# Lecture 7



Wick's theorem

$$T(\phi_1 \dots \phi_n) = :\phi_k \dots \phi_m: + \Delta_F(x_j - x_e)$$

Dyson formula $\quad H_I = \int d^3x \, \mathcal{H}_{int}(t)$

$$U(t, t_0) = T \exp\left(-i \int_{t_0}^{t} H_I(t')\,dt'\right)$$

$$= \mathbb{1} - i \int_{t_0}^{t} H_I \, dt' + \frac{(-ig)^2}{2!} \int_{t_0}^{t} dt' H_I(t') dt' \cdot \int_{t_0}^{t'} dt'' H_I(t'') dt''$$

S-matrix $\quad U(+\infty, -\infty)$.

$$\langle f | S | i \rangle = \langle f | \underbrace{T\exp\left(-i \int d^4x \, \mathcal{H}_{int}\right)}_{\downarrow \text{ series}} | i \rangle$$

$\hookrightarrow$ Feynman rules $\longrightarrow$ need the explicit form of $\mathcal{H}_{int}$

<u>Yukawa theory</u>

$$\mathcal{L} = \partial_\mu \psi^* \partial^\mu \psi - M^2 \psi^* \psi + \frac{1}{2} \partial_\mu \phi \, \partial^\mu \phi - \frac{1}{2} m^2 \phi^2 - g \psi^* \psi \phi$$

§ $H = T + V \qquad \mathcal{L} = T - V$

$\mathcal{H}_{int} = +\underline{g}\,\psi^{*}\psi\phi$

Feynman rules: vertex $-ig(2\pi)^4\delta^4(\sum_i k_i)$

internal line

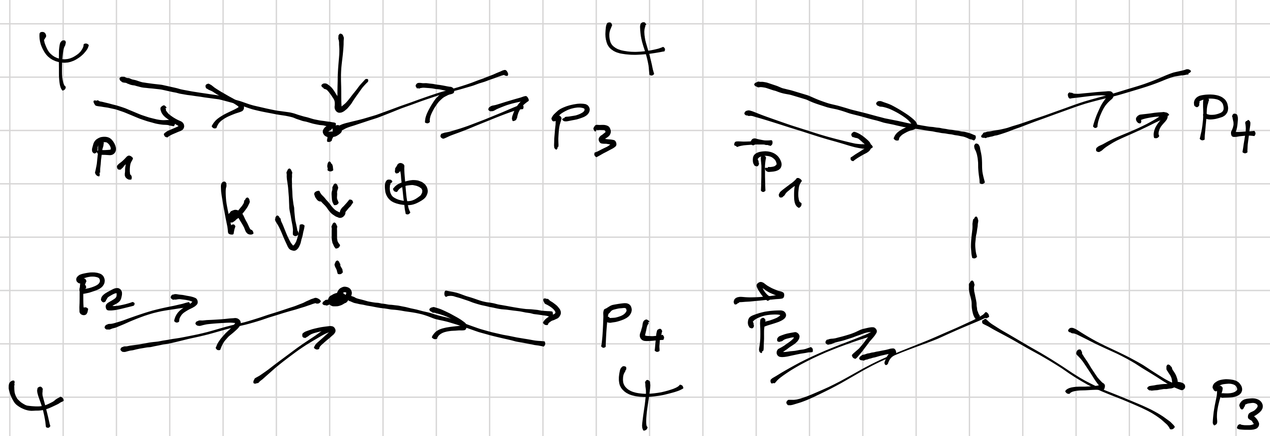
--- $\phi$ $\int \frac{d^4k}{(2\pi)^4}\frac{i}{k^2-m^2+i\varepsilon}$

$\rightarrow$ $\psi$

$\leftarrow$ $\psi^{*}$ $\int \frac{d^4p}{(2\pi)^4}\frac{i}{p^2-M^2+i\varepsilon}$

