⟹ VORLESUNG 1 QM

KAP 1: WELLEN FUNKTION

1.1 SCHRÖDINGER GL.

MIKROSKOPISCHE SYSTEME z.B ATOMKERN

* 8 MeV BINDUNGS ENERGIE p / n IM KERN

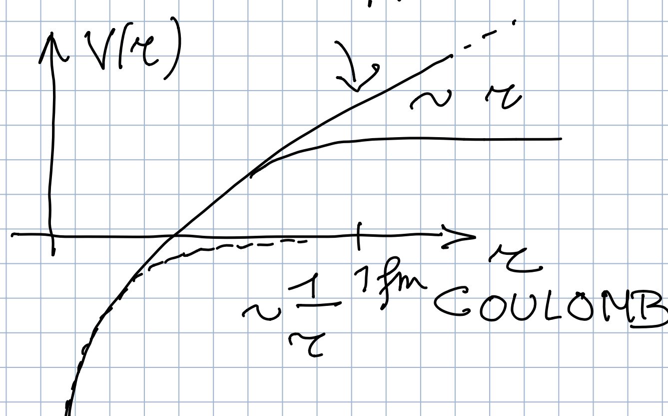
MASSE ~ 938 MeV

BE ~ 1 %

* PROTON ⟶ u, u, d

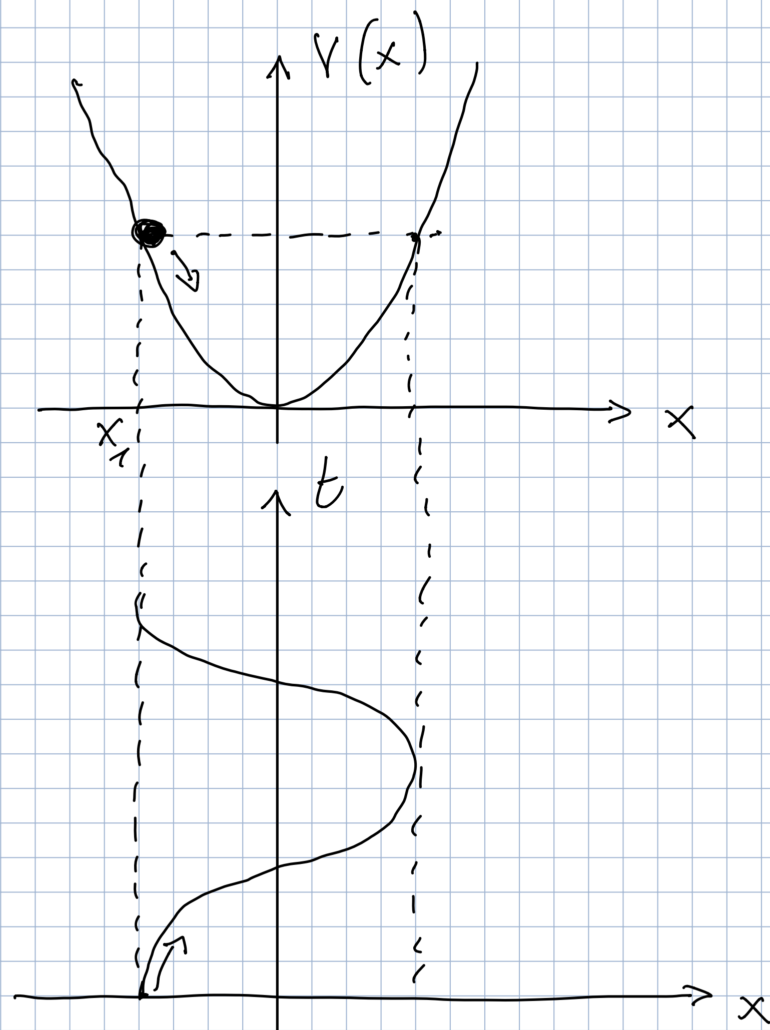
$m_u \sim m_d$: 5 MeV

MASSE ~ PAAR %



BENÖTIGTE ENERGIE $1\ \mathrm{GeV}/\mathrm{fm} \sim 10^{9}\ \mathrm{eV} / 10^{-15}\ \mathrm{m}$

- KLASSISCHE PHYSIK



X(t) NEWTON GL.

