Lecture 8

We derived Feynman rules for scalar theories (Yukawa)

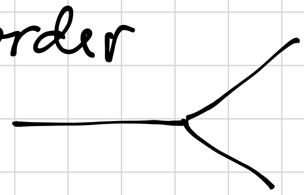
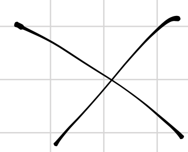
Theories with real scalars

$$\mathcal{L}_{int} = -\frac{\lambda_3}{3!}\phi^3 - \frac{\lambda_4}{4!}\phi^4 - \ldots - \frac{\lambda_n}{n!}\phi^n$$

$$T(\phi(x_1)\ldots\phi(x_n)) \longrightarrow :\underbrace{\phi \ldots \phi}_{\ldots}: + \sum \phi_i \phi_j$$

Typically, $\frac{1}{n!}$ disappears $\longrightarrow (-i\lambda_n)$

1st order

 $-i\lambda_3$  $-i\lambda_4$

General rule: write couplings at each vertex without $\frac{1}{n!}$

$\Rightarrow$ one has to include the symmetry factor $\frac{1}{S}$ for each diagram

$$T\exp(-iH_I) \sim \sum_{V=0}^{\infty} \frac{1}{V!}\left(-i\frac{\lambda_n}{n!}\ldots\right)^V$$

V = # of vertices

$$S = \frac{V!(n!)^V}{R}$$ $\leftarrow$ multiplicity

To calculate R:

- label each external line and draw it as a point
- draw and label the vertices

— to each vertex draw entering lines (n of them)

— count all the ways you can connect external lines to vertices and vertices among them w. propagators

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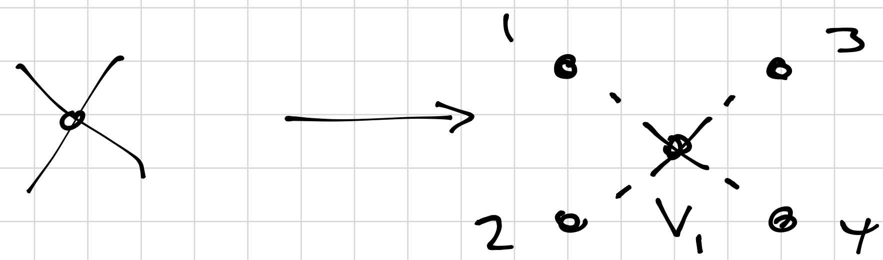
Tree level



Start w. $\underline{1} \longrightarrow \times 4$ ways

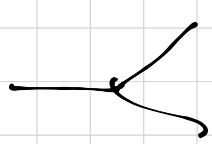
$2 \longrightarrow \times 3$

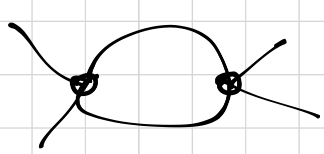
$3 \longrightarrow \times 2$

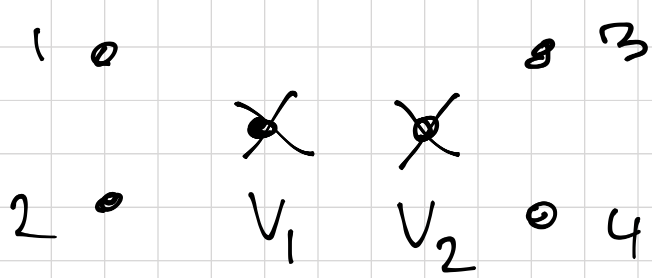
$4 \longrightarrow \times 1$

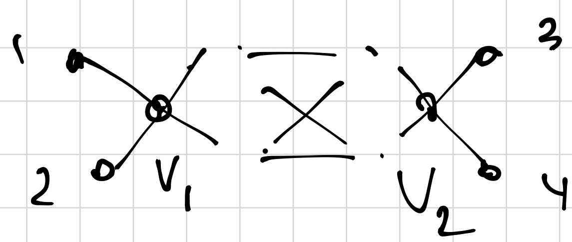
$R = 4 \cdot 3 \cdot 2 \cdot 1 = 4!$

$$S = \frac{1!\,4!}{4!} = \underline{1}$$

 $R = 3!$ $\quad S = 1$

 $V=2 \quad n=4$





$\left.\begin{array}{l} \underline{1} \longrightarrow \times 8 \\ 2 \longrightarrow \times 3 \end{array}\right]$ same V_i