At order $O(g^2)$

$\psi\psi \to \psi\psi$




$$\langle\psi\psi|S-\mathbb{1}|\psi\psi\rangle = \int \frac{d^4k}{(2\pi)^4}\frac{i}{k^2-m^2+i\varepsilon}$$

$$\Big[(-ig)(2\pi)^4\delta^4(p_1-p_3-k)\cdot(-ig)(2\pi)^4\delta(p_2-p_4+k)$$

$$+(-ig)(2\pi)^4\delta(p_1-p_4-k)\cdot(-ig)(2\pi)^4\delta(p_2-p_3+k)\Big]$$

$$\Rightarrow\ i(-ig)^2\left[\frac{1}{(p_1-p_3)^2-m^2+i\varepsilon}+\frac{1}{(p_1-p_4)^2-m^2+i\varepsilon}\right]$$

$$\cdot (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4)$$ (3)

→ External particles → free → on the mass shell
or on-shell : $\underline{p_i^2 = M^2}$

Q. can the intermediate meson go on-shell?

$44 \to 44$ center of momentum
(of mass)

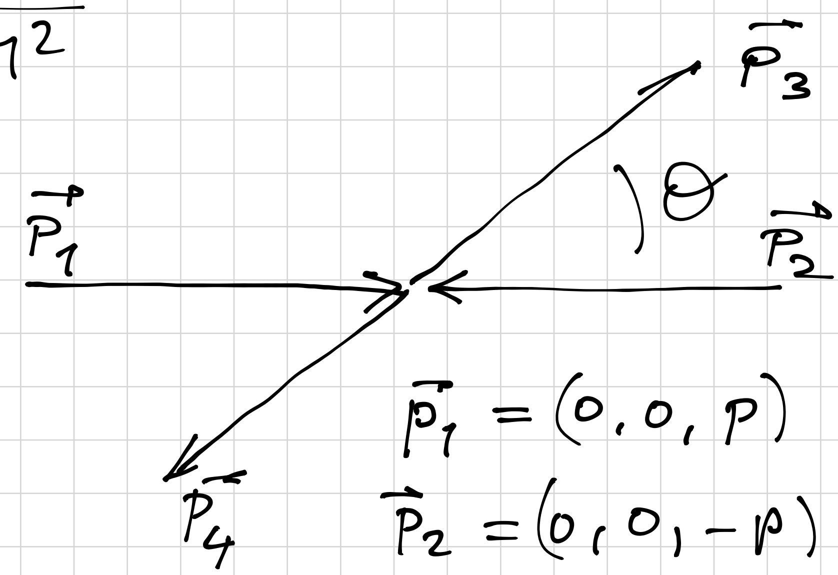
$$\vec{p}_1 + \vec{p}_2 = 0 \qquad M_1 = M_2 = M$$

$$\hookrightarrow E_1 = E_2 = E_3 = E_4$$

$$E = \sqrt{\vec{p}^{\,2} + M^2}$$

$$(p_1 - p_3)^2 - m^2$$

$$= -(\vec{p}_1 - \vec{p}_3)^2 - m^2 < 0$$




$$\vec{p}_1 = (0, 0, p)$$
$$\vec{p}_2 = (0, 0, -p)$$
$$\vec{p}_3 = (p\sin\theta, 0, p\cos\theta)$$
$$\vec{p}_4 = -\vec{p}_3$$

What is the meaning of the above amplitude?

$$\langle f | S - \mathbb{1} | i \rangle = i A (2\pi)^4 \delta^4(\Sigma p)$$

$$A \sim \frac{i g^2}{(\vec{p}_1 - \vec{p}_3)^2 + m^2}$$

$|p| \ll M$ Non-relativistic nucleons

4

$$i\langle \vec{p}_3 | U(\vec{r}) | \vec{p}_1 \rangle \leftrightarrow \frac{-(g/2M)^2}{(\vec{p}_1 - \vec{p}_3)^2 + m^2}$$

$2E_p \to 2M$

$$= \int d^3\vec{r}\, U(\vec{r})\, e^{-i(\vec{p}_1 - \vec{p}_3)\vec{r}}$$

$$U(\vec{r}) = -\left(\frac{g}{2M}\right)^2 \int \frac{d^3\vec{k}}{(2\pi)^3} \frac{e^{i\vec{k}\vec{r}}}{k^2 + m^2}$$