$$m \frac{d^2x}{dt^2} = F = -\frac{\partial V}{\partial x}$$

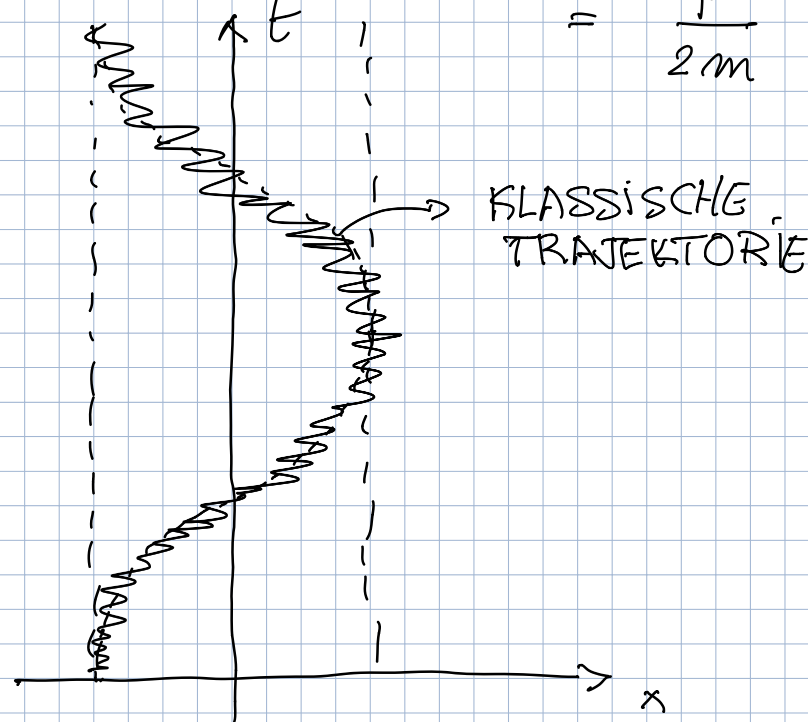
$$x(0) = x_1$$
$$v(0) = 0$$

$$v = \frac{dx}{dt}$$

$x(t)\,,\quad v(t)$

IMPULS $\quad p = m\,v(t)$

KIN. ENERGIE $\quad T = \frac{1}{2} m v^2 = \frac{p^2}{2m}$



$\cancel{x(t)}$

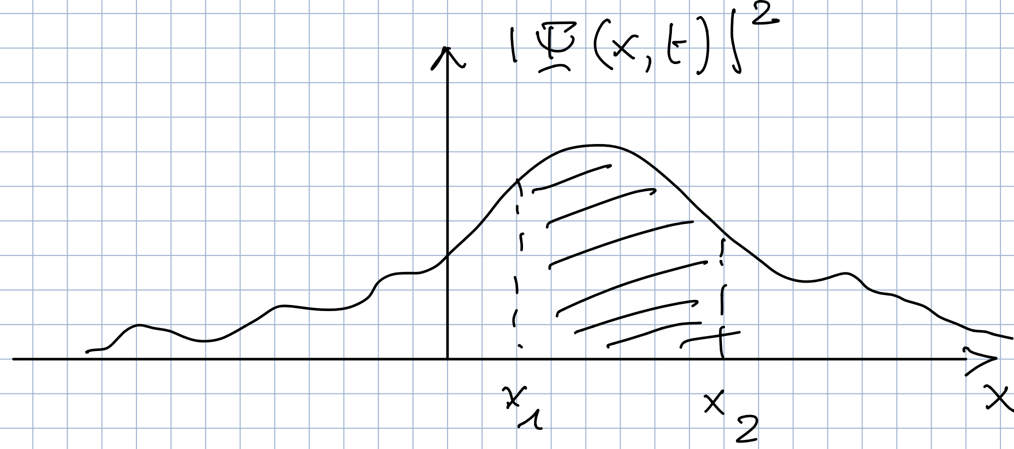
$\Psi(x,t) \Rightarrow$ WAHRSCHEIN-LICHKEITS AMPLITUDE

$\uparrow$

WELLEN FUNKTION

$|\Psi(x,t)|^2$ $\longrightarrow$ WAHRSCHEIN-LICHKEITS DICHTE

KOMPLEXE ZAHL



$$P(x_1 \le x \le x_2) = \int_{x_1}^{x_2} dx \quad |\Psi(x,t)|^2$$

$\Psi(x,t)$: LÖSUNG DER SCHRÖDINGER GL.

