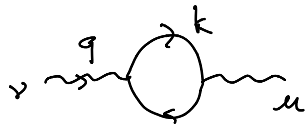
⇒ LECTURE 23

VI 6) RENORMALIZABILITY : GENERAL

↳ QED @ 1 LOOP

- 

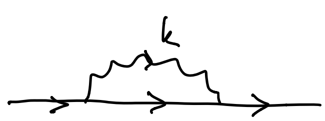
$$\int d^4k \; \frac{1}{k} \; \frac{1}{k} \qquad \frac{k}{k^2} \qquad \underbrace{DD = 2}$$

SUPERFICIAL DEGREE OF DIVERGENCE (DD)

DD = # NUMERATOR MOMENTA
 − # DENOMINATOR "

$$\Pi^{\mu\nu}(q) = (q^2 g^{\mu\nu} - q^\mu q^\nu) \; \underbrace{\Pi(q^2)}_{\text{LOG. DIV.}}$$

$$e_B^2 = \underbrace{Z_3^{-1}}_{\substack{\text{LOG. DIV.} \\ \frac{1}{\varepsilon}}} \; \underbrace{e_R^2}_{\uparrow \, \text{FINITE}}$$

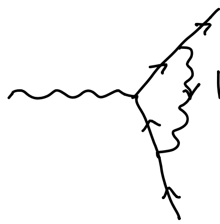
- 

$$\int d^4k \; \frac{1}{k^2} \; \frac{1}{k}$$

$$DD = 4 - 3 = 1$$

$$m_B = Z_m \, m_R$$

$$\psi_B = Z_2^{1/2} \, \psi_R$$

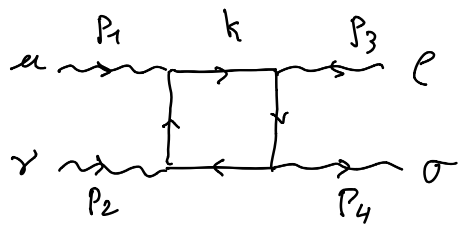
- 

$$\int d^4k \ \frac{1}{k^2} \ \frac{1}{\not k} \ \frac{1}{\not k}$$

$DD = 0$

$\underline{\text{QED}}$: $\underset{\sim\sim\sim\sim\sim}{Z_1 = Z_2}$ (GAUGE INV.)

- 4- POINT FUNCTION



$$\int d^4k \ \frac{1}{\not k} \ \frac{1}{\not k} \ \frac{1}{\not k} \ \frac{1}{\not k}$$

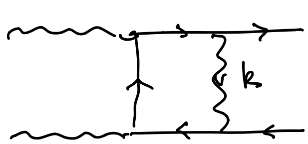
$DD = 0$

$$M^{\mu\nu\rho\sigma} = a\left(g^{\mu\nu} g^{\rho\sigma} + g^{\mu\rho} g^{\nu\sigma} + g^{\mu\sigma} g^{\nu\rho}\right)$$

+ PROP. TO $\underline{\text{MOMENTA}}$ (FINITE)

a : DIV.

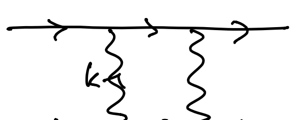
$$(p_1)_\mu \ M^{\mu\nu\rho\sigma} = 0 \quad \Longrightarrow \quad \underline{a = 0}$$

- 

$$\int d^4k \ \frac{1}{k^2} \ \frac{1}{\not k} \ \frac{1}{\not k} \ \frac{1}{\not k}$$

$DD = -1$

$\underline{\text{FINITE}}$!