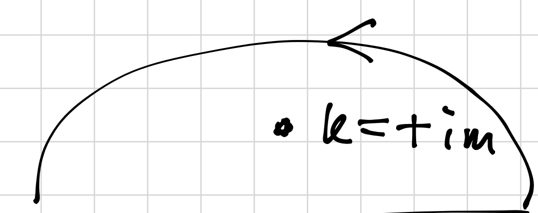
We take the $\int$ to complex $k$-plane

$\longrightarrow$ pick the pole at $k = \pm im$

$$\int \frac{k^2 dk\, d\cos\theta\, d\varphi}{(2\pi)^3} \frac{e^{ikr\cos\theta}}{k^2 + m^2}$$

$$= \frac{1}{ir} \frac{1}{4\pi^2} \int \frac{k\,dk}{k^2 + m^2} \left(e^{ikr} - e^{-ikr}\right)$$

$e^{ikr}$




$$\int_{C_+} \frac{k\,dk}{(k+im)(k-im)} e^{ikr} = 2\pi i \frac{im}{2im} e^{-mr}$$

$$\int_{C_-} \frac{k\,dk\; e^{-ikr}}{(k+im)(k-im)} = (-)2\pi i \frac{+im}{+2im} e^{-mr}$$

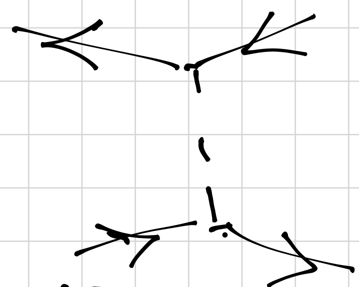
$$U(r) = \frac{-(g/2M)^2}{4\pi r} e^{-mr}$$ Force of range $1/m$

$$F = \frac{\partial U}{\partial r} = + \frac{(g/2M)^2}{4\pi r^2} e^{-mr} (1+mr)$$

Attractive

If we consider $\psi\bar{\psi} - \psi\bar{\psi}$

the same sign



Universally scalar exchange $\rightarrow$ attractive potential

spin 1 (photon) $\rightarrow$ charge inverts $e^+, e^-$

spin 2 $\rightarrow$ again universally attr.

↳ graviton (gravity)

↳ no antigravity

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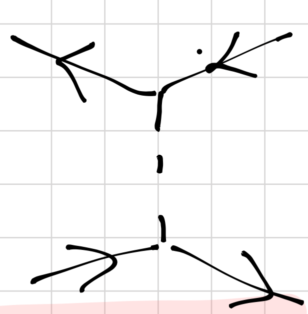
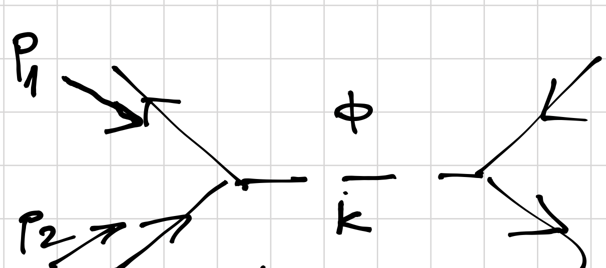
Is $\psi\bar{\psi} \rightarrow \psi\bar{\psi}$ exactly the same as $\psi\psi \rightarrow \psi\psi$ ?



the same




$$\frac{-ig^2}{(p_1+p_2)^2 - m^2} \longrightarrow \text{can go on-shell}$$

↳ can create a particle

Higgs boson discovery:



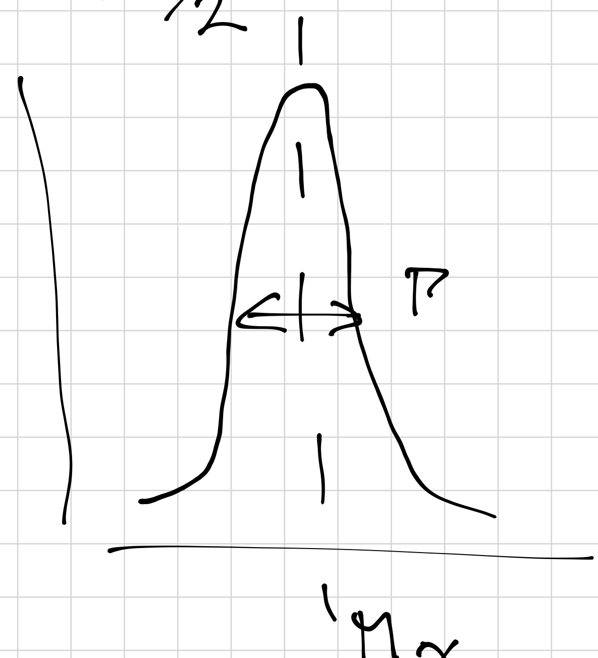
Imagine we include a heavy meson $\chi$

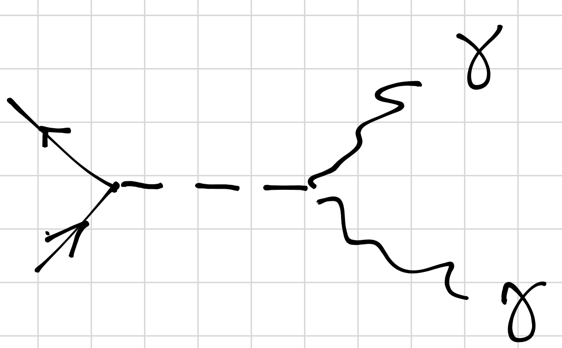
$$\mathcal{L}_\chi = \frac{1}{2}\partial_\mu\chi\,\partial^\mu\chi - \frac{1}{2}M_\chi^2\chi^2 - g'\psi^*\psi\chi$$

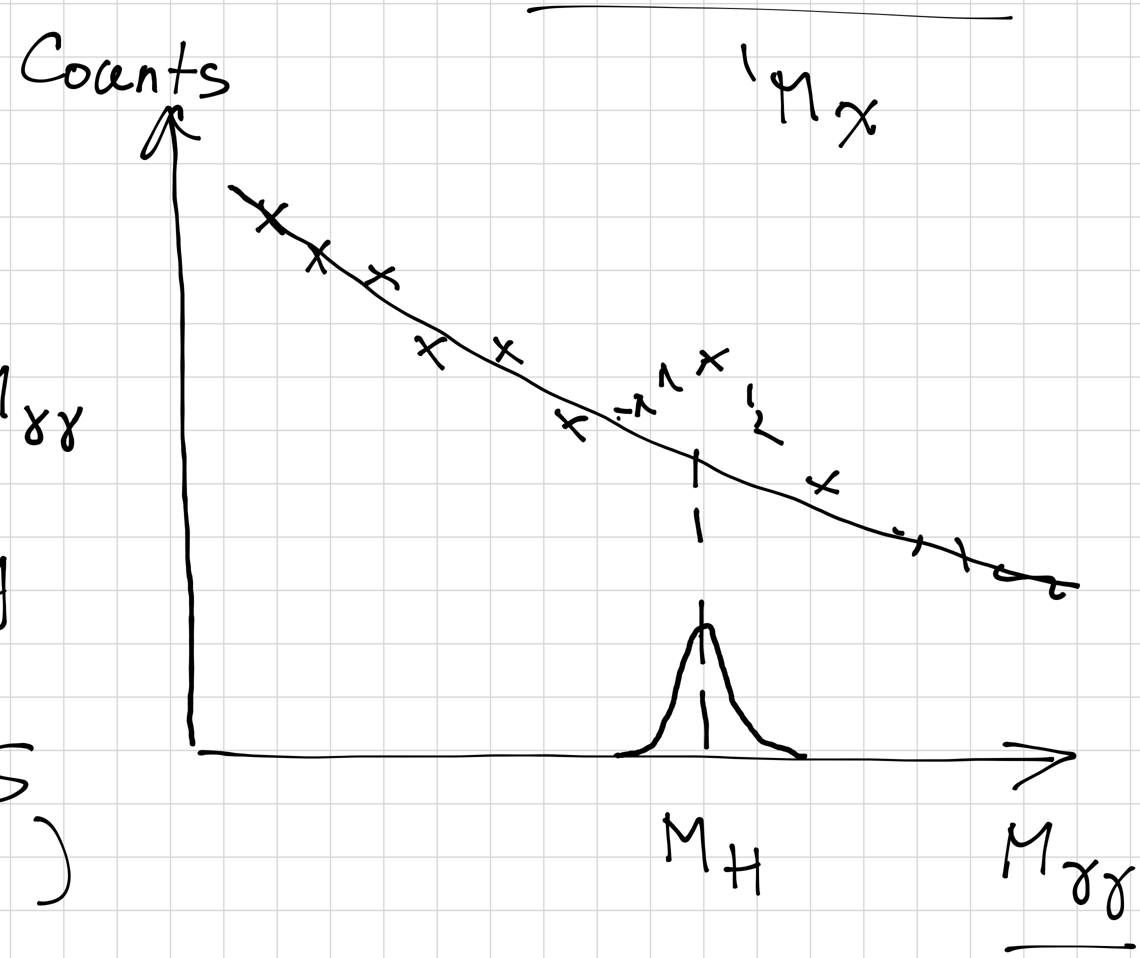
$$\frac{-ig'^2}{(p_1+p_2)^2 - \underline{M_\chi^2}} \qquad M_\chi \gg M$$

