Lecture 1

Nov. 2

Quantum Field Theory
- Classical Field Theory
- Scalar Field
- Dirac Fields
- Interacting Fields
- Renormalization

Books
- QFT and Standard Model by Matthew Schwartz
- Peskin and Schroeder
- Steven Weinberg The Quantum Theory of Fields I and II
- QFT by David Tong
- QFT by Tobias Osborne

Intro Why QFT?

Theory of Everything

SUSY; Quantum Gravity; String; Extra Dimensions

⇓ Lower Energies / Larger scale

Relativistic QFT

Standard Model
Electromagnetic
Weak
Strong Interactions

⇓ Lower Energies

Non-Relativistic QFT

⇓ Lower Energies / Larger Scales

Decoherence

⇓

Classical Field Theory

QM works with fixed # of particles

Relativistic QFT ?

$E = mc^2$ $\qquad \Delta E \geq 2 m_e c^2$

An e^+e^- pair can pop out of vacuum.

$$\lambda = \frac{\hbar}{mc}$$

$\boxed{1}$ Relativity is incompatible with fixed number of particles

$\boxed{2}$ All particles accross the Universe are indistinguishable

Protons from cosmic rays; nuclear spectra in stars and on Earth

$\boxed{3}$ Concept of the force acting at distance

Force carrier: photon $\leftrightarrow$ electromagn.
graviton $\leftrightarrow$ gravity
Z, W boson $\leftrightarrow$ weak
(gluon) pion $\leftrightarrow$ strong

mass $= 0$
Range $= \infty$

mass $\neq 0$ $\quad M_{Z,W} \sim 80$ GeV
$M_\pi \sim 140$ MeV

Natural units $\hbar = c = 1$

4

$$[c] = \frac{L}{T} \qquad [\hbar] = \frac{L^2 M}{T}$$

$$\hookrightarrow [M] = [L]^{-1} = [T]^{-1} = [E] = [p]$$

Typical unit electron-Volt $eV = [M]$

Typical scale of the atom $R_B \sim (\alpha m_e)^{-1}$

$m_e = 511 \text{ keV} \qquad \alpha = (137,03599\ldots)^{-1}$

Atom scale $\sim 10^{-10} m = 10^5 fm$

$1 fm = 10^{-15} m$

Nucleus $\sim 1 fm \sim \frac{1}{200 \text{ MeV}}$

$\updownarrow$

Pion mass $m_\pi \cong 140 \text{ MeV}$

Gravitational constant $[G] = M^{-2}$

$\hookrightarrow G = \frac{\hbar c}{M_P^2}$

$M_P \cong 10^{19} \text{ GeV} = 10^{28} eV$

$l_P \cong 10^{-33} cm$

$m_\nu \sim 10^{-2} eV$

$m_e \sim 0.5 \text{ MeV}$

$m_\mu \sim 100 \text{ MeV}$

$m_\pi \sim 140 \text{ MeV}$

$m_{p,n} \sim 1 \text{ GeV}$

$m_\tau \sim 1.7 \text{ GeV}$

$M_{Z,W} \sim 80-90 \text{ GeV}$

$m_H \sim 126 \text{ GeV}$

QFT: joins Quantum Mechanics with Special Relativity

Invariance under Lorentz transformation
$\longleftrightarrow$ Equations are the same in all reference frames

Lorentz transformations

Minkowski space $\mathbb{R}(3,1)$
4-vector

$$x^\mu = \begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix} \qquad x_\mu = \begin{pmatrix} t \\ -x \\ -y \\ -z \end{pmatrix}$$

$$x^2 = x^\mu \cdot x_\mu = t^2 - \vec{x}^2$$ Lorentz scalar

Metric tensor $g^{\mu\nu} = \begin{pmatrix} 1 & & & 0 \\ & -1 & & \\ & & -1 & \\ 0 & & & -1 \end{pmatrix}$

Scalar product

$$(a \cdot b) \equiv g^{\mu\nu} a_\mu b_\nu = a^0 b^0 - \vec{a}\,\vec{b}$$

Vector is space-like if $a^2 = a^{0\,2} - \vec{a}^{\,2} < 0$
time-like if $a^2 = a^{0\,2} - \vec{a}^{\,2} > 0$
light-like if $a^2 = 0$

$x^\mu \longrightarrow$ contravariant
$x_\mu \longrightarrow$ covariant

Covariant derivative

$$\partial^\mu = \begin{pmatrix} \partial/\partial t \\ \partial/\partial x \\ \partial/\partial y \\ \partial/\partial z \end{pmatrix}$$

Lorentz transformation

$$\Lambda : \mathbb{R}(3,1) \longrightarrow \mathbb{R}(3,1)$$

$$x^\mu \longrightarrow x'^\mu = \Lambda^\mu{}_\nu \, x^\nu$$

Definition

$$\boxed{\Lambda^{\mu\nu} \Lambda^{\alpha\beta} g_{\nu\beta} = g^{\mu\alpha}}$$

$$\Lambda^T g \Lambda = g$$

3d Rotation + Boost

e.g.

$$\begin{pmatrix} 1 & & & \\ & \cos\theta & -\sin\theta & \\ & \sin\theta & \cos\theta & \\ & & & 1 \end{pmatrix}$$

Rotation by θ around z-axi

Boost in the z-direction

$$\begin{pmatrix} \cosh\beta & & & \sinh\beta \\ & 1 & 0 & \\ & 0 & 1 & \\ \sinh\beta & & & \cosh\beta \end{pmatrix}$$

$$\cosh\beta = \frac{1}{\sqrt{1-v^2}} = \gamma \quad \text{rel. factor}$$

$$\sinh\beta = \frac{v}{\sqrt{1-v^2}}$$

$$x^2 \xrightarrow{\Lambda} x'^2 = \left(\Lambda^{\mu}{}_{\nu} x^{\nu}\right) g_{\mu\mu'} \left(\Lambda^{\mu'}{}_{\nu'} x^{\nu'}\right)$$

$$= \underbrace{\Lambda^{\mu}{}_{\nu}\, g_{\mu\mu'}\, \Lambda^{\mu'}{}_{\nu'}}_{\overset{\shortparallel}{g_{\nu\nu'}}} \; x^{\nu} x^{\nu'} = x^2$$

Fields $\varphi(\vec{x}, t) = \varphi(x)$ scalar

Active Lorentz transformation

$$\Lambda: \quad \begin{aligned} &x \to x' \\ &\varphi(x) \to \varphi(\Lambda^{-1} x) \end{aligned}$$