

1. Consider an arbitrary transformation

$$\phi \rightarrow \phi + \delta\phi$$

$$\delta\mathcal{L} = \frac{\partial\mathcal{L}}{\partial\phi} \delta\phi + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)} \partial_\mu\delta\phi$$

$$= \left[\frac{\partial\mathcal{L}}{\partial\phi} - \partial_\mu \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)} \right] \delta\phi \xrightarrow{\text{E.O.M.}} 0$$

$$+ \partial_\mu \left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)} \delta\phi \right)$$

2. Transf. is a symmetry of \mathcal{L}
if $\delta\mathcal{L} = \partial_\mu f^\mu$ i.e. total derivative

\Downarrow

$$\partial_\mu \left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)} \delta\phi - f^\mu \right) = 0$$

conserved current $J^\mu: \partial_\mu J^\mu = 0$

\Downarrow

3. Find symmetries of \mathcal{L} (i.e. such that $\delta\mathcal{L} = \partial_\mu f^\mu$)

Translation $x^\mu \rightarrow x^\mu + \epsilon^\mu$

$$\phi \rightarrow \phi(x + \epsilon) = \phi + \epsilon^\mu \partial_\mu \phi$$

$$\mathcal{L} \rightarrow \mathcal{L} + \epsilon^\mu \partial_\mu \mathcal{L}$$

Total ∂

$$f^\mu = \mathcal{L} \epsilon^\mu$$

$$\frac{\partial\mathcal{L}}{\partial(\partial_\nu\phi)} \epsilon^\nu \partial_\nu\phi - \mathcal{L} \epsilon^\mu$$

Arbitrary ε , arbitrary \mathcal{L} !

$$T^{\mu\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)} \partial^\nu \varphi - g^{\mu\nu} \mathcal{L}$$

conserved

$$K-G \Rightarrow T^{\mu\nu} = \partial^\mu \varphi \partial^\nu \varphi - g^{\mu\nu} \mathcal{L}$$

$$\text{Dirac} \quad \mathcal{L} = \bar{\Psi} (i \partial_\mu \gamma^\mu - m) \Psi$$

$$\downarrow$$
$$T^{\mu\nu} = \bar{\Psi} i \gamma^\mu \partial^\nu \Psi - g^{\mu\nu} \mathcal{L}$$

And so on