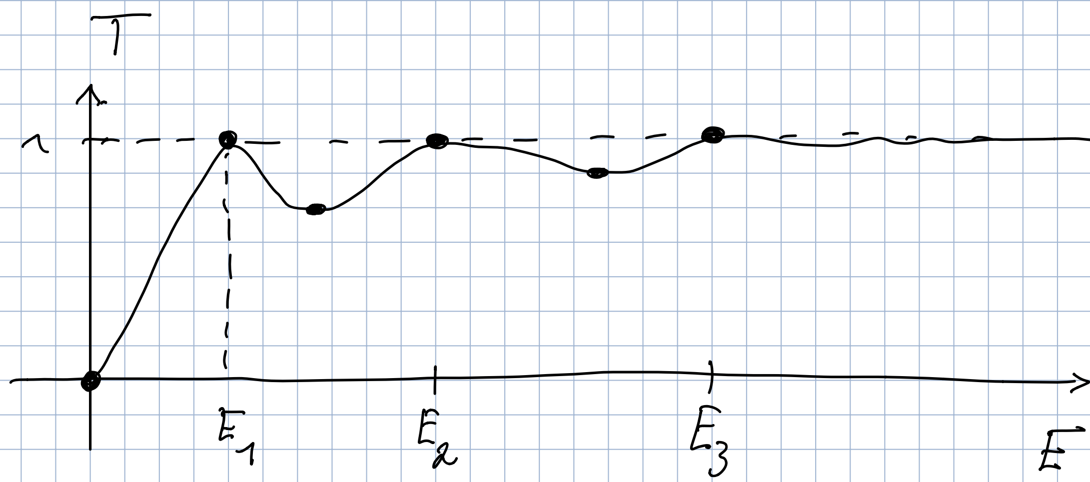
$$\underbrace{2a}\,\underbrace{\ell} = n\pi$$

$$\ell = \frac{2\pi}{\lambda}$$

$$(2a)\,\frac{2\pi}{\lambda} = n\pi \qquad n = 1, 2, 3, \ldots$$

$$\boxed{2a = n\left(\frac{\lambda}{2}\right)} \qquad n = 1, 2, 3, \ldots$$





$$\frac{2a}{\hbar}\sqrt{2m(E+V_0)} = n\pi$$

$$E_n = -V_0 + \frac{\hbar^2}{2m}\, n^2 \left(\frac{\pi}{2a}\right)^2.$$