$\left.\begin{array}{l} 3 \longrightarrow \times 4 \\ 4 \longrightarrow \times 3 \end{array}\right]$ same V

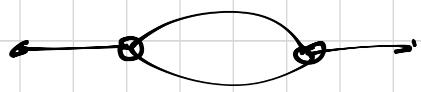
$\times\, 2$ ways to connect V_1 to V_2

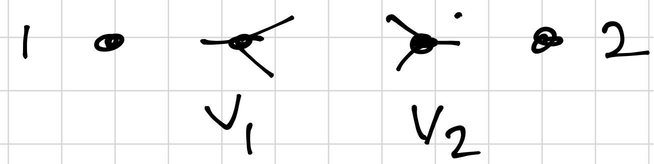
$R = 8 \cdot 3 \cdot 4 \cdot 3 \cdot 2$

$$S = \frac{2!\,(4!)^2}{\underbrace{8 \cdot 3}_{4!} \cdot \underbrace{4 \cdot 3 \cdot 2}_{4!}} = 2$$

Write $\frac{1}{2}$ in front of the Feynman diag.

$V=2 \quad n=3$

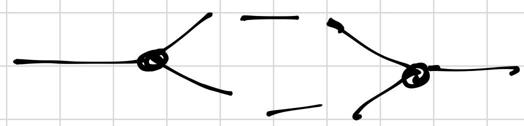




$1 \rightarrow \times 6$ ways

$2 \rightarrow \times 3$ ways

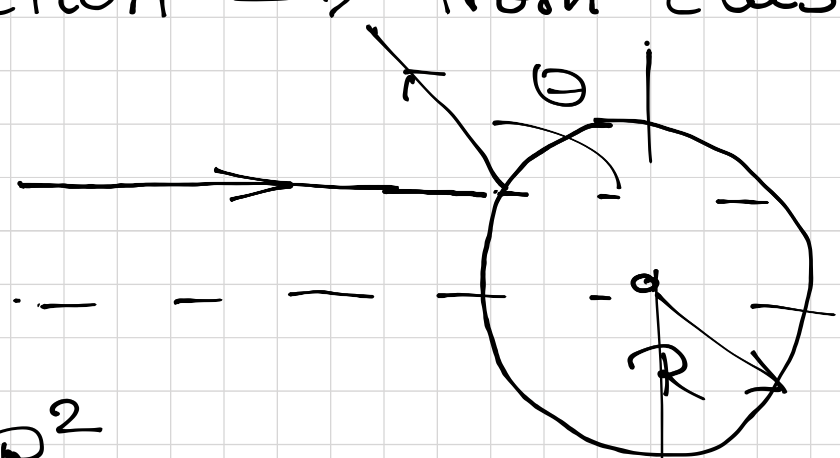
V_1 to V_2 $\times 2$ ways



$$S = \frac{2!(3!)^2}{6\cdot 3\cdot 2} = 2$$

Cross sections and decay rates

Cross section $\rightarrow$ from classical scatt. th.



$$\sigma = \pi R^2$$

$$\sigma = \frac{\#\text{ of particles scattered}}{\text{time}\cdot\#\text{ density of the beam}\cdot\text{velocity}}$$

$$= \frac{N}{T\cdot\Phi} \qquad \text{Flux } \Phi = \frac{|\vec{v}_1 - \vec{v}_2|}{V}$$

Classically : # of events $\rightarrow$ 100% or 0

projectile hits Target

— " — misses the Target

QM : $\quad P = \dfrac{N}{N_{inc}}$

$$d\sigma = \frac{V}{T}\,\frac{1}{|\bar{v}_1 - \bar{v}_2|}\,d\mathcal{P}$$

$$d\mathcal{P} = \frac{|\langle f|T|i\rangle|^2}{\langle f|f\rangle\langle i|i\rangle}\cdot d\Pi$$

$\longrightarrow$ $S = \mathbb{1} + iT$ $\quad \langle f|T|i\rangle = (2\pi)^4\delta(\Sigma \mathcal{P})\,A_{fi}$

