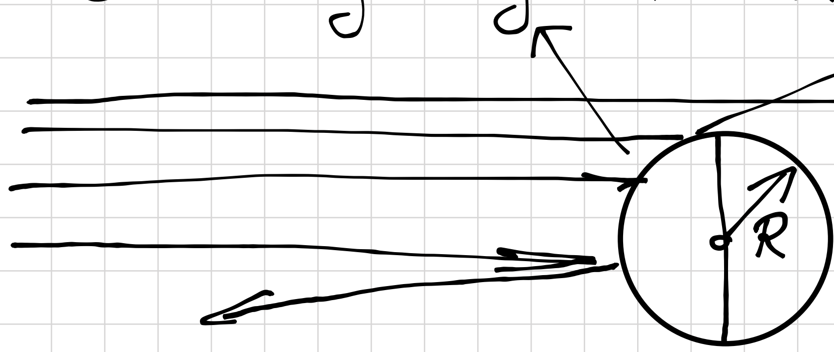
Lecture 9 30/11/2020

Cross section

Classically: geometric interpretation



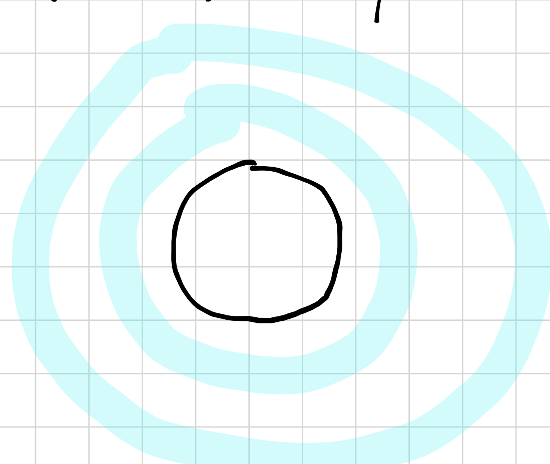
$\sigma = \pi R^2$

Quantum mechanics: velocity of particles $\vec{k}$

Factor 4: you sum over diffraction pattern

$k \ll$

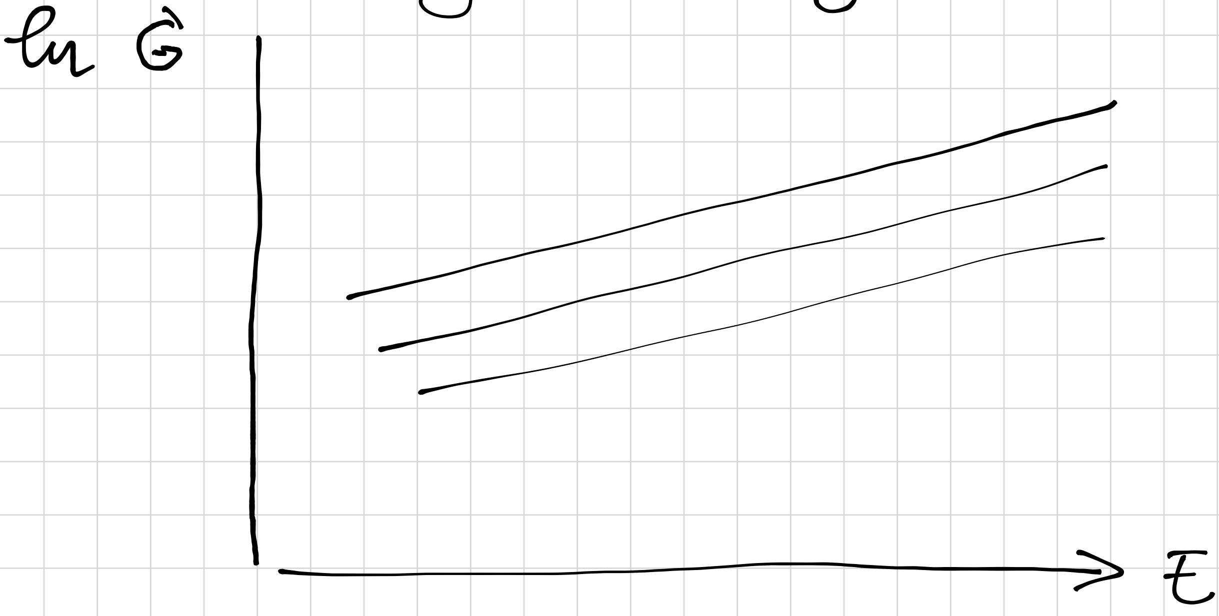
$\sigma = \underline{4}\pi R^2$



HE limit $k \gg$

$\sigma = 2\pi R^2$

What is the asymptotic behavior of particle scattering cross sections at high energies?



