

⇒ VORLESUNG 1 QM

KAP 1: WELLENFUNKTION

1.1 SCHRÖDINGER GL.

MIKROSKOPISCHE SYSTEME z.B. ATOMKERN

* 8 MeV BINDUNGS ENERGIE
p / m IM KERN

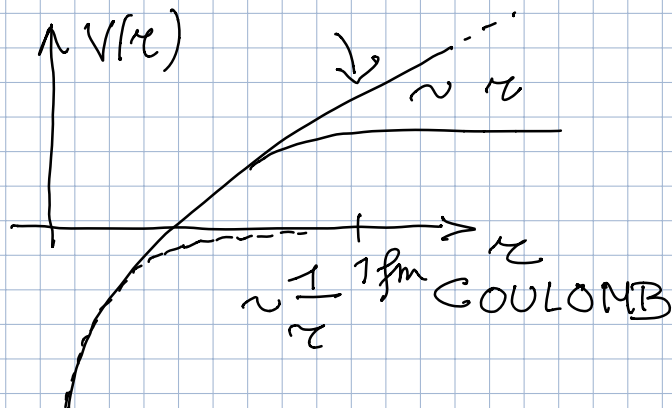
MASSE ~ 938 MeV

BE $\sim 1\%$

* PROTON \rightarrow u, u, d

$m_u \sim m_d : 5$ MeV

MASSE \sim PAAR 90

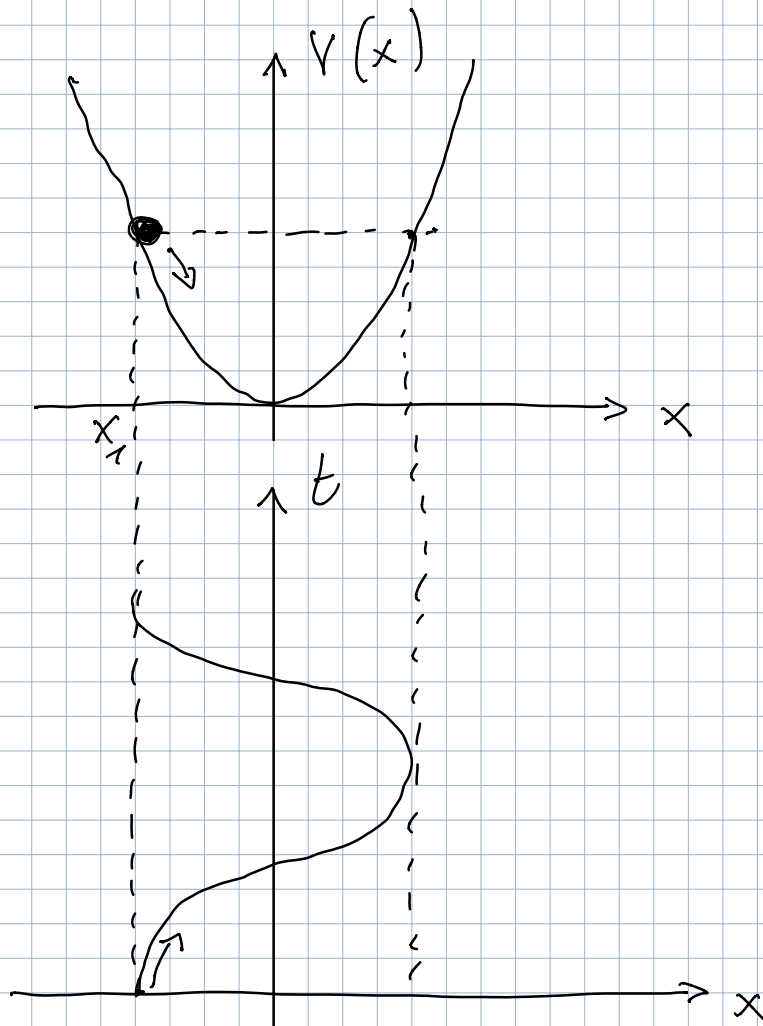


BENÖTIGTE
ENERGIE

$$1 \text{ GeV} / \underset{\text{fm}}{\sim} \sim 10^9 \text{ eV} / 10^{-15} \text{ m}$$

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
5

• KLASSISCHE PHYSIK



$x(t)$ NEWTON GL.