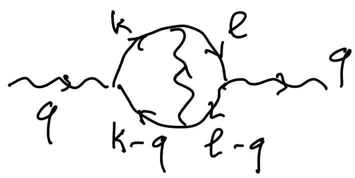
- 

$$\int d^4k \ \frac{1}{k^2} \ \frac{1}{k^2} \ \frac{1}{\not k} \ \frac{1}{\not k}$$

$DD = -2$

↳ QED @ 2 LOOP

- 2 POINT $\langle AA \rangle$

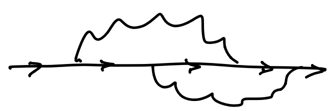


$$\int d^4k \int d^4\ell \; \frac{1}{\not k}\,\frac{1}{\not k}\,\frac{1}{\not \ell}\,\frac{1}{\not \ell}\;\frac{1}{(\ell-k)^2}$$

$$DD = 8 - 6 = 2$$

$$e_B^2 = Z_3^{-1}\, e_R^2$$

↳ Z_3 TO $O(\alpha_{em}^2)$

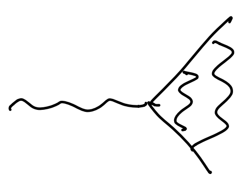
- 

$$DD = 8 - 4 - 3 = 1$$

$$m_B = Z_m\, m_R$$

$$\psi_B = Z_2^{1/2}\, \psi_R$$

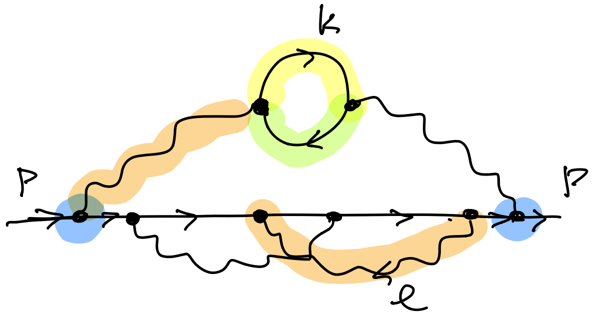
→ Z_m, Z_2 TO $O(\alpha_{em}^2)$

- 

$$DD = 0$$

$$Z_1 = Z_2$$

↳ GENERAL



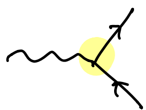
$O(\alpha^4)$ in Z_m, Z_2

$$DD \doteq 4 - \frac{3}{2}\,2 - 0 = 1$$

1) VERTEX OF TYPE i $\qquad \delta_i = d_i + \frac{3}{2} f_i + b_i - 4$

- SPINOR QED

$\delta_1 = 0$



$\mathcal{L}_{INT} = e\,\bar{\psi}\gamma^\mu\psi A_\mu$

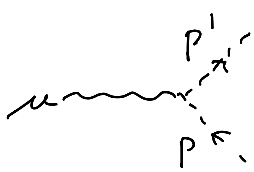
$ie\gamma^\mu$

$d_1 = 0 \qquad b_1 = 1$

$\uparrow$ # DERIVATIVES $\qquad f_1 = 2$

- SCALAR QED

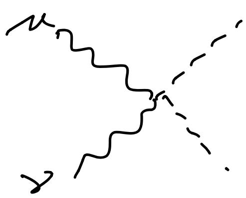
$\delta_1 = 0$



$-iq\,(p+p')^\mu$

$d_1 = 1$, $b_1 = 3$, $f_1 = 0$

$\delta_2 = 0$



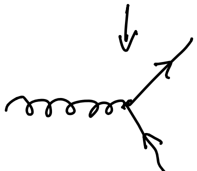
$2iq^2 g^{\mu\nu}$

$d_2 = 0$, $b_2 = 4$, $f_2 = 0$

- QCD

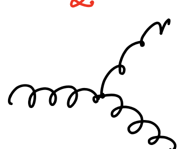
$\downarrow$

$\delta_1 = 0$



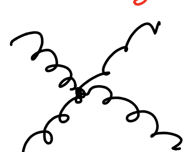
$d_1 = 0,\ b_1 = 1$

$\delta_2 = 0$



$d_2 = 1,\ b_2 = 3$

$\delta_3 = 0$



$d_3 = 0,\ b_3 = 4$

2) INTERNAL FERMION LINES

$$\int d^4k \ \frac{1}{\not k} \longrightarrow DD = 3$$

FOR EACH VERTEX OF TYPE i $\Longrightarrow \frac{3}{2}$

$\rightsquigarrow \frac{3}{2} \cdot f_i$

↳ # FERMION LINES ENTERING VERTEX i

spinor QED $\quad f_i = 2$

3) INTERNAL BOSON LINE