$\longrightarrow$ $d\Pi = \prod_j \frac{d^3\vec{p}_j}{(2\pi)^3}\cdot V$

$|\langle\ \rangle|^2 \to [\delta(\ )]^2 ?$

$$\langle p|q\rangle = (2\pi)^3\, 2E_p\, \delta^3(\bar{p}-\bar{q})$$

$\hookrightarrow$ $\langle i|i\rangle = (2\pi)^3 2E_i \underbrace{\delta^3(0)}_{\infty}$

$$\delta^3(\vec{p}) = \int \frac{d^3\vec{x}}{(2\pi)^3}\, e^{i\vec{p}\vec{x}} \longrightarrow \delta^3(0) = \frac{V}{(2\pi)^3}$$

Keep V finite $\quad \left(\int^L \frac{d^3x}{(2\pi)^3}\Big|_{L\to\infty}\right)$

$$(2\pi)^3\delta^3(0) \longrightarrow V$$
$$(2\pi)^4\delta^4(0) \longrightarrow V\cdot T$$

$$|\langle f|T|i\rangle|^2 = (2\pi)^4\delta^{(4)}(\Sigma p_i - \Sigma p_f)\cdot\underbrace{(2\pi)^4\delta^{(4)}(0)}_{\text{"}T\cdot V}\,|A_{fi}|^2$$

$$\langle i|i\rangle = \prod_{j\in i}(2E_j V) \qquad \langle f|f\rangle = \prod_{k\in f}(2E_k V)$$

$$|i\rangle = |p_1\rangle |p_2\rangle$$

$$\downarrow$$

$$dP = \frac{(2\pi)^4 \delta^4(\Sigma p_i - \Sigma p_f)\, T V |A_{fi}|^2}{(2E_1 V)(2E_2 V) \prod_{j\in f} (2E_j \cancel{V})} \cdot \prod_k \left(\frac{d^3\vec{p}_k}{(2\pi)^3} \cancel{V} \right)$$

$$d\sigma = \frac{\cancel{V}}{\cancel{T}} \frac{1}{|\vec{v}_1 - \vec{v}_2|} \cdot (2\pi)^4 \delta^4(\Sigma p)\, |A_{fi}|^2 \cdot \frac{\cancel{T}\cancel{V}}{(2E_1\cancel{V})(2E_2\cancel{V})} \cdot d\Pi_{LI}$$

All T and V cancel out $\longrightarrow$ expand to $\underline{\mathbb{R}^4}$

$$d\Pi_{LI} = \prod_j \frac{d^3\vec{p}_j}{(2\pi)^3} \frac{1}{2E_j} (2\pi)^4 \delta^4(\Sigma p)$$

$$\vec{v} = \frac{\vec{p}}{E} \qquad \vec{v}_1 - \vec{v}_2 = \frac{E_2\vec{p}_1 - E_1\vec{p}_2}{E_1 E_2}$$

$$\Downarrow$$

$$\boxed{d\sigma = \frac{1}{|E_2\vec{p}_1 - E_1\vec{p}_2|} |A_{fi}|^2 \, d\Pi_{LI}}$$

Cross-section $2 \to 2$

$$d\Pi_{LI} = (2\pi)^4 \delta(p_1 + p_2 - p_3 - p_4) \cdot \frac{d^3\vec{p}_3}{(2\pi)^3 2E_3} \frac{d^3\vec{p}_4}{(2\pi)^3 2E_4}$$

Center of momentum : $\vec{p}_1 + \vec{p}_2 = 0$

Mandelstaum variable $s = (p_1 + p_2)^2 = (E_1 + E_2)^2$

$$E_1 + E_2 = \sqrt{s}$$

$$d\Pi_{LI} = 2\pi\, \delta(\sqrt{s} - E_3 - E_4)$$

$$E_3 = \sqrt{M_3^2 + \vec{p}_f^{\,2}}$$

$$E_4 = \sqrt{M_4^2 + \vec{p}_f^{\,2}}$$

$$d^3\vec{p}_3 = p_f^2\, dp_f\, d\Omega$$

$$d\Pi_{LI} = \frac{1}{16\pi^2} d\Omega \int \frac{p_f^2\, dp_f}{E_3 E_4} \delta(\sqrt{s} - E_3 - E_4)$$