$$m \frac{d^2 x}{dt^2} = F = - \frac{\partial V}{\partial x}$$

$$x(0) = x_1$$

$$v(0) = 0$$

$$v = \frac{dx}{dt}$$

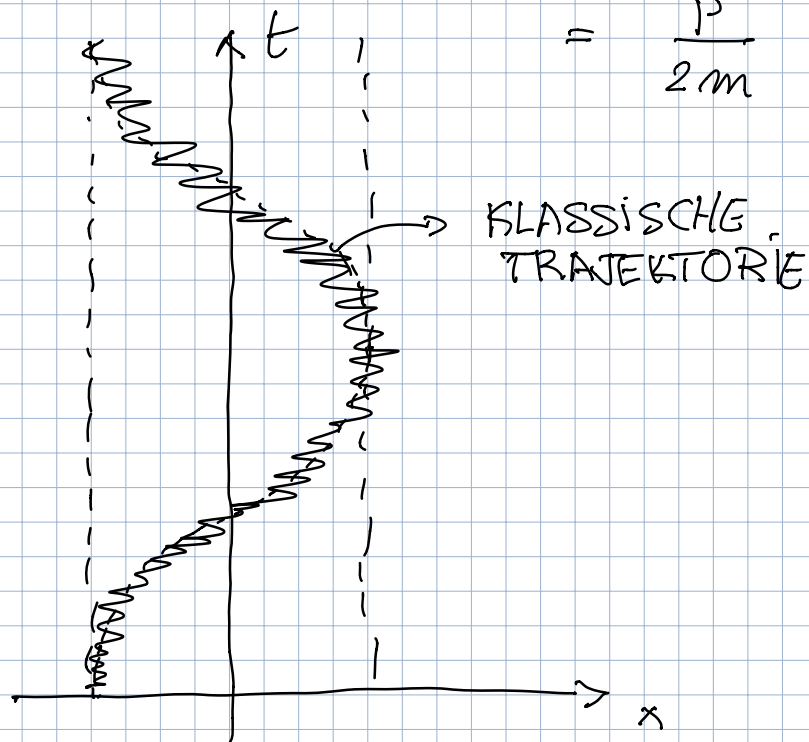
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50

↓

$$x(t), v(t)$$

$$\text{IMPULS } p = m v(t)$$

$$\text{KIN. ENERGIE } T = \frac{1}{2} m v^2$$
$$= \frac{p^2}{2m}$$



~~$x(t)$~~

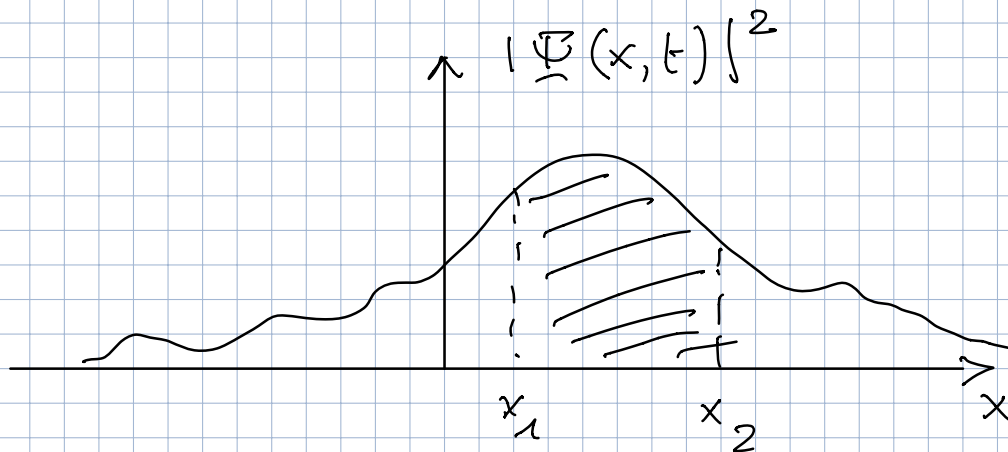
$\Psi(x, t) \Rightarrow$ WAHRSCHEIN-
LICHKEITS

↑
WELLEN FUNKTION AMPLITUDE

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50

$|\Psi(x, t)|^2 \rightarrow$ WAHRSCHEINLICHKEITSDICHTE
 $\uparrow \uparrow$

KOMPLEXE ZAHL



$$P(x_1 \leq x \leq x_2) = \int_{x_1}^{x_2} dx |\Psi(x, t)|^2$$

$\Psi(x, t)$: LÖSUNG DER
 SCHRÖDINGER GL.