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} + V\Psi = i\hbar \frac{\partial \Psi}{\partial t}$$

KIN EN   POT EN   GESAMMT. ENERGIE

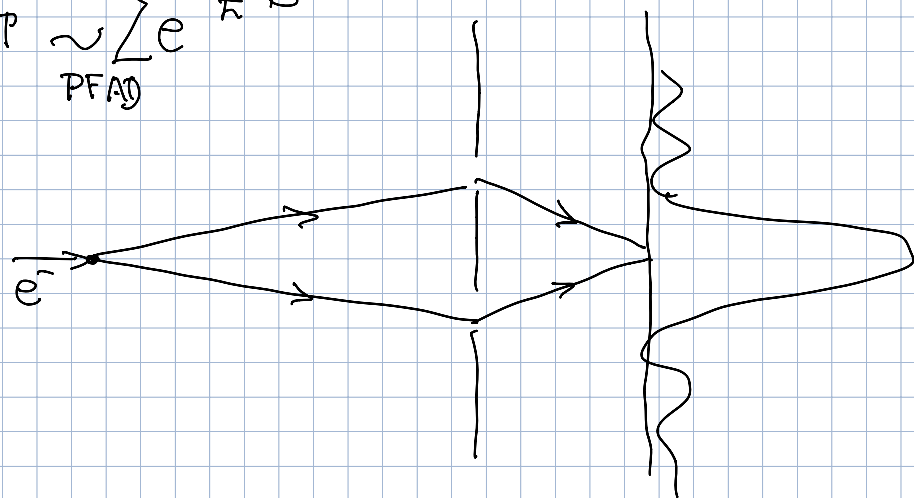
$\hbar$ PLANCK KONSTANTE

$\hbar \simeq 1.05 \; 10^{-34}$ J. s

WIRKUNG

$$S = \int_{t_1}^{t_2} dt \; L(q, \dot{q}, t)$$

$$P \sim \sum_{\text{PFAD}} e^{\frac{i}{\hbar} S}$$



1.2 WAHRSCHEINLICHKEITS LEHRE

- **DISKREET**

$j = 0, 1, 2, 3, \ldots$ Z.B ALTER IN JAHRE

$N(j)$ ANZAHL MIT ALTER j

N GESAMMT ANZAHL

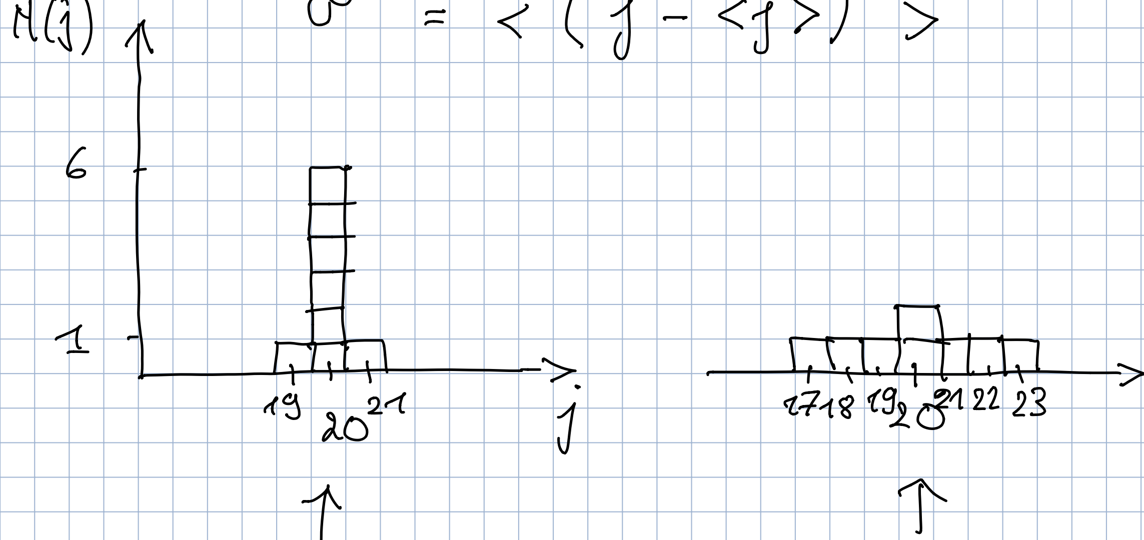
$$P(j) = \frac{N(j)}{N}$$

↳ MITTELWERT

$$\langle j \rangle = \sum_{j=0}^{\infty} j \, P(j) = \sum_j j \frac{N(j)}{N}$$

↳ VARIANZ

$$\sigma^2 = \langle (j - \langle j \rangle)^2 \rangle$$



$$\sigma^2 = \langle j^2 - 2j\langle j\rangle + \langle j\rangle^2 \rangle$$

$$= \langle j^2\rangle - 2\langle j\rangle^2 + \langle j\rangle^2$$

$$= \langle j^2\rangle - \langle j\rangle^2$$

↳ STANDARD ABWEICHUNG

$$\sigma = \sqrt{\sigma^2} = \sqrt{\langle j^2\rangle - \langle j\rangle^2}$$

- **KONTINUIERLICH**

x GRÖSSE

DICHTE $\rho(x)$

$$P(x, x+dx) = \rho(x)\, dx$$