c.m. $\vec{p}_1 + \vec{p}_2 = 0 \implies \underline{4(M^2 + \vec{p}^{\,2}) = M_\chi^2}$

$$M_\chi \longrightarrow M_\chi - i\,\Gamma/2$$



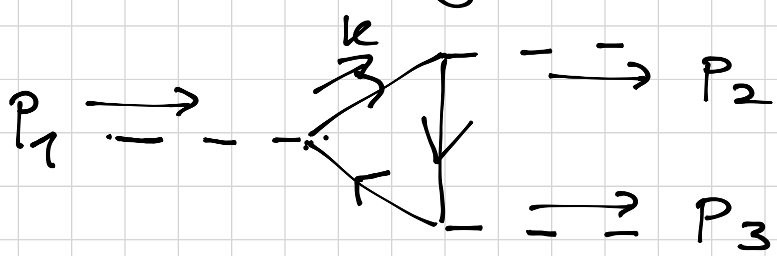




$$\sqrt{(p_1^\gamma + p_2^\gamma)^2} = M_{\gamma\gamma}$$

Higgs discovery
2012 CERN
(ATLAS and CMS collaborations)

Meson – meson interaction?

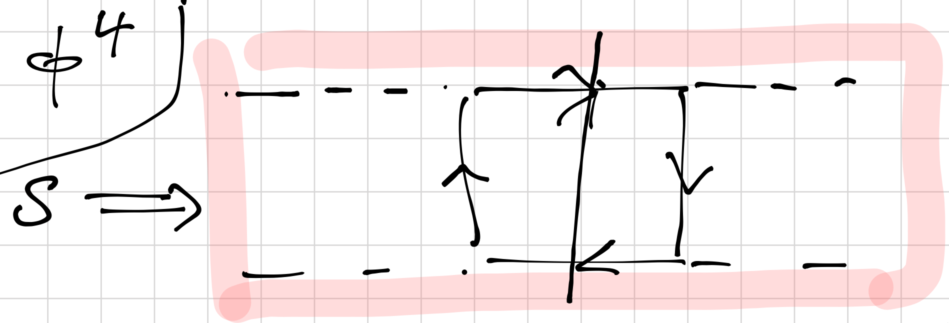
$\longrightarrow$ will only appear at higher orders through nucleon loops




$$(-ig)^3 \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2-M^2+i\varepsilon} \frac{i}{(k-p_2)^2-M^2+i\varepsilon} \frac{i}{(k-p_1)^2-M^2+i\varepsilon}$$

$\frac{1}{3!}\lambda\phi^3$ — not in the original $\mathcal{L}$!