$\sigma \sim s^{\alpha}$

Valid for pp

$p\bar{p}$

πp

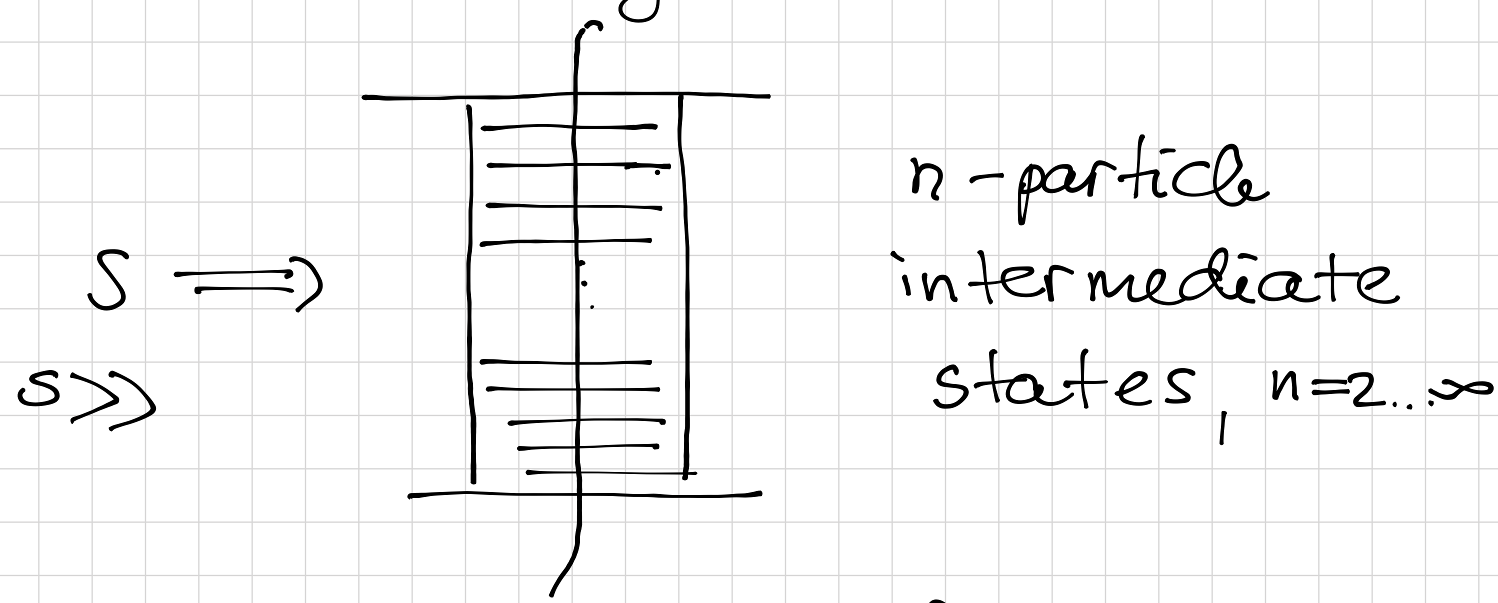
Kp

$\pi\pi$

$\vdots$

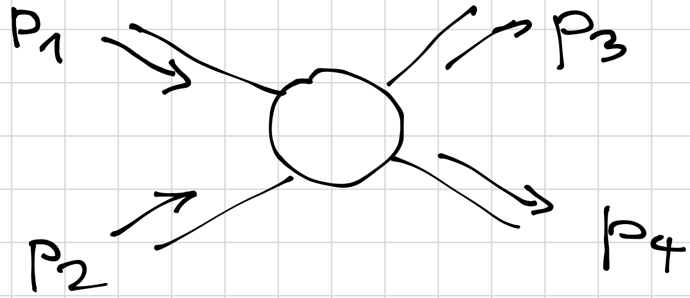
<u>Anticipating</u>: multiparticle production is responsible for growing σ

