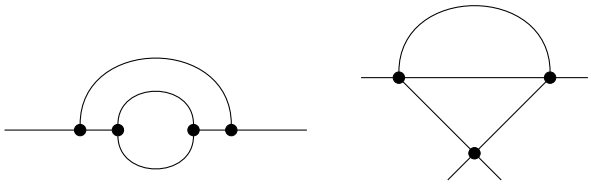
Relativistic QFT (Theo 6a): Exercise Sheet 4
Total: 100 points

27/11/2020

1. More Feynman Diagrams and Symmetry Factors (15 points)

Write the analytical expressions for the diagrams in ϕ^3 (left diagram) and ϕ^4 (right diagram) theories and obtain their symmetry factors. Elimimnate as many 4-fold momentum integrations as possible with the δ-functions.



(1)

2. Relativistic Muons at Fermilab (10 points)

Knowing that the muon's mean lifetime in the rest frame $\tau_\mu \approx 2.20$ μs and the mass $m_\mu \approx 106$ MeV, calculate the mean lifetime of the muon beam with the energy $E_\mu \approx 750$ GeV circulating in the muon accelerator ring at Fermilab.

3. $\sigma \to \pi\pi$ decays (30 points)

The $\sigma - \pi\pi$ interaction Lagrangian term reads

$$\mathcal{L}_{\sigma\pi} = -g_{\sigma\pi} M_\sigma \sigma(\pi_1^2 + \pi_2^2 + \pi_3^2). \tag{2}$$

Here σ is a real scalar field, and so are the components of the isospin triplet pion $\vec{\pi} = (\pi_1, \pi_2, \pi_3)^T$. The physically observed charge eigenstates are their linear combinations, $\pi^\pm = \frac{1}{\sqrt{2}}(\pi_1 \pm i\pi_2)$, $\pi_0 = \pi_3$.

Calculate the decay width Γ for the reaction $\sigma \to \pi^+\pi^-$. Compare the results with the case $\sigma \to \pi^0\pi^0$

4. Nucleon Scattering in Yukawa Theory (45 points)

Consider the nucleon-nucleon ($NN \to NN$) and nucleon-antinucleon ($N\bar{N} \to N\bar{N}$) scattering processes in Yukawa theory $\mathcal{L}_{int} = -g\phi\psi^*\psi$ in the center-of-momentum frame of the incomming particles.

(a) With the masses of the particles M for the nucleon and m for the meson, obtain expressions for the differential and total cross sections for these processes (**30 points**)

(b) Compare the angular dependence of the differential cross sections $d\sigma/d\Omega$ at low ($s \to 4M^2_{\text{nucleon}}$) and high ($s \to \infty$) energy limits. How do the total cross sections $\sigma = \int d\Omega\, (d\sigma/d\Omega)$ behave with s at high energies, $s \gg M^2$, m^2? (**15 points**)