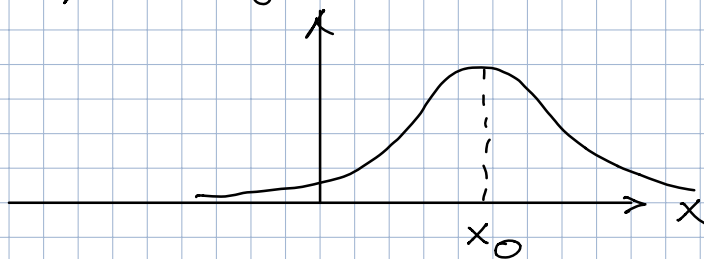


⇒ VORLESUNG 2 QM

$$P(x) = A e^{-\lambda (x-x_0)^2}$$

$$\int_{-\infty}^{+\infty} dx P(x) = 1 \Rightarrow A = \sqrt{\frac{\lambda}{\pi}}$$

$$\langle x \rangle = x_0$$



$$\sigma^2 = \langle (x - \langle x \rangle)^2 \rangle$$

$$= \sqrt{\frac{\lambda}{\pi}} \int_{-\infty}^{+\infty} dx (x - x_0)^2 e^{-\lambda (x - x_0)^2}$$

$$\downarrow \quad x' = x - x_0$$
$$= \sqrt{\frac{\lambda}{\pi}} \int_{-\infty}^{+\infty} dx' x'^2 e^{-\lambda x'^2}$$

$$\int_{-\infty}^{+\infty} dx e^{-\lambda x^2} = \sqrt{\frac{\pi}{\lambda}}$$

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$$\int_{-\infty}^{+\infty} dx x^2 e^{-\lambda x^2} = - \frac{d}{d\lambda} \underbrace{\int_{-\infty}^{+\infty} dx e^{-\lambda x^2}}_{\sqrt{\frac{\pi}{\lambda}}}$$

$$= \frac{1}{2} \sqrt{\frac{\pi}{\lambda}} \frac{1}{\lambda}$$

$$\sigma^2 = \sqrt{\frac{\lambda}{\pi}} \cdot \frac{1}{2} \sqrt{\frac{\pi}{\lambda}} \frac{1}{\lambda}$$

$$\sigma^2 = \frac{1}{2\lambda} \quad \lambda = \frac{1}{2\sigma^2}$$

$$P(x) = \sqrt{\frac{\lambda}{\pi}} e^{-\lambda (x - \langle x \rangle)^2}$$

$$= \frac{1}{\sqrt{2\pi}} \frac{1}{\sigma} e^{-\frac{1}{2\sigma^2} (x - \langle x \rangle)^2}$$



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$$P_{1\sigma} \quad x \in [\langle x \rangle - \sigma, \langle x \rangle + \sigma]$$

$$= \int_{\langle x \rangle - \sigma}^{\langle x \rangle + \sigma} dx \rho(x)$$

$$P_{1\sigma} \sim 68\%$$

$$P_{2\sigma} \sim 95\%$$

$$P_{3\sigma} \sim 99.7\%$$

1.3 NORMIERUNG

$$\Psi(x, t)$$

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} + V \Psi = i\hbar \frac{\partial \Psi}{\partial t}$$

$$\rho(x, t) = |\Psi(x, t)|^2 = \Psi^* \Psi$$

WAHRSCHEINLICHKEIT

$$\underline{\Psi} = |\Psi| e^{i\phi}$$

$$\text{Re } \underline{\Psi} = |\Psi| \cos \phi$$

$$\text{Im } \underline{\Psi} = |\Psi| \sin \phi$$

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$$\int_{-\infty}^{+\infty} dx |\Psi(x,t)|^2 = 1$$