$$\int d^4\ell \ \frac{1}{\ell^2} \qquad DD = 2$$

FOR EACH VERTEX OF TYPE $i \Rightarrow 1$

$\rightsquigarrow 1 \cdot b_i$

↳ # BOSON LINES ENTERING VERTEX i

4) ENERGY - MOMENTUM CONSERVATION AT EACH VERTEX i

$$\delta^4 (k_1 + k_2 + k_3)$$

$$DD = -4$$

INDEX OF DIVERGENCE

$$\delta_i = d_i + \frac{3}{2} f_i + b_i - 4$$

$$DD = \sum_{\substack{i \\ \text{TYPES}}} n_i \delta_i - \frac{3}{2} F - B + 4$$

n_i: # VERTICES OF TYPE i

4: GLOBAL EN-MOM CONS

F # EXT. FERMION LINES

B # EXT. BOSON LINES

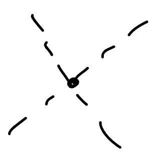
FOR $\delta_i = 0$: RENORMALIZABLE THEORIES

$$DD = 4 - \frac{3}{2} F - B$$

ONLY DEPENDS ON EXT. LINES

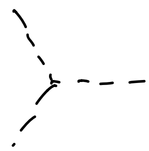
$\Rightarrow \quad \mathcal{L}_{INT} = \frac{\lambda}{4!} \phi^4 \qquad [\lambda] = 0$

! DIMENSIONLESS

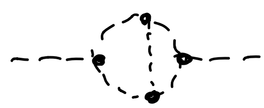


$(i\lambda) \qquad \delta = 0$

$$\Rightarrow \quad \mathcal{L}_{INT} = g\,\phi^3 \qquad [g] = 1$$



$$\delta = -1$$



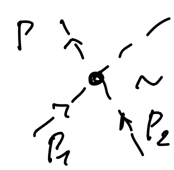
$$DD = n(-1) + 4 - B$$

$$DD_{1LOOP} = -2 + 4 - 2 = 0$$

$$DD_{2LOOP} = -2$$

$\delta < 0$: SUPER RENORMALIZABLE

$$\Rightarrow \quad \mathcal{L}_{INT} = \frac{g}{4!}\,\phi^2(\Box\,\phi^2)$$

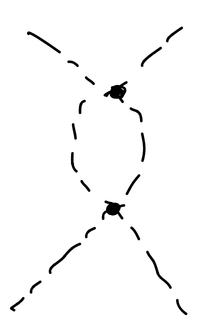


$$\sim g\,\underbrace{p_1 \cdot p_2}$$

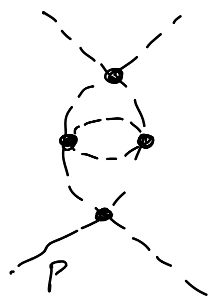
$$\delta = d + \frac{3}{2}f + b - 4 \stackrel{?}{=} 2 + 0 + 4 - 4 = 2$$

$\delta > 0$: NON- RENORMALIZABLE

$$DD = \underbrace{n}(\delta) + 4 - B$$



$$DD_{1LOOP} = 2 \cdot 2 + 4 - 4 = \underbrace{4}$$



$$DD_{2\,LOOP} = 4 \cdot 2 + 4 - 4$$
$$= 8$$

↳ EFFECTIVE FIELD THEORY

$$p < M$$

$$O\left(\frac{p}{M}\right)$$

$$\begin{array}{ccccccc} \mathcal{L} & = & \mathcal{L}_{SM} & + & g_5\mathcal{L}_5 & + & g_6\mathcal{L}_6 \\ & & & & \uparrow & & \uparrow \\ \text{DIM} & & 4 & & 5 & & 6 \end{array}$$

$$[g_5] = -1 \quad [g_6] = -2$$