<u>Plan</u> : to show that resumming a class of diagrams (ladder diagrams) leads to a strong modification of HE behavior $2 \to 2$ scattering



We will need to develop a few tools

* ϕ^3 theory $\longrightarrow$ calculate the box diag. to deal w. HE asymptotics
  $\longrightarrow$ introduce Sudakov's light-like vectors
* learn how to estimate HE behavior of a diagram
* perform 4-fold integration in the complex plane
* integration region method
* Cutkosky's rules (cutting rules)
* Dispersion relations for Feynman d.
* generalize to n^{th} order of PT
  $\hookrightarrow$ resum ladder

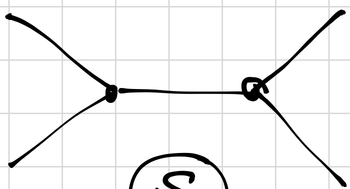
$S \Longrightarrow$ 

$s = (p_1+p_2)^2 \gg m^2$

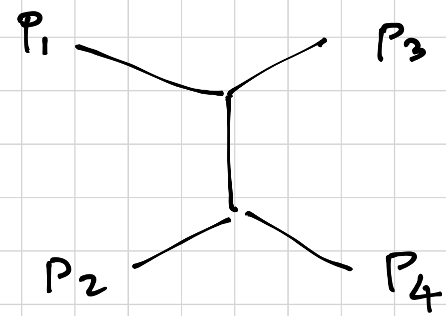
$t = (p_1-p_3)^2; \quad |t| \ll s$

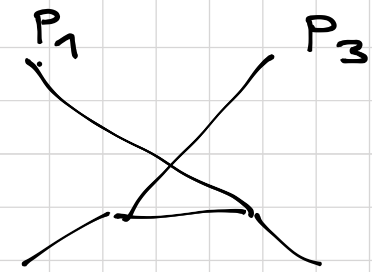
$s+u+t = 4m^2$ EX.

$-u \sim s$



S



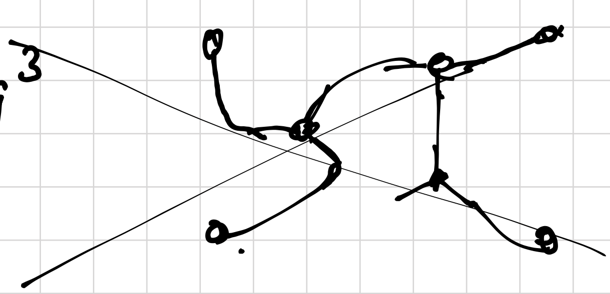


$$iA_s^{(2)} = \frac{(-ig)^2 i}{s-m^2+i\varepsilon} \qquad iA_t^{(2)} = \frac{(-ig)^2 i}{t-m^2+i\varepsilon} \qquad iA^{(2)} = \frac{(-ig)^2 i}{u-m^2+i\varepsilon}$$

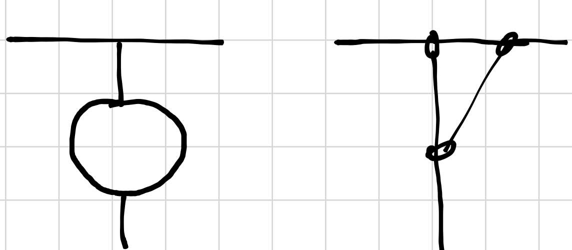
$$\frac{g^2}{m^2} < 1 \quad \Big| \quad \underline{\left|A_{s,u}^{(2)}\right| \sim \frac{g^2}{s}} \qquad \underline{\left|A_t^{(2)}\right| \sim \frac{g^2}{|m^2-t|}}$$

We want to go to higher orders in PT (g^n, $n>2$) and look for corrections that are substantial at high energies