$\Psi \leadsto A\Psi$ AUCH LÖSUNG

QUADRATISCH INTEGRIERBARE
 0 LÖSUNG

$$\int_{-\infty}^{+\infty} dx |\Psi(x,0)|^2 = 1$$

$$\int_{-\infty}^{+\infty} dx |\Psi(x,t)|^2 = 1$$

$$\frac{d}{dt} \int_{-\infty}^{+\infty} dx \frac{\partial}{\partial t} |\Psi(x,t)|^2 = 0$$

$$\begin{aligned} \frac{\partial}{\partial t} |\Psi|^2 &= \frac{\partial}{\partial t} (\Psi^* \Psi) \\ &= \frac{\partial \Psi^*}{\partial t} \Psi + \Psi^* \frac{\partial \Psi}{\partial t} \end{aligned}$$

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$$\rightarrow i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} + V \Psi$$

$$\rightarrow \underbrace{-i\hbar \frac{\partial \Psi^*}{\partial t}}^* = -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi^*}{\partial x^2} + V^* \Psi^*$$

V REAL

$$V^* = V$$

$$\frac{\partial \Psi^*}{\partial t} \Psi = -\frac{i\hbar}{2m} \frac{\partial^2 \Psi^*}{\partial x^2} \Psi + \cancel{\frac{i}{\hbar} V \Psi^* \Psi}$$

$$\Psi^* \frac{\partial \Psi}{\partial t} = +\frac{i\hbar}{2m} \Psi^* \frac{\partial^2 \Psi}{\partial x^2} - \cancel{\frac{i}{\hbar} V \Psi^* \Psi}$$

$$\frac{\partial}{\partial t} |\Psi|^2 = \frac{i\hbar}{2m} \left(\Psi^* \frac{\partial^2 \Psi}{\partial x^2} - \frac{\partial^2 \Psi^*}{\partial x^2} \Psi \right)$$

$$= \frac{i\hbar}{2m} \frac{\partial}{\partial x} \left(\Psi^* \frac{\partial \Psi}{\partial x} - \frac{\partial \Psi^*}{\partial x} \Psi \right)$$

$$\int_{-\infty}^{+\infty} dx \frac{\partial}{\partial t} |\Psi|^2 = \frac{i\hbar}{2m} \int_{-\infty}^{+\infty} dx \frac{\partial}{\partial x} \left(\dots \right)$$

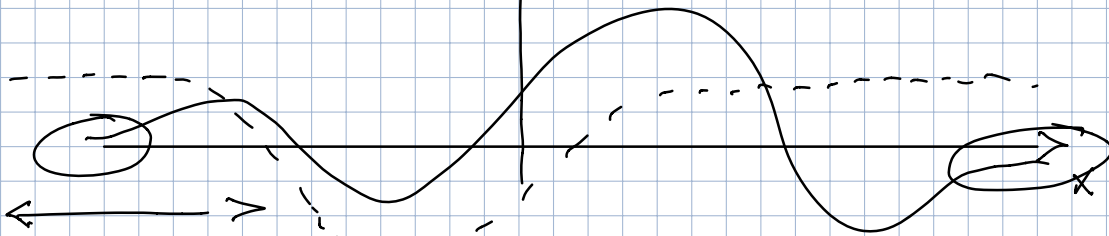
$$= \frac{i\hbar}{2m} \left[\Psi^* \frac{\partial \Psi}{\partial x} - \frac{\partial \Psi^*}{\partial x} \Psi \right]_{-\infty}^{+\infty}$$

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$$\Psi(+\infty, t) = \Psi(-\infty, t) = 0$$

$$\int_{-\infty}^{+\infty} dx \frac{\partial}{\partial t} |\Psi|^2 = 0$$

$\text{Re}(\Psi(x, t))$



$$\int_{-\infty}^{+\infty} dx |\Psi(x, t)|^2 = \infty$$

$$= 1$$

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1.4 IMPULS

$$\underline{\Psi}(x, t)$$

$$\langle x \rangle = \int_{-\infty}^{+\infty} dx \ x \ |\underline{\Psi}(x, t)|^2$$

$$\langle v \rangle \equiv \frac{d}{dt} \langle x \rangle$$

$$= \int_{-\infty}^{+\infty} dx \ x \ \frac{\partial}{\partial t} |\underline{\Psi}(x, t)|^2$$