$\frac{1}{4!}\lambda\phi^4$




Imagine that $M \gg m, |\vec{p}_i|$

$$(-ig)^3 \int^{\Lambda} \frac{d^4k}{(2\pi)^4} \frac{i^3}{(-M^2)^3} \sim \frac{g^3}{M^2} \frac{1}{(2\pi)^4} \frac{\Lambda^4}{M^4}$$

When $M \to \infty$ (very low energies
$\longrightarrow$ you can only see $\phi$'s)

The heavy fields decouple from the theory

$\hookrightarrow$ at low energies you operate with an effective theory with just mesons

$\longrightarrow$ Go to very high energies try to get close $(2M)^2$ 

$\rightarrow$ This is the bases for precision tests of Standard Model at low energies
You look for deviations from your low-energy theory that doesn't know anything about particles + of heavy mass $M$

Experiments on Parity-Violating electron scattering

SM prediction $\longrightarrow$ $< 1\%$ of the signal

Measurement: $1,5\%$ measurement of a tiny asymmetry $(10^{-8})$ !

$\longrightarrow$ Set limits of unknown particles at $\sim 50$ TeV
(compare to LHC $\sim 14$ TeV)

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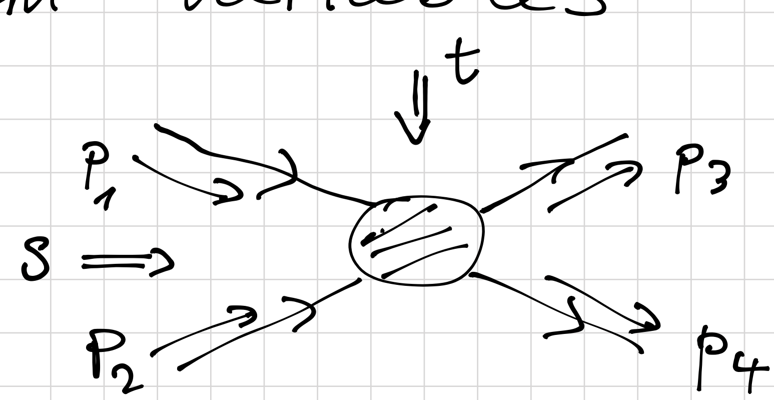
Mandelstam variables

Lorentz scalars out of $p_{1 \dots 4}$




$s = (p_1 + p_2)^2 = (p_3 + p_4)^2$   c.m. $\frac{(E_1 + E_2)^2 = s}{\vec{p}_1 + \vec{p}_2 = 0}$

$t = (p_1 - p_3)^2 = (p_2 - p_4)^2$

$u = (p_2 - p_3)^2 = (p_1 - p_4)^2$ $\qquad p_i^2 = M_i^2$

$$s + t + u = \sum_i M_i^2$$

$\psi\psi \to \psi\psi \qquad A = -ig^2\left(\frac{1}{t-m^2} + \frac{1}{u-m^2}\right)$

$\psi\bar{\psi} \to \psi\bar{\psi} \qquad A = -ig^2\left(\frac{1}{t-m^2} + \frac{1}{s-m^2}\right)$

Similarity between different reaction channels

Crossing $=$ swapping some outgoing and incoming momenta

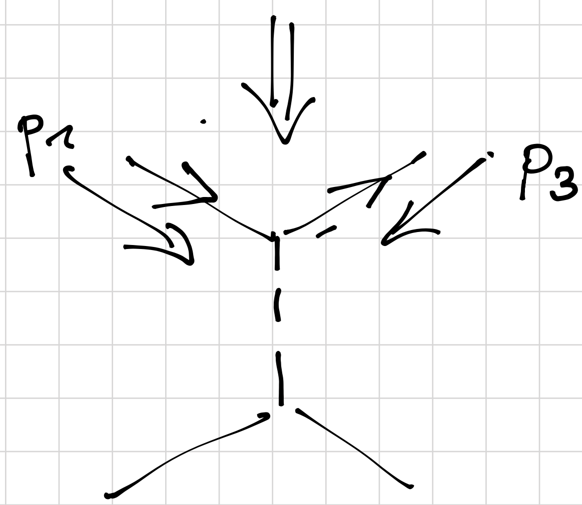
s-channel ( $E_1 + E_2 = \sqrt{s}$ )