Substantial : $\sim \left[\frac{g^2}{m^2} \ln \frac{s}{m^2}\right]^n$

g^3 

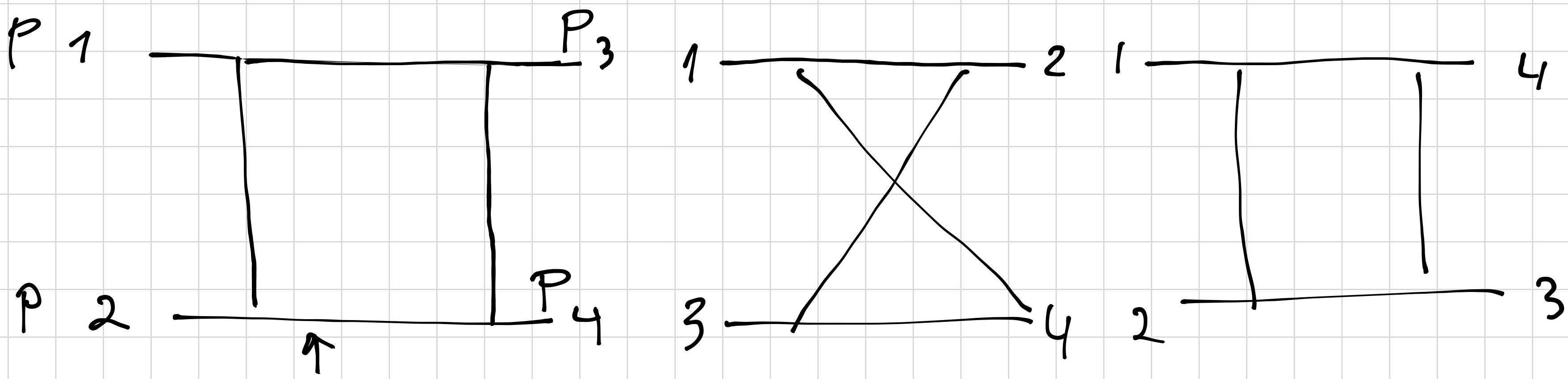
no way to connect 4 external legs to 3 vertices

g^4  + crossed

↳ These diagrams do not lead to the needed behavior

look like the tree level ones ($A^{(2)}$)

The only topology that is new



* Direct box ~~for~~ $s \gg$

* Introduce Sudakov's vectors

$$n^\mu = \sqrt{\frac{s}{2}}(1,0,0,1) \qquad n^2 = \bar{n}^2 = 0$$
$$\bar{n}^\mu = \sqrt{\frac{s}{2}}(1,0,0,-1) \qquad 2(n\bar{n}) = s$$

Any 4-vector can be written as

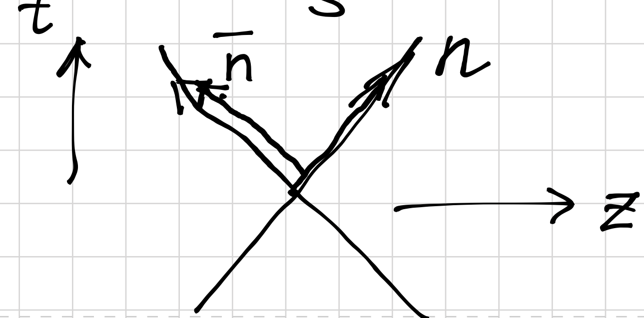
$$a^\mu = (\alpha, \beta, \bar{a}_\perp) \equiv \alpha \cdot n^\mu + \beta \bar{n}^\mu + (0, a_\perp^x, a_\perp^y, 0)$$

$$a^2 = \alpha\beta s - \vec{a}_\perp^{\,2}$$
$$a_\mu b^\mu = \frac{s}{2}(\alpha_1\beta_2 + \alpha_2\beta_1) - \bar{a}_\perp \bar{b}_\perp \qquad \begin{cases} a = (\alpha_1, \beta_1, a_\perp) \\ b = (\alpha_2, \beta_2, b_\perp) \end{cases}$$

Let's fix the external kinematics

$$p_1^\mu = (\alpha_1, \beta_1, \vec{0}_\perp) \qquad p_1^2 = p_2^2 = m^2$$
$$p_2^\mu = (\alpha_2, \beta_2, \vec{0}_\perp) \qquad (p_1 + p_2)^2 = s$$