$$\frac{\partial}{\partial t} |\underline{\Psi}|^2 = \frac{i\hbar}{2m} \left(\underline{\Psi}^* \frac{\partial^2 \underline{\Psi}}{\partial x^2} - \frac{\partial^2 \underline{\Psi}^*}{\partial x^2} \underline{\Psi} \right)$$

$$= \frac{i\hbar}{2m} \frac{\partial}{\partial x} \left(\underline{\Psi}^* \frac{\partial \underline{\Psi}}{\partial x} - \frac{\partial \underline{\Psi}^*}{\partial x} \underline{\Psi} \right)$$

$$\langle v \rangle = \frac{i\hbar}{2m} \int_{-\infty}^{+\infty} dx \ x \ \frac{\partial}{\partial x} \left(\underline{\Psi}^* \frac{\partial \underline{\Psi}}{\partial x} - \frac{\partial \underline{\Psi}^*}{\partial x} \underline{\Psi} \right)$$

$$= - \frac{i\hbar}{2m} \int_{-\infty}^{+\infty} dx \ \left(\underline{\Psi}^* \frac{\partial \underline{\Psi}}{\partial x} - \frac{\partial \underline{\Psi}^*}{\partial x} \underline{\Psi} \right) + \text{TERM}$$

~~$\int_{-\infty}^{+\infty}$~~

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$$= -\frac{i\hbar}{2m} \int_{-\infty}^{+\infty} dx \left(\Psi^* \frac{\partial \Psi}{\partial x} + \Psi \frac{\partial \Psi^*}{\partial x} \right) + \text{TERM} \textcircled{2}$$

$$\langle v \rangle = \frac{d}{dt} \langle x \rangle = -\frac{i\hbar}{m} \int_{-\infty}^{+\infty} dx \Psi^* \frac{\partial \Psi}{\partial x}$$

↳ IMPULS $p = m v$

$$\langle p \rangle \equiv m \langle v \rangle$$

$$= m \frac{d}{dt} \langle x \rangle$$

$$= -i\hbar \int_{-\infty}^{+\infty} dx \Psi^* \frac{\partial \Psi}{\partial x}$$

$$= \int_{-\infty}^{+\infty} dx \Psi^* \left(-i\hbar \frac{\partial}{\partial x} \right) \Psi$$

\hat{p} OPERATOR

↳ KINETISCHE ENERGIE

$$T = \frac{p^2}{2m}$$

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$$\langle T \rangle = \int_{-\infty}^{+\infty} dx \Psi^* \left(\frac{\hat{P}^2}{2m} \right) \Psi$$

$$= \int_{-\infty}^{+\infty} dx \Psi^* \left(-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \right) \Psi$$

$$\langle V \rangle = \int_{-\infty}^{+\infty} dx \Psi^* V \Psi$$

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} + V \Psi = i\hbar \frac{\partial \Psi}{\partial t}$$

$$\int dx \Psi^*$$

$$\langle T \rangle + \langle V \rangle = \langle E \rangle$$

$$= \int_{-\infty}^{+\infty} dx \Psi^* \left(i\hbar \frac{\partial}{\partial t} \right) \Psi$$

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$$\langle p \rangle = m \frac{d}{dt} \langle x \rangle = \int_{-\infty}^{+\infty} dx \Psi^* \left(-i\hbar \frac{\partial \Psi}{\partial x} \right)$$

$$\text{(KL.) } \frac{dp}{dt} = - \frac{\partial V}{\partial x}$$

$$\frac{d}{dt} \langle p \rangle = -i\hbar \int_{-\infty}^{+\infty} dx \left(\frac{\partial \Psi^*}{\partial t} \frac{\partial \Psi}{\partial x} + \Psi^* \frac{\partial}{\partial x} \frac{\partial \Psi}{\partial t} \right)$$

$$i\hbar \frac{\partial \Psi}{\partial t} = - \frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} + V \Psi$$

