Relativistic QFT (Theo 6a): Exercise Sheet 3
Total: 100 points

20/11/2020

1. Wick's theorem and Feynman diagrams in $g\psi^*\psi\phi$ theory (50 points)

The $g\psi^*\psi\phi$ Yukawa theory for mesons and nucleons is described by the Lagrangian

$$\mathcal{L} = \partial_\mu \psi^* \partial^\mu \psi - M^2 \psi^* \psi + \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2 - g\psi^*\psi\phi, \tag{1}$$

where the first pair of terms corresponds to a free complex "nucleon" field ψ, the second pair of terms - to a free scalar "meson" field ϕ, and the last term - to the interaction between these fields. Since $\mathcal{H} = 2\partial_0\psi^*\partial^0\psi + \partial_0\phi\partial^0\phi - \mathcal{L}$, the interaction part of Hamiltonian is $\mathcal{H}_{int} = g\psi^*\psi\phi$.

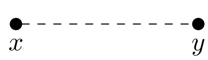
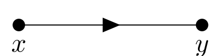
(a) Consider the theory at the second order of perturbation series, e.g. the second term in the S-matrix expansion:

$$S^{(2)} = \frac{(-ig)^2}{2!} \int d^4x \int d^4y \, T\left[\phi(x)\psi(x)\psi^*(x)\phi(y)\psi(y)\psi^*(y)\right]. \tag{2}$$

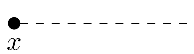
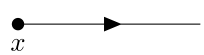
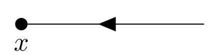
Applying the Wick's theorem, expand the T-ordered product

$$T\left[\phi(x)\psi(x)\psi^*(x)\phi(y)\psi(y)\psi^*(y)\right] \tag{3}$$

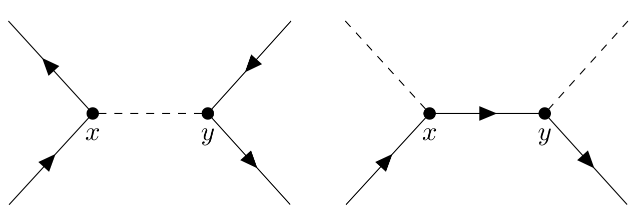
using the following conventions for the graphical representation of Wick's contractions

$\overline{\phi(x)\phi(y)} =$  $\overline{\psi(x)\psi^*(y)} =$ 

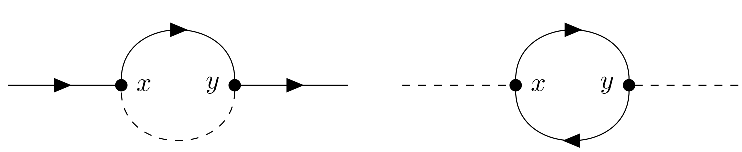
and for the normal ordered terms

$: ...\phi(x)... :=$  $: ...\psi(x)... :=$  $: ...\psi^*(x)... :=$ 

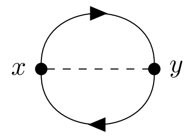
Show, that T-product (3) produces the following set of the Feynman diagrams (**25 points**):



(4)



(5)



(6)

(b) Obtain the matrix elements (**25 points**)

$$\langle p_f, k_f|S^{(2)}|p_i, k_i\rangle, \quad \langle p_f|S^{(2)}|p_i\rangle, \quad \langle k_f|S^{(2)}|k_i\rangle, \tag{7}$$

where $|p_i\rangle$ and $\langle p_f|$ are the initial and final states of the "fermion" particle ψ with corresponding momenta p_i and p_f, $|k_i\rangle$ and $\langle k_f|$ are the initial and final states of the "meson" particle ϕ with corresponding momenta k_i and k_f ($k_i \neq k_f$ and $p_i \neq p_f$ is assumed). The steps are following:

i. choose the diagram in set (4)-(5), which corresponds to the correct initial and final state,
ii. write the expression for it using Feynman rules from the last lecture,
iii. substitute this expression into (2) and take the integration over momenta as far as it is possible.

2. Feynman rules in ϕ^3 and ϕ^4 theories (50 points)

Let us consider the Lagrangian $\mathcal{L}$ for the real scalar field ϕ

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 + \mathcal{L}_{int} \tag{8}$$

with interaction term

$$\mathcal{L}_{int} = \begin{cases} \frac{-\lambda}{3!}\phi^3 & \text{for } \phi^3 \text{ theory} \\ \frac{-\lambda}{4!}\phi^4 & \text{for } \phi^4 \text{ theory} \end{cases} \tag{9}$$

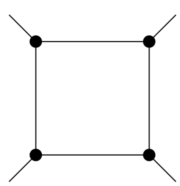
or, alternatively, the interaction part of Hamiltonian

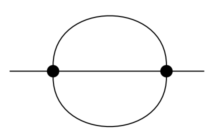
$$\mathcal{H}_{int} = \begin{cases} \frac{\lambda}{3!}\phi^3 & \text{for } \phi^3 \text{ theory} \\ \frac{\lambda}{4!}\phi^4 & \text{for } \phi^4 \text{ theory} \end{cases} \tag{10}$$

The parameter λ is real and positive.

(a) Write down the Feynman rules for both ϕ^3 and ϕ^4 theories.(**10 points**)

(b) Find the expressions for the diagrams below. All external momenta are assumed to be incoming.(**20 points**)





(11)

(c) Obtain the symmetry multiplier of the diagrams. (**20 points**)