$$\delta(f(x)) = \frac{1}{|f'(x_0)|} \delta(x_0)$$

$$\delta\left(\sqrt{s} - \sqrt{M_3^2 + p_f^2} - \sqrt{M_4^2 + p_f^2}\right)$$

$$|f'| = \frac{p_f}{E_3} + \frac{p_f}{E_4} = \frac{\sqrt{s}\, p_f}{E_3 E_4}$$

$$p_f^0 = \frac{\Lambda(s, M_3, M_4)}{2\sqrt{s}}$$

$$\Lambda(a,b,c) = \sqrt{[a - (b+c)^2][a - (b-c)^2]}$$

$$= \frac{\sqrt{[s - (M_3 + M_4)^2][s - (M_3 - M_4)^2]}}{2\sqrt{s}}$$

$$s \geq (M_3 + M_4)^2$$

$$\frac{d\sigma}{d\Omega_{c.m.}} = \frac{1}{16\pi^2}\,\frac{|\vec{p}_f^{\,\circ}|}{\sqrt{s}}\,\frac{\Theta(\sqrt{s}-M_3-M_4)}{4\underbrace{|E_2\vec{p}_1 - E_1\vec{p}_2|}_{\sqrt{s}\cdot|\vec{p}_i^{\,\circ}|}}\,|A_{fi}|^2$$

$$\boxed{\frac{d\sigma}{d\Omega_{c.m.}} = \frac{1}{64\pi^2 s}\,\frac{|\vec{p}_f^{\,\circ}|}{|\vec{p}_i^{\,\circ}|}\,|A_{fi}|^2\,\Theta(\sqrt{s}-M_3-M_4)}$$

Decay rates

differential decay rate $d\Gamma = \frac{dP}{T}$
(probability for a particle to decay over time T)

Use dP

$$dP = \frac{1}{2E_i}\,|A_{fi}|^2\,d\Pi_{LI}$$

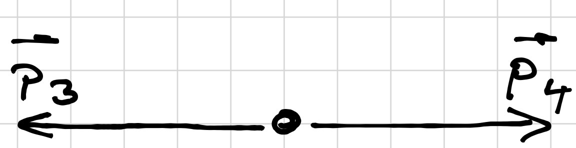
Define Γ in the particle rest frame

$$d\Gamma = \frac{1}{2M}\,|A_{fi}|^2\,d\Pi_{LI}$$

$1 \to 2$

$$d\Pi^{(2)}_{LI} = \frac{1}{16\pi^2}\,\frac{|\vec{p}_f^{\,\circ}|}{M}\,\Theta(M-M_3-M_4)$$

$$\sqrt{s} = M$$



$d\Omega \longrightarrow 4\pi$

If particles 3 and 4 identical $\longrightarrow \int d\Omega = 2\pi$

For n identical particles $\rightarrow$

$$\int d\Omega = \frac{1}{n!} 4\pi$$

$$\Gamma = \frac{1}{8\pi} \frac{|\vec{p}_f|}{M^2} |A_{fi}|^2$$ non-identical

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$$\Gamma = \frac{1}{16\pi} \frac{|\vec{p}_f|}{M^2} |A_{fi}|^2$$ identical

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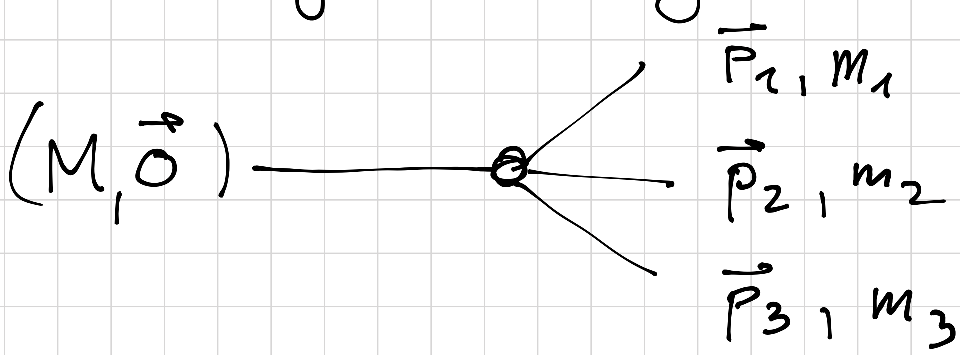
3- body decay