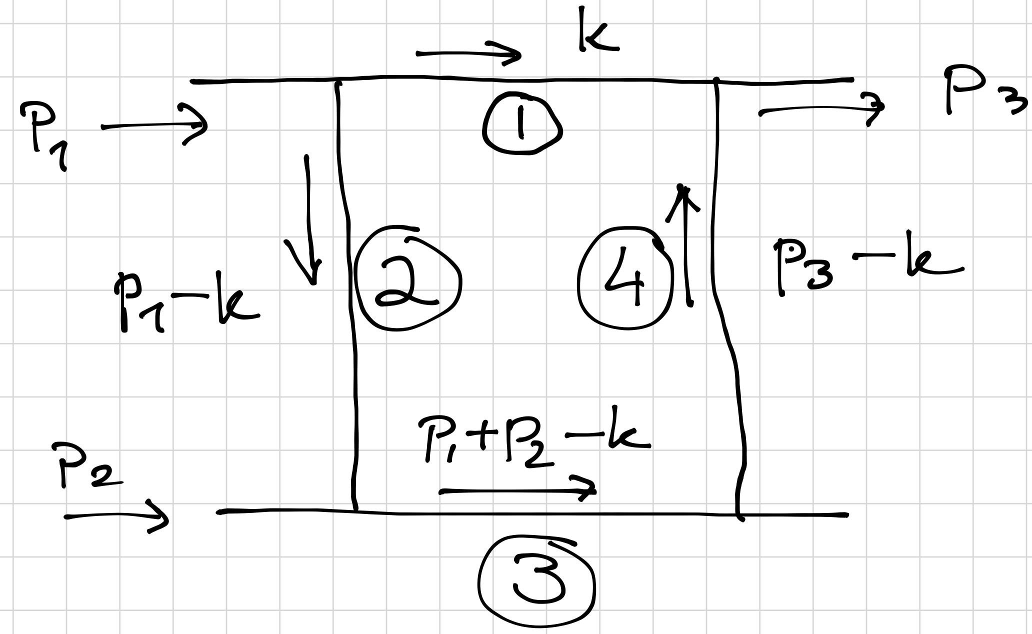
$$p_1^\mu \simeq (\underline{1}, \gamma, \vec{0}_\perp) \qquad \gamma = \frac{m^2}{s}$$
$$p_2^\mu \simeq (\underline{\gamma}, 1, \vec{0}_\perp)$$



$$t = (p_1 - p_3)^2 = -\vec{\Delta}_\perp^2 \qquad \vec{\Delta}_\perp^2 = -t \ll s$$

$$p_3^\mu \cong (1,\ \gamma + \gamma_\Delta,\ \vec{\Delta}_\perp) \qquad \gamma_\Delta = \frac{\Delta_\perp^2}{s}$$

$$p_4^\mu \cong (\gamma + \gamma_\Delta,\ 1,\ -\vec{\Delta}_\perp)$$



$$J = \int \frac{d^4k}{(2\pi)^4} \underbrace{\frac{1}{k^2 - m^2 + i\varepsilon}}_{①} \underbrace{\frac{1}{(p_1-k)^2 - m^2 + i\varepsilon}}_{②} \underbrace{\frac{1}{(p_1+p_2-k)^2 - m^2 + i\varepsilon}}_{③} \cdot \underbrace{\frac{1}{(p_3-k)^2 - m^2 + i\varepsilon}}_{④}$$

We want to rewrite the $\int$ in terms of our new coordinates $(\alpha, \beta, k_\perp)$

$$\int d^4k = \frac{s}{2} \int_{-\infty}^{\infty} d\alpha \int_{-\infty}^{\infty} d\beta \int d^2\vec{k}_\perp$$

Denominators

① $= k^2 - m^2 + i\varepsilon = \alpha\beta s - k_\perp^2 - m^2 + i\varepsilon$

② $= (p_1-k)^2 - m^2 + i\varepsilon = (1-\alpha)(\gamma-\beta)s - k_\perp^2 - m^2 + i\varepsilon$

③ $= (p_1+p_2-k)^2 - m^2 + i\varepsilon = (1+\gamma-\alpha)(1+\gamma-\beta)s - \vec{k}_\perp^2 - m^2 + i\varepsilon$

④ $= (p_3-k)^2 - m^2 + i\varepsilon = (1-\alpha)(\gamma+\gamma_\Delta-\beta)s - (\vec{k}_\perp - \vec{\Delta}_\perp)^2 - m^2 + i\varepsilon$