$$P(x_1, x_2) = \int_{x_1}^{x_2} dx\, \rho(x)$$

$$1 = \int_{-\infty}^{+\infty} dx\, \rho(x)$$

↳ MITTELWERT

$$\langle x \rangle = \int dx \; x \; \rho(x)$$

$$\langle f(x) \rangle = \int dx \; f(x) \; \rho(x)$$

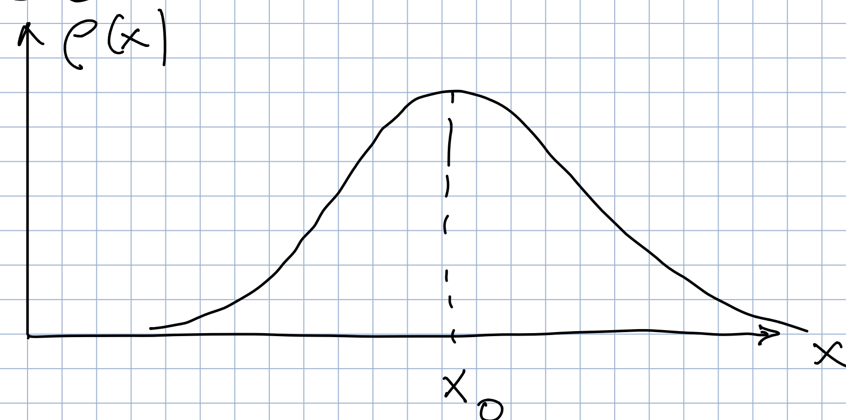
↳ VARIANZ

$$\sigma^2 = \langle (x - \langle x \rangle)^2 \rangle$$

$$= \int dx \; (x - \langle x \rangle)^2 \; \rho(x)$$

$$= \langle x^2 \rangle - \langle x \rangle^2$$

- BEISPIEL : GAUSS VERTEILUNG



$$\rho(x) = \underbrace{A} \; e^{-\lambda (x - x_0)^2}$$

↳ NORMIERUNG

$$\int_{-\infty}^{+\infty} dx \; \rho(x) = 1$$

$$1 = A \int_{-\infty}^{+\infty} dx \; e^{-\lambda (x - x_0)^2}$$

$$x' \stackrel{!}{=} x - x_0$$

$$= A \underbrace{\int_{-\infty}^{+\infty} dx' \; e^{-\lambda x'^2}}_{\sqrt{\frac{\pi}{\lambda}}}$$

$$\Downarrow$$

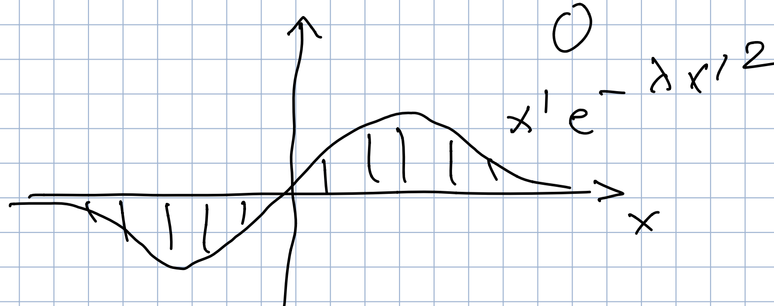
$$A = \sqrt{\frac{\lambda}{\pi}}$$

↳ MITTELWERT

$$\underset{\sim\sim\sim}{\langle x \rangle} = A \int_{-\infty}^{+\infty} dx \; x \; e^{-\lambda (x-x_0)^2}$$

$$x' = x - x_0$$

$$= A \int_{-\infty}^{+\infty} dx' \; (\underbrace{\cancel{x'}}_{0} + x_0) \; e^{-\lambda x'^2}$$



$$\overset{!}{=} \; x_0$$