Additional 3- fold $\int$

But all 3-momenta are in one plane

$\hookrightarrow$ only $2\int$ remain

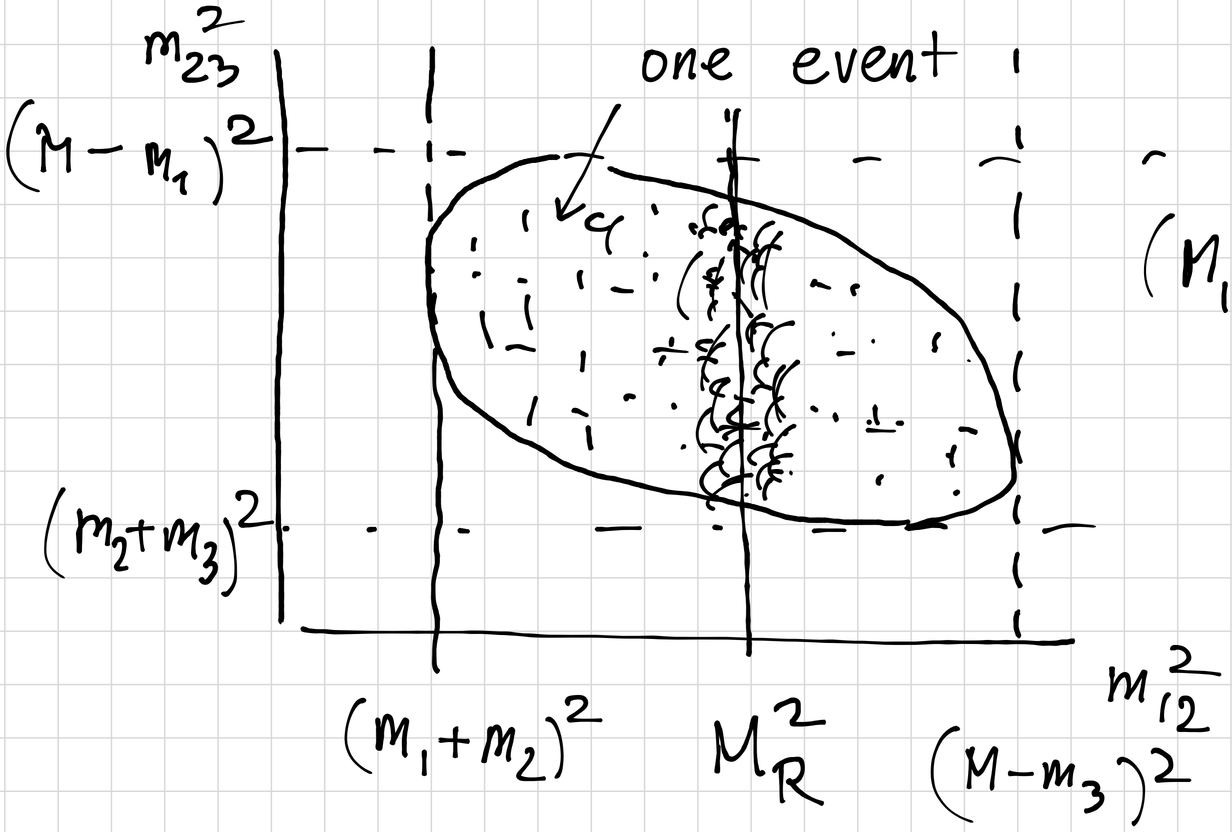
Standard choice:

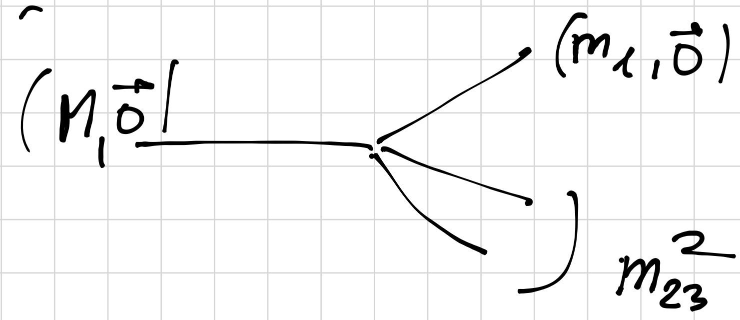
$$m_{12}^2 \equiv (p_1 + p_2)^2 = m_1^2 + m_2^2 + 2 p_1 p_2$$

$$m_{23}^2 = (p_2 + p_3)^2$$

$$d\Gamma = \frac{1}{(2\pi)^3} \frac{1}{32 M^3} |A|^2 \, dm_{12}^2 \, dm_{23}^2 \, \Theta(\dots)$$

$\hookrightarrow$ Dalitz plot





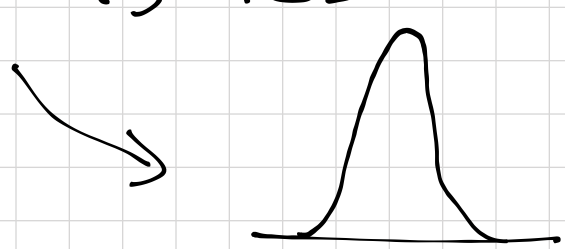
$$\underline{m_{23}^2 = (M-m_1)^2}$$

Dalitz plot $\longrightarrow$ is <u>observable</u>

Experiment: detect $(1,2,3)$

If $|A_{fi}|^2 =$ constant $\Longrightarrow$ uniformly populated

If $|A_{fi}|^2$ is resonant (has an on-shell particle with a width)



$\rightarrow$ This is one way to look for new resonances

<u>Example</u> "σ meson" $\longrightarrow$ 2 "neutral pions"

$$\sigma \longrightarrow 2\pi^0$$

$\sigma \longleftrightarrow f_0(500)$ spin-0 (0^{++}) state

Mass $\underset{M_\sigma}{[400-500]\,\text{MeV}} - i\underset{\text{''}\ \Gamma_\sigma/2}{[200-350]\,\text{MeV}}$

π^0 : spin - 0 (0^{-+}) $\quad m_\pi = 134,97$ MeV

$$\mathcal{L}_{\sigma\pi^0} = - g_{\sigma\pi^0} \underline{M_\sigma} \, \sigma \pi^0 \pi^0$$

$2\pi^0$ are identical

$$A = - M_\sigma g_{\sigma\pi^0} \times \underline{2}$$

C.M. frame $\quad -\vec{p} \longleftarrow \underset{M_\sigma}{\circ} \longrightarrow \vec{p}$

$$E_1 = E_2 = \sqrt{m_\pi^2 + \vec{p}^{\,2}} = \frac{M_\sigma}{2}$$

$$|\vec{p}| = \sqrt{\frac{M_\sigma^2}{4} - m_\pi^2} \qquad \theta(M_\sigma - 2m_\pi)$$

$$d\Gamma = \frac{1}{32\pi^2} \frac{|\vec{p}|}{M_\sigma^2} |A|^2 \underbrace{d\Omega} \longrightarrow \frac{1}{2} 4\pi$$

$$\Gamma_{\sigma\to\pi^0\pi^0} = \frac{1}{4\pi} \sqrt{\frac{M_\sigma^2}{4} - m_\pi^2} \; g_{\sigma\pi^0}^2$$

$\sigma \longrightarrow \psi \bar{\psi}$ $\qquad \underline{\mathcal{L} = - M_\sigma g_{\sigma N} \sigma \psi^* \psi}$

$$A = - M_\sigma g_{\sigma N} \qquad d\Omega \to 4\pi$$

$$\Gamma_{\sigma\to N\bar{N}} = \frac{1}{8\pi} \sqrt{\frac{M_\sigma^2}{4} - m_N^2} \; g_{\sigma N}^2$$