$$\underbrace{-\frac{\hbar^2}{2m}}_{\text{KIN EN}} \underbrace{\frac{\partial^2 \Psi}{\partial x^2}}_{\text{KIN EN}} + \underbrace{V \Psi}_{\text{POT EN}} = i\hbar \underbrace{\frac{\partial \Psi}{\partial t}}_{\text{GESAMT. ENERGIE}}$$

\hbar PLANCK KONSTANTE

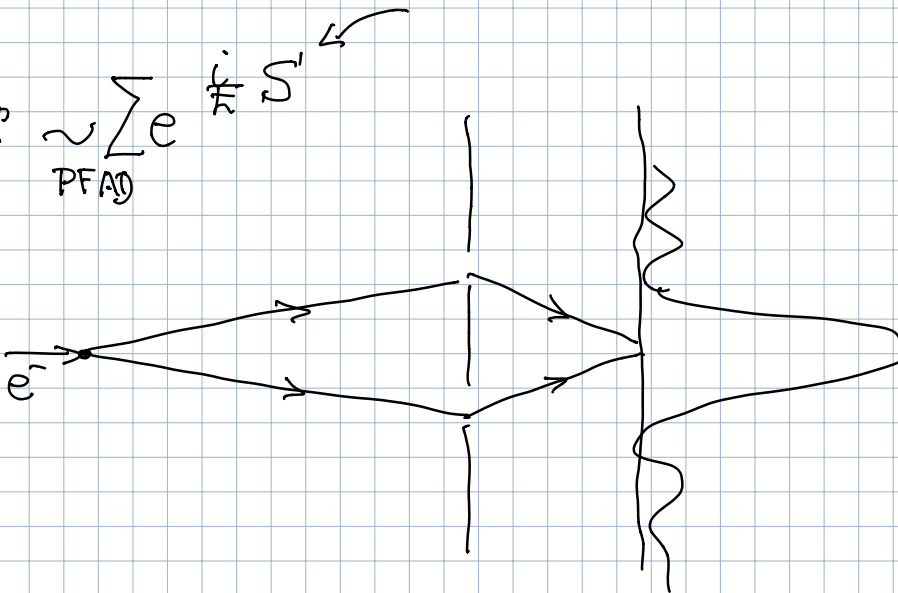
$$\hbar \approx 1.05 \cdot 10^{-34} \text{ J} \cdot \text{s}$$

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50

WIRKUNG

$$S = \int_{t_1}^{t_2} dt L(\varphi, \dot{\varphi}, t)$$

$$P \sim \sum_{\text{PFAD}} e^{i \frac{1}{\hbar} S}$$



1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50

1.2 WAHRSCHEINLICHKEITS LEHRE

• DISKREET

$j = 0, 1, 2, 3, \dots$ z.B. ALTER IN
JAHRE

$N(j)$ ANZAHL MIT ALTER j

N GESAMMT ANZAHL

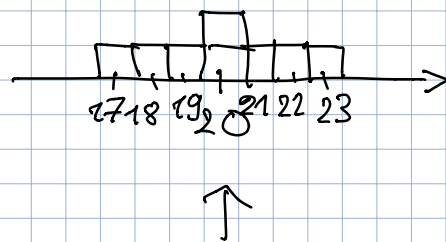
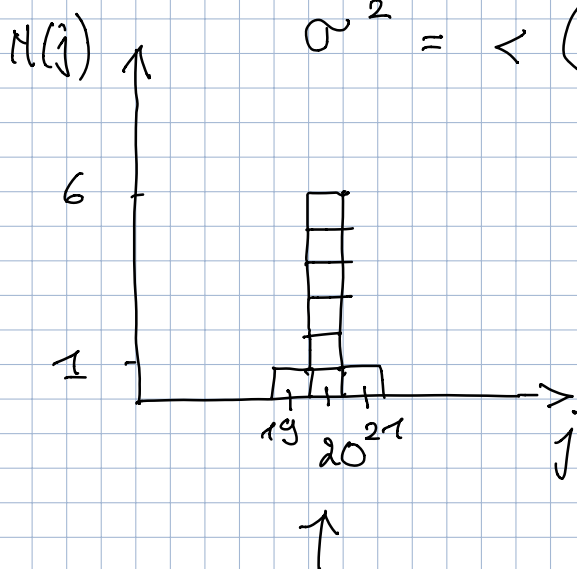
$$P(j) = \frac{N(j)}{N}$$

