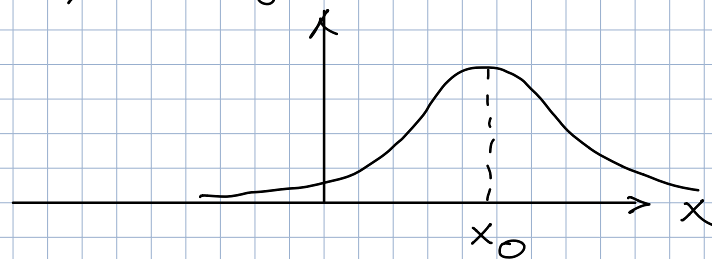
$\Longrightarrow$ VORLESUNG 2 QM

$$\rho(x) = A \, e^{-\lambda (x - x_0)^2}$$

$$\int_{-\infty}^{+\infty} dx \, \rho(x) = 1 \quad \Longrightarrow \quad A = \sqrt{\frac{\lambda}{\pi}}$$

$$\langle x \rangle = x_0$$



$$\sigma^2 = \langle (x - \langle x \rangle)^2 \rangle$$

$$= \sqrt{\frac{\lambda}{\pi}} \int_{-\infty}^{+\infty} dx \, (x - x_0)^2 \, e^{-\lambda (x - x_0)^2}$$

$$\Big\downarrow \quad x' = x - x_0$$

$$= \sqrt{\frac{\lambda}{\pi}} \int_{-\infty}^{+\infty} dx' \, x'^2 \, e^{-\lambda x'^2}$$

$$\int_{-\infty}^{+\infty} dx \, e^{-\lambda x^2} = \sqrt{\frac{\pi}{\lambda}}$$

$$\int_{-\infty}^{+\infty} dx \; x^2 \, e^{-\lambda x^2} = -\frac{d}{d\lambda} \underbrace{\int_{-\infty}^{+\infty} dx \; e^{-\lambda x^2}}_{\sqrt{\frac{\pi}{\lambda}}}$$

$$= \frac{1}{2} \sqrt{\frac{\pi}{\lambda}} \; \frac{1}{\lambda}$$

$$\sigma^2 = \sqrt{\frac{\lambda}{\pi}} \cdot \frac{1}{2} \sqrt{\frac{\pi}{\lambda}} \; \frac{1}{\lambda}$$

$$\sigma^2 = \frac{1}{2\lambda} \qquad \lambda = \frac{1}{2\sigma^2}$$

$$\rho(x) = \sqrt{\frac{\lambda}{\pi}} \; e^{-\lambda \left(x - \langle x \rangle\right)^2}$$

$$= \frac{1}{\sqrt{2\pi}} \; \frac{1}{\sigma} \; e^{-\frac{1}{2\sigma^2}\left(x - \langle x \rangle\right)^2}$$

$$P_{1\sigma} \quad x \in \left[\langle x \rangle - \sigma , \langle x \rangle + \sigma \right]$$

$$= \int_{\langle x \rangle - \sigma}^{\langle x \rangle + \sigma} dx \; \rho(x)$$

$$P_{1\sigma} \sim 68\,\%$$

$$P_{2\sigma} \sim 95\,\%$$

$$P_{3\sigma} \sim 99.7\,\%$$

1.3 NORMIERUNG

$$\Psi(x,t)$$

$$- \frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} + V \Psi = i\hbar \frac{\partial \Psi}{\partial t}$$

$$\rho(x,t) = |\Psi(x,t)|^2 = \Psi^* \Psi$$

WAHRSCHEINLICHKEIT

$$\Psi = |\Psi| e^{i\phi}$$

$$\mathrm{Re}\,\Psi = |\Psi| \cos\phi$$

$$\mathrm{Im}\,\Psi = |\Psi| \sin\phi$$

$$\int_{-\infty}^{+\infty} dx \, |\Psi(x,t)|^2 = 1$$

$\Psi \curvearrowright A\Psi$ AUCH LÖSUNG

∇ QUADRATISCH INTEGRIÈRBARE

0 LÖSUNG

$$\int_{-\infty}^{+\infty} dx \, |\Psi(x,0)|^2 = 1$$

$$\int_{-\infty}^{+\infty} dx \, |\Psi(x,t)|^2 = 1$$

$\frac{d}{dt}$

$$\int_{-\infty}^{+\infty} dx \, \frac{\partial}{\partial t} |\Psi(x,t)|^2 = 0$$

$$\frac{\partial}{\partial t} |\Psi|^2 = \frac{\partial}{\partial t} (\Psi^* \Psi)$$

$$= \frac{\partial \Psi^*}{\partial t} \Psi + \Psi^* \frac{\partial \Psi}{\partial t}$$

$$\rightarrow \quad i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} + V \Psi$$

$$\Big\downarrow \, *$$

$$\rightarrow \quad \underbrace{-i\hbar} \frac{\partial \Psi^*}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi^*}{\partial x^2} + V^* \Psi^*$$