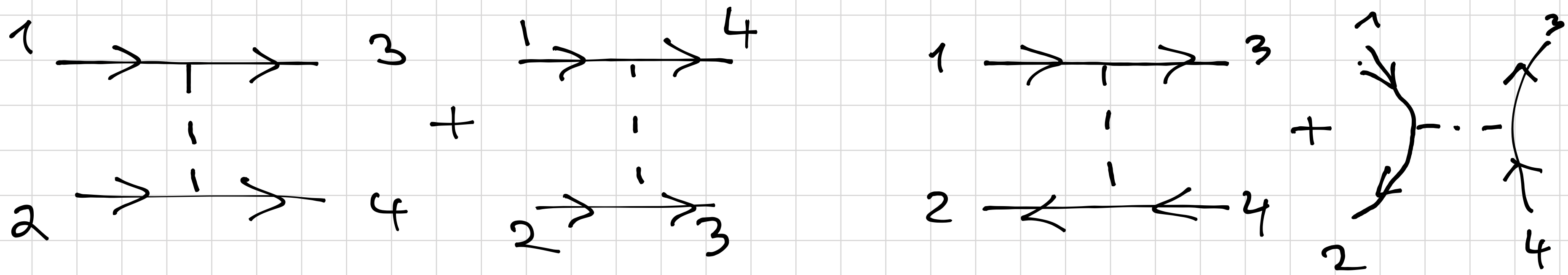
$2\times$ probability for a decay into 2 indistinguishable particles

$$\psi\psi \to \psi\psi \Longleftrightarrow \psi\bar{\psi} \to \psi\bar{\psi}$$

What will the $\frac{d\sigma}{d\Omega}(\theta)$ look like for both?

$$|A_{NN\to NN}|^2 \quad \text{vs} \quad |A_{N\bar{N}\to N\bar{N}}|^2$$

$$\left[\frac{1}{t-m^2}+\frac{1}{u-m^2}\right]^2 \qquad \left[\frac{1}{t-m^2}+\frac{1}{s-m^2}\right]^2$$



$$E_1 = E_2 = E_3 = E_4 = \frac{\sqrt{s}}{2} \qquad |\vec{p}_1| = |\vec{p}_2| = |\vec{p}_3| = |\vec{p}_4| = |\vec{p}|$$

$$t = (p_1 - p_3)^2 = -2\vec{p}^{\,2}(1-\cos\theta)$$

$$u = (p_1 - p_4)^2 = -2\vec{p}^{\,2}(1+\cos\theta)$$

$$S = s = 4E^2$$

$$\frac{1}{t-m^2}+\frac{1}{u-m^2} = -\frac{4p^2+2m^2}{(2p^2+m^2)^2-4p^4\cos^2\theta}$$