Next step: evaluate $\int d\alpha$ in complex plane using Cauchy's theorem

$\rightarrow$ pick a pole (or poles)

$\oint \rightarrow 2\pi i \sum$ residues

We need to analyze the position of these poles

$$\alpha_1 = \frac{k_\perp^2 + m^2 - i\varepsilon}{\beta s} = \frac{k_\perp^2 + m^2}{\beta s} - \text{sign}(\beta)\, i\varepsilon$$

$$\alpha_2 = 1 - \frac{k_\perp^2 + m^2}{(\gamma-\beta)s} + \text{sign}(\gamma-\beta)\, i\varepsilon$$

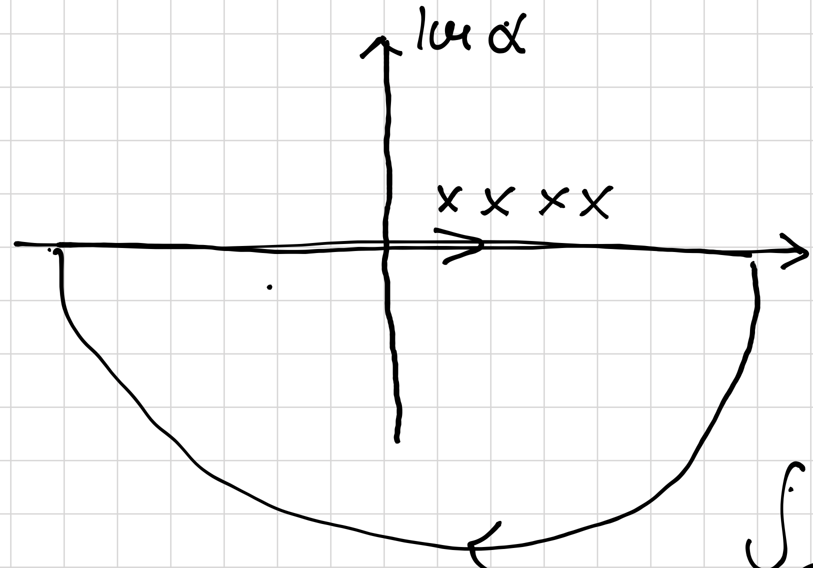
$$\alpha_3 = 1 + \gamma - \frac{k_\perp^2 + m^2}{(1+\gamma-\beta)s} + \text{sign}(1+\gamma-\beta)\, i\varepsilon$$

$$\alpha_4 = 1 - \frac{(\bar{k}_\perp - \bar{\Delta}_\perp)^2 + m^2}{(\gamma+\gamma_\Delta-\beta)s} + \text{sign}(\gamma+\gamma_\Delta-\beta)\, i\varepsilon$$

Keep track of sign of $i\varepsilon$

1). $\beta < 0 \Rightarrow$ all poles in the upper plane

$\oint = 0$

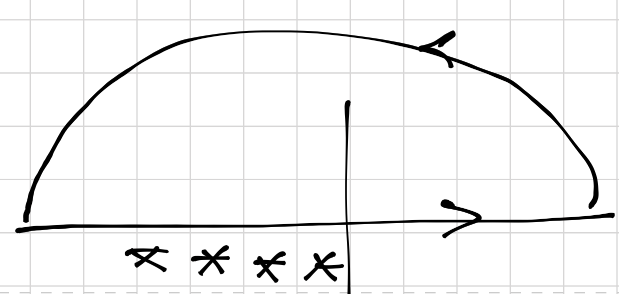


$\int_{C_R} \rightarrow 0 \quad R \rightarrow \infty$

$$\int_{C_R} d\alpha \frac{1}{\alpha^4} \rightarrow 0$$

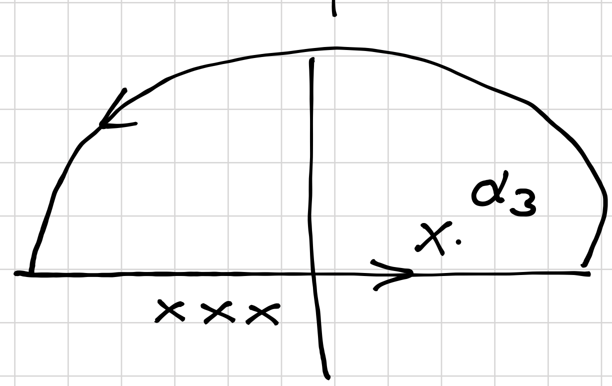
2). $\beta > 1+\gamma$ all poles lie below