V REAL

$$-i\hbar \frac{\partial \Psi^*}{\partial t} = - \frac{\hbar^2}{2m} \frac{\partial^2 \Psi^*}{\partial x^2} + V \Psi^*$$

$$\frac{d}{dt} \langle p \rangle = \int_{-\infty}^{+\infty} dx \left\{ \left(- \frac{\hbar^2}{2m} \frac{\partial^2 \Psi^*}{\partial x^2} + V \Psi^* \right) \frac{\partial \Psi}{\partial x} - \Psi^* \frac{\partial}{\partial x} \left(- \frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} + V \Psi \right) \right\}$$

$$\frac{\partial^2 \Psi^*}{\partial x^2} \frac{\partial \Psi}{\partial x} \rightarrow - \frac{\partial \Psi^*}{\partial x} \frac{\partial^2 \Psi}{\partial x^2} \rightarrow \Psi^* \frac{\partial^3 \Psi}{\partial x^3}$$

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$$\frac{d}{dt} \langle p \rangle = \int_{-\infty}^{+\infty} dx \left(\cancel{\Psi^*} V \cancel{\frac{\partial \Psi}{\partial x}} - \Psi^* \frac{\partial}{\partial x} (V \Psi) \right)$$

$$\frac{\partial V}{\partial x} \Psi + V \cancel{\frac{\partial \Psi}{\partial x}}$$

$$\frac{d}{dt} \langle p \rangle = \int_{-\infty}^{+\infty} dx \Psi^* \left(- \frac{\partial V}{\partial x} \right) \Psi = \langle - \frac{\partial V}{\partial x} \rangle$$

KLASSISCH $\frac{dp}{dt} = - \frac{\partial V}{\partial x}$

↓

EHRENFEST THEOREM

1.5 UNSCHÄRFE PRINZIP

$$\Psi(x, t)$$

$$p \longleftrightarrow \lambda$$

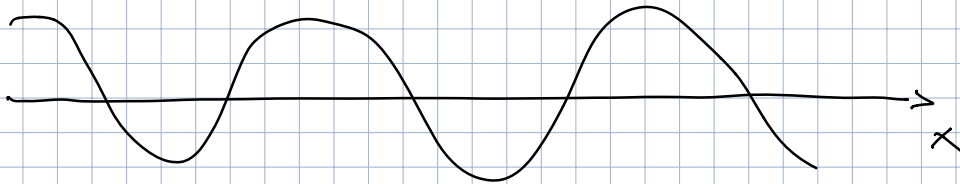
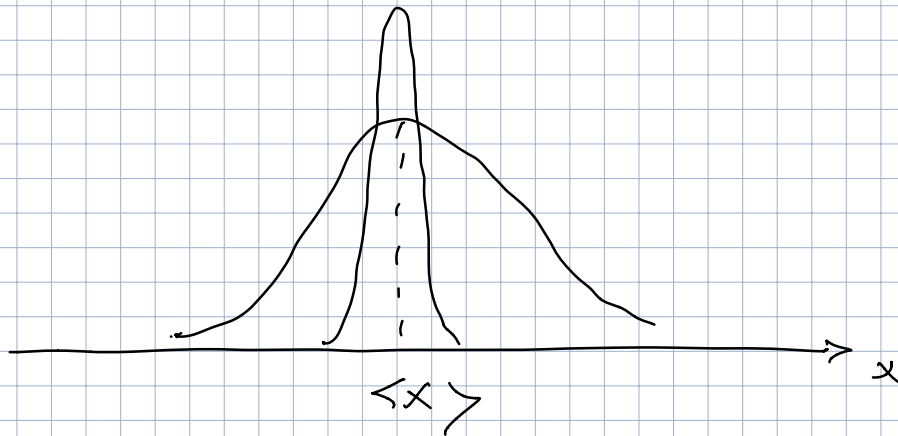
DE BROGLIE

$$p = \frac{h}{\lambda} = \frac{2\pi \hbar}{\lambda}$$

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$$\sigma_x \cdot \sigma_p \geq \frac{\hbar}{2}$$

$$\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$$



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