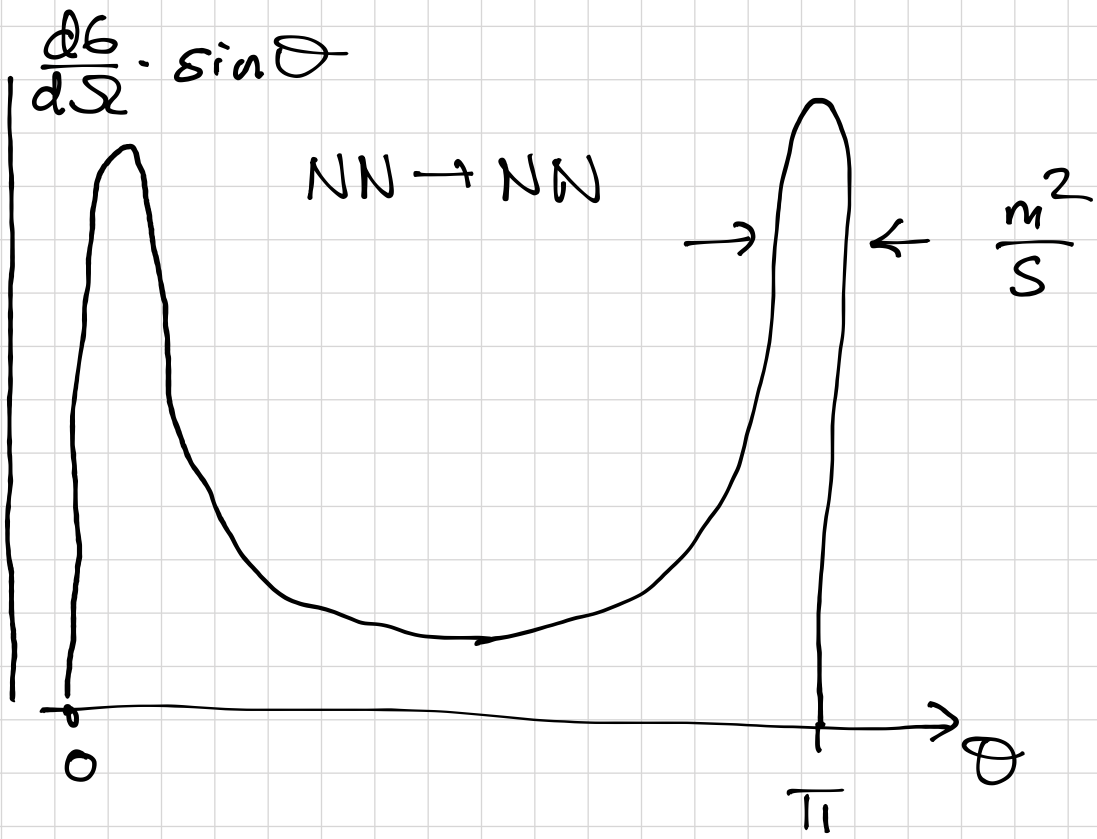
symmetric $\theta \to \pi - \theta$

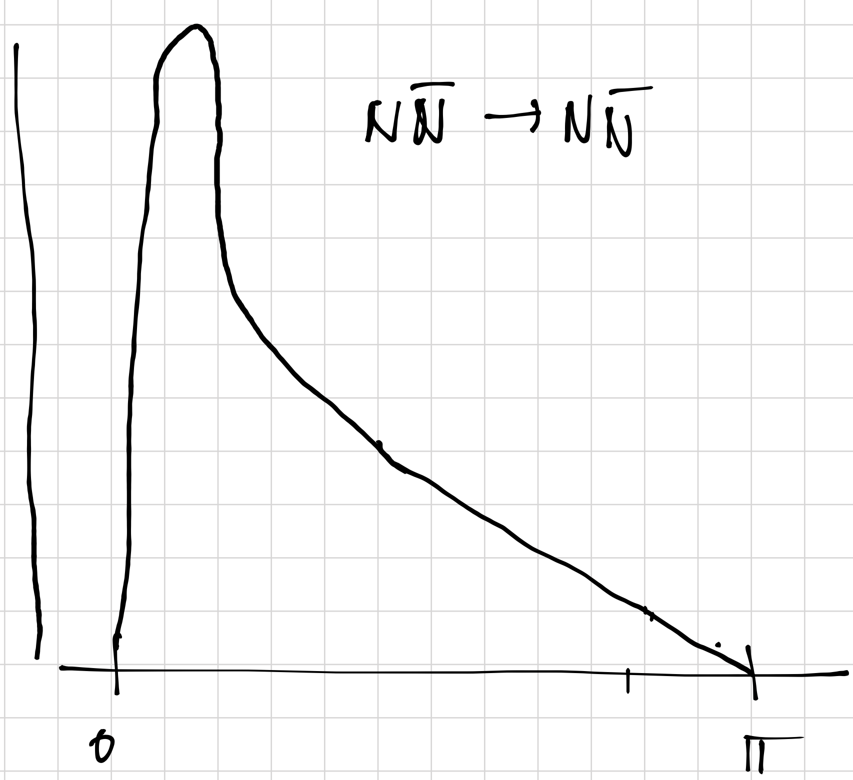
$$\frac{1}{t-m^2}+\frac{1}{s-m^2} = \frac{1}{4E^2-m^2} - \frac{1}{2p^2(1+\cos\theta)+m^2}$$

High energy $s \gg M$ $\quad |\vec{p}| = \sqrt{\frac{s}{4}-M^2} \approx \frac{\sqrt{s}}{2}$

Let's keep m finite

$\frac{d\sigma}{d\Omega}(\theta)$ $\qquad$ $\frac{d\sigma}{d\Omega} \cdot \sin\theta$ $\qquad$ $\frac{4E^2 - 4E^2}{\cdots} \to 0$





ILC

Bhabha scattering

$e^+e^- \to e^+e^-$

at large angles

→ look for signals from heavy particles (in SM the cross section is small and can be calculated reliably)