↳ MITTELWERT

$$\langle j \rangle = \sum_{j=0}^{\infty} j P(j) = \sum_j j \frac{N(j)}{N}$$

↳ VARIANZ

$$\sigma^2 = \langle (j - \langle j \rangle)^2 \rangle$$



1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50

$$\begin{aligned}
 \sigma^2 &= \langle j^2 - 2j\langle j \rangle + \langle j \rangle^2 \rangle \\
 &= \langle j^2 \rangle - \underbrace{2\langle j \rangle^2 + \langle j \rangle^2} \\
 &= \langle j^2 \rangle - \langle j \rangle^2
 \end{aligned}$$

↳ STANDARD ABWEICHUNG

$$\sigma = \sqrt{\sigma^2} = \sqrt{\langle j^2 \rangle - \langle j \rangle^2}$$

• KONTINUIERLICH

x GRÖSSE

DICHTE $P(x)$

$$P(x, x+dx) = P(x) dx$$

$$P(x_1, x_2) = \int_{x_1}^{x_2} dx P(x)$$

$$1 = \int_{-\infty}^{+\infty} dx P(x)$$

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50

↳ MITTELWERT

$$\langle x \rangle = \int dx \ x \ P(x)$$

$$\langle f(x) \rangle = \int dx \ f(x) \ P(x)$$

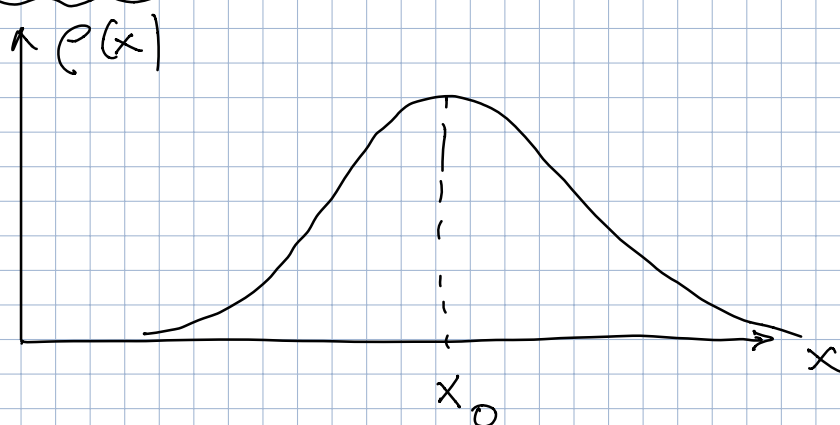
↳ VARIANZ

$$\sigma^2 = \langle (x - \langle x \rangle)^2 \rangle$$

$$= \int dx \ (x - \langle x \rangle)^2 \ P(x)$$

$$= \langle x^2 \rangle - \langle x \rangle^2$$

• BEISPIEL : GAUSS VERTEILUNG



1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50

$$p(x) = \underbrace{A}_{\text{normierung}} e^{-\lambda (x-x_0)^2}$$

↳ NORMIERUNG

$$\int_{-\infty}^{+\infty} dx p(x) = 1$$

$$1 = A \int_{-\infty}^{+\infty} dx e^{-\lambda (x-x_0)^2}$$

$$= A \int_{-\infty}^{+\infty} dx' e^{-\lambda x'^2}$$

$x' = x - x_0$

$\sqrt{\frac{\pi}{\lambda}}$

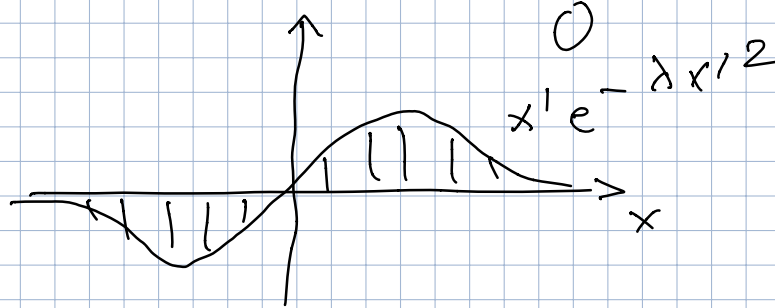
$$A = \sqrt{\frac{\lambda}{\pi}}$$

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50

↳ MITTELWERT

$$\langle x \rangle = A \int_{-\infty}^{+\infty} dx x e^{-\lambda(x-x_0)^2}$$

$$= A \int_{-\infty}^{+\infty} dx' (x' + x_0) e^{-\lambda x'^2}$$



$$= x_0$$

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50