V $\underline{\underline{\text{REAL}}}$

$V^* = V$

$$\frac{\partial \Psi^*}{\partial t} \Psi = -\frac{i\hbar}{2m} \frac{\partial^2 \Psi^*}{\partial x^2} \Psi + \cancel{\frac{i}{\hbar} V \Psi^* \Psi}$$

$$\Psi^* \frac{\partial \Psi}{\partial t} = +\frac{i\hbar}{2m} \Psi^* \frac{\partial^2 \Psi}{\partial x^2} - \cancel{\frac{i}{\hbar} V \Psi^* \Psi}$$

+

$$\frac{\partial}{\partial t} |\Psi|^2 = \frac{i\hbar}{2m} \left(\Psi^* \frac{\partial^2 \Psi}{\partial x^2} - \frac{\partial^2 \Psi^*}{\partial x^2} \Psi \right)$$

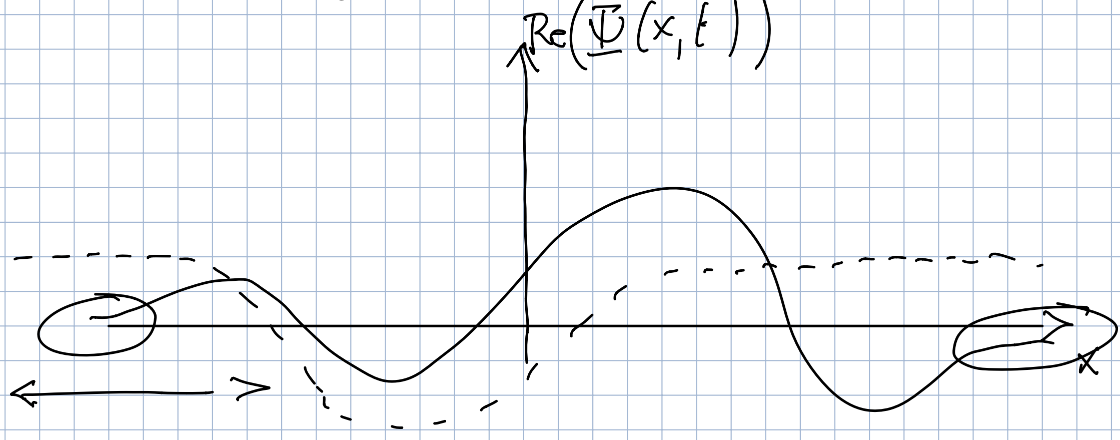
$$= \frac{i\hbar}{2m} \frac{\partial}{\partial x} \left(\Psi^* \frac{\partial \Psi}{\partial x} - \frac{\partial \Psi^*}{\partial x} \Psi \right)$$

$$\int_{-\infty}^{+\infty} dx \, \frac{\partial}{\partial t} |\Psi|^2 = \frac{i\hbar}{2m} \int_{-\infty}^{+\infty} dx \, \underbrace{\frac{\partial}{\partial x}} (\cdots)$$

$$= \frac{i\hbar}{2m} \left[\Psi^* \frac{\partial \Psi}{\partial x} - \frac{\partial \Psi^*}{\partial x} \Psi \right]_{-\infty}^{+\infty}$$

$$\Psi(+\infty, t) = \Psi(-\infty, t) = 0$$

$$\therefore \quad \int_{-\infty}^{+\infty} dx \, \frac{\partial}{\partial t} |\Psi|^2 \overset{!}{=} 0$$



$$\int_{-\infty}^{+\infty} dx \, |\Psi(x,t)|^2 \cancel{= \infty}$$

$$= 1$$

1.4 IMPULS

$$\Psi(x,t)$$

$$\langle x \rangle = \int_{-\infty}^{+\infty} dx \; x \; |\Psi(x,t)|^2$$

$$\langle v \rangle = \frac{d}{dt} \langle x \rangle$$

$$= \int_{-\infty}^{+\infty} dx \; x \; \frac{\partial}{\partial t} |\Psi(x,t)|^2$$

$$\frac{\partial}{\partial t} |\Psi|^2 = \frac{i\hbar}{2m} \left(\Psi^* \frac{\partial^2 \Psi}{\partial x^2} - \frac{\partial^2 \Psi^*}{\partial x^2} \Psi \right)$$

$$= \frac{i\hbar}{2m} \frac{\partial}{\partial x} \left(\Psi^* \frac{\partial \Psi}{\partial x} - \frac{\partial \Psi^*}{\partial x} \Psi \right)$$

$$\langle v \rangle = \frac{i\hbar}{2m} \int_{-\infty}^{+\infty} dx \; x \; \frac{\partial}{\partial x} \left(\Psi^* \frac{\partial \Psi}{\partial x} - \frac{\partial \Psi^*}{\partial x} \Psi \right)$$

$$= -\frac{i\hbar}{2m} \int_{-\infty}^{+\infty} dx \left(\Psi^* \frac{\partial \Psi}{\partial x} - \frac{\partial \Psi^*}{\partial x} \Psi \right) + \cancel{\text{TERM @ } -\infty \;\&\; +\infty}$$

$$= -\frac{i\hbar}{2m}\int_{-\infty}^{+\infty} dx \left(\Psi^* \frac{\partial \Psi}{\partial x} + \Psi^* \frac{\partial \Psi}{\partial x} \right) + \cancel{\text{TERM}}$$