$\oint = 0$



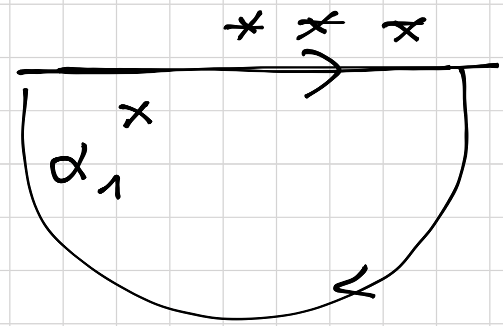
$$\boxed{0 < \beta < 1+\gamma}$$

3). $\gamma+\gamma_\Delta \leq \beta \leq 1+\gamma$

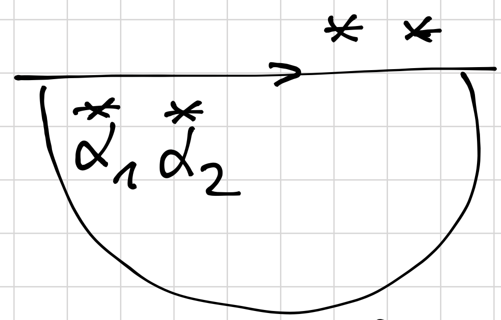


$\oint \neq 0$

4). $0 \leq \beta \leq \gamma$



5). $\gamma \leq \beta \leq \gamma_\Delta$



$$\gamma = \frac{m^2}{s} \ll 1$$
$$\gamma_\Delta = \frac{\Delta_\perp^2}{s} \ll 1$$

While $\int d\alpha$ is finite in 4). and 5).

$\int d\beta$ is constrained to a tiny region $\rightarrow \ll 1$

$$\int \frac{d^4k}{(2\pi)^4} \frac{1}{\textcircled{1}\textcircled{2}\textcircled{3}\textcircled{4}} = \frac{s}{2}\int_{-\infty}^{\infty} d\alpha \int_{-\infty}^{\infty} d\beta \int d^2k_\perp \frac{(2\pi)^{-4}}{\textcircled{1}\textcircled{2}\textcircled{3}\textcircled{4}}$$

$$\cong \frac{s}{2}\int_{\gamma+\gamma_\Delta}^{1+\gamma} d\beta \int d^2k_\perp \frac{1}{(2\pi)^4} \cdot 2\pi i \operatorname*{res}_{\alpha=\alpha_3} \frac{1}{\textcircled{1}\textcircled{2}\textcircled{4}}$$

$$\cdot \frac{1}{(1+\gamma-\beta)s}$$

$$\oint \frac{d\alpha}{\alpha-\alpha_3} f(\alpha) = 2\pi i\, f(\alpha_3)$$

$$= \frac{i}{16\pi^3} \int_{\gamma+\gamma_\Delta}^{1+\gamma} \frac{d\beta}{1+\gamma-\beta} \int d^2k_\perp \frac{1}{\textcircled{1}\textcircled{2}\textcircled{4}}$$

$$① = (1+\gamma)\left[\beta s - \frac{k_\perp^2+\mu^2}{1+\gamma-\beta} + i\varepsilon\right]$$

$$② = -\gamma(\gamma-\beta)\,s - \frac{k_\perp^2+\mu^2}{1+\gamma-\beta} + i\varepsilon$$

$$④ = -\gamma(\gamma+\gamma_\Delta-\beta)\,s - \frac{\gamma+\gamma_\Delta-\beta}{1+\gamma-\beta}(k_\perp^2+\mu^2)$$

$$\sim \frac{m^4}{s} \qquad -(\bar{k}_\perp-\bar{\Delta}_\perp)^2 - M^2 + i\varepsilon$$

$$\int_0^1 \frac{d\beta}{\beta s - k_\perp^2 - \mu^2 + i\varepsilon} \sim \frac{1}{s}\ln\left(\frac{s}{m^2}\right)$$

$$\gamma = \frac{m^2}{s} \ll \beta \ll 1$$

$$\cong \frac{-i}{16\pi^3}\underbrace{\int\frac{d^2k_\perp}{[k_\perp^2+m^2][(k_\perp-\Delta_\perp)^2+\mu^2]}}_{f(\Delta_\perp)}\underbrace{\int_\gamma^1\frac{d\beta}{-\beta s+m^2-i\varepsilon}}_{\frac{1}{s}\ln\frac{m^2-s-i\varepsilon}{m^2}}$$