$p_1 + p_2 \longrightarrow p_3 + p_4$

$p_2 \leftrightarrow -p_3$

$s_t = (p_1 + p_3)^2$

$t_t = (p_1 - p_2)^2$

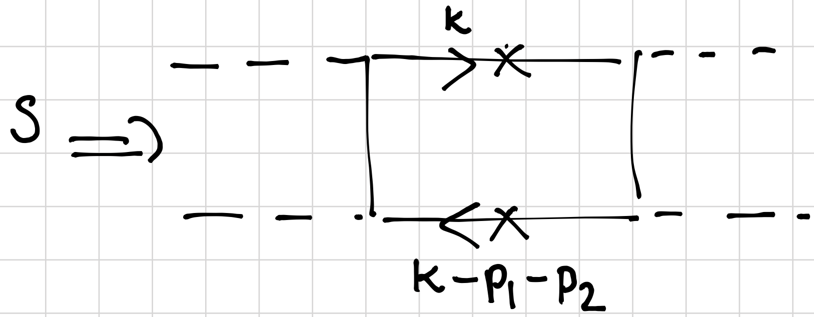



establish symmetries between different reactions

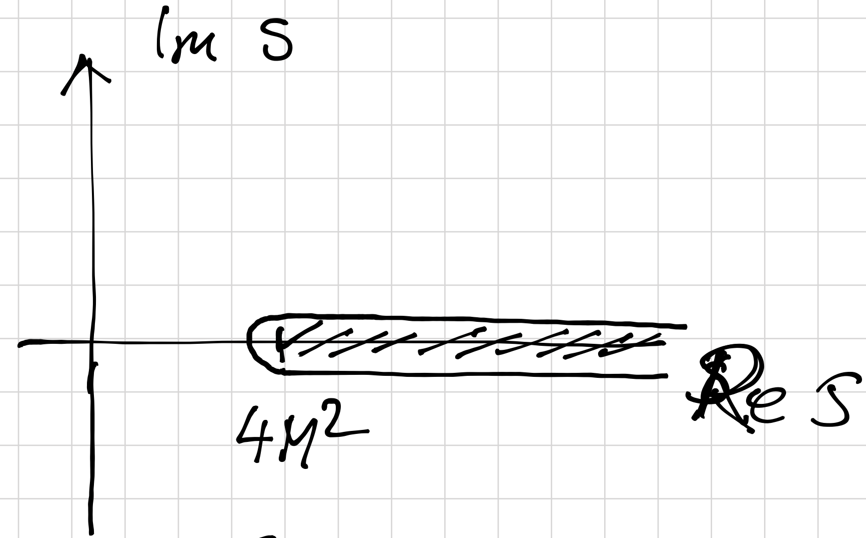
Also: simple analytical structure:

$$A \sim \frac{1}{t-m^2}\ ;\ \frac{1}{s-m^2}\ ;\ \frac{1}{u-m^2}$$

If you have loops $\longrightarrow$ another type of singularities (branch cut)




$\psi\bar{\psi}$ pair can be on-shell starting from $\sqrt{s} = 2M$




$$\log(4M^2 - s \pm i\varepsilon) = \pm i\pi + \ln|s - 4M^2|$$

$$\sqrt{4M^2 - s \pm i\varepsilon} = \pm i\sqrt{s - 4M^2}$$

Analytical properties of scatt. ampls.

→ poles and branch cuts along the real axis (correspond to on-shell part. production)

→ crossing symmetry