∂

$-\infty$ & $+\infty$

$$\langle v \rangle = \frac{d}{dt}\langle x \rangle = -\frac{i\hbar}{m}\int_{-\infty}^{+\infty} dx \; \Psi^* \frac{\partial \Psi}{\partial x}$$

↳ IMPULS $\quad p = m v$

$$\langle p \rangle \equiv m \langle v \rangle$$

$$= m \frac{d}{dt} \langle x \rangle$$

$$= -i\hbar \int_{-\infty}^{+\infty} dx \; \Psi^* \frac{\partial \Psi}{\partial x}$$

$$= \int_{-\infty}^{+\infty} dx \; \Psi^* \underbrace{\left(-i\hbar \frac{\partial}{\partial x} \right)}_{\hat{p}} \Psi$$

$\hat{p}$ OPERATOR

↳ KINETISCHE ENERGIE

$$T = \frac{p^2}{2m}$$

$$\langle T \rangle = \int_{-\infty}^{+\infty} dx \; \Psi^* \left(\frac{\hat{p}^2}{2m} \right) \Psi$$

$$= \int_{-\infty}^{+\infty} dx \; \Psi^* \left(-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \right) \Psi$$

$$\langle V \rangle = \int_{-\infty}^{+\infty} dx \; \Psi^* \; V \; \Psi$$

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} + V \Psi = i\hbar \frac{\partial \Psi}{\partial t}$$

$$\int dx \, \Psi^*$$

$$\langle T \rangle + \langle V \rangle = \langle E \rangle$$

$$= \int_{-\infty}^{+\infty} dx \; \Psi^* \left(i\hbar \frac{\partial}{\partial t} \right) \Psi$$

$$\langle p \rangle = m \frac{d}{dt}\langle x \rangle = \int_{-\infty}^{+\infty} dx \; \Psi^* \left(-i\hbar \frac{\partial \Psi}{\partial x}\right)$$

KL. $\frac{dp}{dt} = -\frac{\partial V}{\partial x}$

$$\frac{d}{dt}\langle p \rangle = -i\hbar \int_{-\infty}^{+\infty} dx \left(\frac{\partial \Psi^*}{\partial t} \frac{\partial \Psi}{\partial x} + \Psi^* \frac{\partial}{\partial x} \frac{\partial \Psi}{\partial t} \right)$$

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} + V\Psi$$

V REAL

$$-i\hbar \frac{\partial \Psi^*}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi^*}{\partial x^2} + V\Psi^*$$

$$\frac{d}{dt}\langle p \rangle = \int_{-\infty}^{+\infty} dx \left\{ \left(\cancel{-\frac{\hbar^2}{2m} \frac{\partial^2 \Psi^*}{\partial x^2}} + V\Psi^* \right) \frac{\partial \Psi}{\partial x} - \Psi^* \frac{\partial}{\partial x} \left(\cancel{-\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2}} + V\Psi \right) \right\}$$

$$\frac{\partial^2 \Psi^*}{\partial x^2} \frac{\partial \Psi}{\partial x} \rightarrow -\frac{\partial \Psi^*}{\partial x} \frac{\partial^2 \Psi}{\partial x^2} \rightarrow \Psi^* \frac{\partial^3 \Psi}{\partial x^3}$$

$$\frac{d}{dt}\langle p\rangle = \int_{-\infty}^{+\infty} dx \left(\Psi^* \cancel{V \frac{\partial \Psi}{\partial x}} - \Psi^* \frac{\partial}{\partial x}(V\Psi) \right)$$

$$\frac{\partial V}{\partial x}\Psi + \cancel{V\frac{\partial \Psi}{\partial x}}$$

$$\frac{d}{dt}\langle p\rangle = \int_{-\infty}^{+\infty} dx \, \Psi^* \left(-\frac{\partial V}{\partial x}\right)\Psi = \left\langle -\frac{\partial V}{\partial x}\right\rangle$$

KLASSISCH $\quad \frac{dp}{dt} = -\frac{\partial V}{\partial x}$

EHRENFEST THEOREM

1.5 UNSCHÄRFE PRINZIP

$\Psi(x,t)$

$p \longleftrightarrow \lambda$

DE BROGLIE

$$p = \frac{h}{\lambda} = \frac{2\pi\hbar}{\lambda}$$

$$\sigma_x \cdot \sigma_p \geq \frac{\hbar}{2}$$

$$\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$$