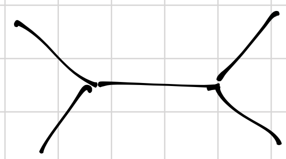
$$\Delta_\perp \to 0 \Longrightarrow \int\frac{k_\perp dk_\perp d\varphi}{(k_\perp^2+\mu^2)^2} = \frac{\pi}{m^2}$$

$$= \frac{-i}{16\pi^3} f(\Delta_\perp)\cdot\frac{1}{s}\ln\left(-\frac{s}{m^2}-i\varepsilon\right)$$

$$iA_s^{(4)} = (-ig)^4\cdot i^4\cdot J$$

$$\boxed{A_s^{(4)} = -\frac{f(\Delta_\perp)}{16\pi^3}\frac{g^4}{s}\ln\left(-\frac{s}{m^2}-i\varepsilon\right)}$$

We obtained the HE asymptotics of the box graph.

$$A_s^{(2)}$$  $$= \frac{-g^2}{s-m^2} \approx \frac{-g^2}{s}$$

$$A_s^{(4)}(t=0) = -\frac{g^2}{s}\cdot\frac{g^2}{16\pi^2 m^2}\ln\left(-\frac{s}{m^2}-i\varepsilon\right)$$

$$t = -\Delta_\perp^2$$

We started from $\underbrace{\frac{g^2}{16\pi^2 m^2} < 1}$

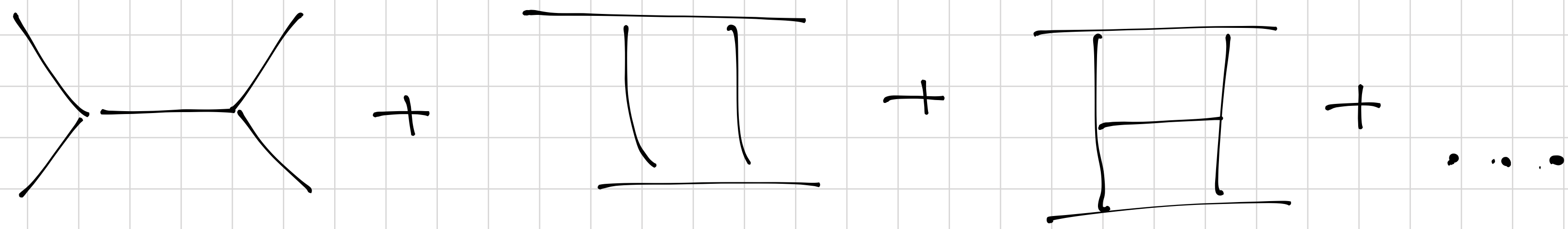
$$\ln\left|\frac{s}{m^2}\right| \gg 1$$

At sufficienly large s PT breaks down!

↳ This is the substantial modification of the HE asymptotics due to perturbative corrections

$$-\frac{g^2}{s}\sum_{n=0}^{\infty}\left(\frac{g^2}{16\pi^2 m^2}\ln\frac{s}{m^2}\right)^n\cdot\frac{1}{n!} = -\frac{g^2}{m^2}\frac{m^2}{s}\exp\left(\ln\left(\frac{s}{m^2}\right)^\eta\right)$$

$$\underbrace{\eta = \frac{g^2}{16\pi^2 m^2}} \qquad = -g^2\cdot\left(\frac{s}{m^2}\right)^{-1+\eta}$$



For high s you tend to produce an ifinite # of particles (virtual)

$\frac{m}{s} \sim \tau \longrightarrow$ long-laving fluctuation of ∞ number of virtual particles