⇒ LECTURE 22

VI 4) e^- SELF-ENERGY IN QED

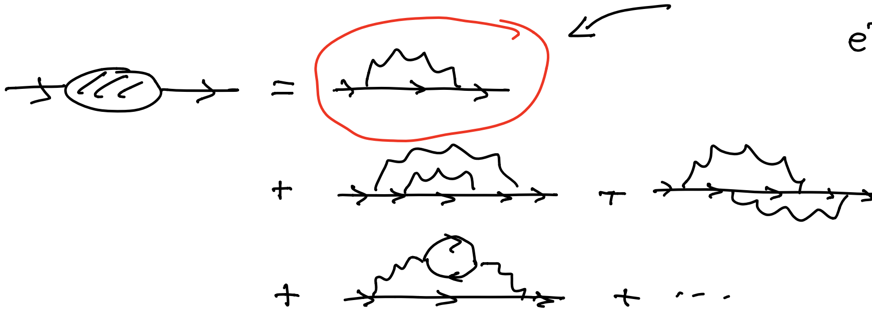
$$iS_0(p) = i\,\frac{\not p + m}{p^2 - m^2 + i\varepsilon} = \frac{i}{\not p - m}$$

$+ \dots$

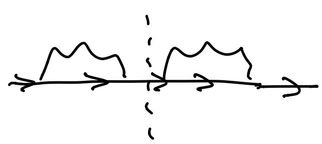
$\Uparrow$

$-i\Sigma(p)$

e^- SELF-ENERGY

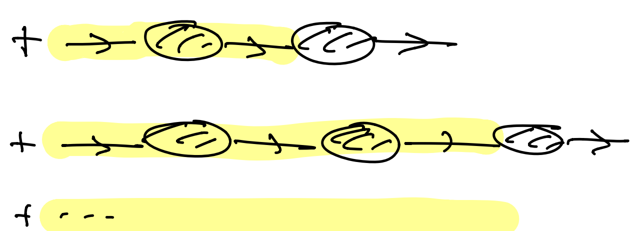


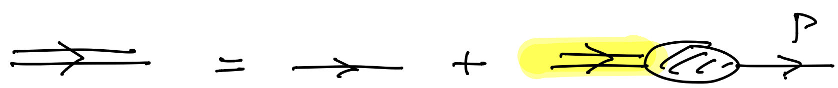
1 - PARTICLE IRREDUCIBLE DIAGRAMS



1 - PARTICLE REDUCIBLE

$$iS(\not p) \qquad iS_0 \qquad iS_0(-i\Sigma)\,iS_0$$





$$iS = iS_0 \qquad iS_0\,(\underbrace{-i\Sigma})\; iS$$

$\downarrow$ MULTIPLY WITH S_0^{-1}

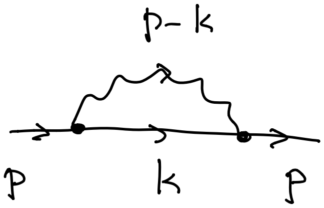
$$S_0^{-1} S = 1 + \Sigma S$$

$$\boxed{S(p) = \frac{1}{S_0^{-1} - \Sigma} = \frac{1}{\not p - m - \Sigma(\not p)}}$$

$$\Longrightarrow \Sigma(\not p) = \not p \, \underbrace{f_2(p^2)} + m \, \underbrace{f_m(p^2)}$$

$$S(p) = \frac{1}{\not p \left(1 - f_2(p^2)\right) - m \left(1 + f_m(p^2)\right)}$$

↳ CALCULATION OF $\Sigma(\not p)$ AT 1-LOOP LEVEL



$$-i\Sigma(\not p) = \int \frac{d^4k}{(2\pi)^4} \frac{(ie\gamma^\mu)\, i(\not k + m)(-i g_{\mu\nu})(ie\gamma^\nu)}{\underbrace{[k^2-m^2+i\varepsilon]}_{A}\,\underbrace{[(p-k)^2+i\varepsilon]}_{B}}$$

$\Big\downarrow$ FEYNMAN PARAM.

$$\frac{1}{AB} = \int_0^1 dx \frac{1}{[A+(B-A)x]^2}$$

$$\Sigma(\not p) = -ie^2\int_0^1 dx \int \frac{d^4k}{(2\pi)^4}\frac{\gamma^\mu(\not k+m)\gamma_\mu}{\underbrace{[k^2-m^2+x(p^2-2p\cdot k+m^2)+i\varepsilon]}_{(k-px)^2-\cancel{p^2x^2}-m^2(1-x)+p^2x(1-x)}^2}$$

$\Big\downarrow \quad k \to k-px$

$$\Sigma(\not p) = -ie^2\int_0^1 dx\int\frac{d^4k}{(2\pi)^4}\frac{\gamma^\mu(\cancel{\not k}+\not p x+m)\gamma_\mu}{[k^2-\Delta+i\varepsilon]^2}$$

$$\Delta \equiv (m^2-p^2x)(1-x)$$

↳ DIM. REG.

$$\Sigma(\not p) = -ie^2 \mu^{2\varepsilon} \int_0^1 dx \int \frac{d^Dk}{(2\pi)^D} \frac{\gamma^\mu(\not p x + m)\gamma_\mu}{[k^2 - \Delta + i\varepsilon]^2}$$

$$\gamma^\mu \not p \gamma_\mu = (2-D)\not p = -2(1-\varepsilon)\not p$$

$$\varepsilon = 2 - D/2$$

$$\gamma^\mu \gamma_\mu = D = 4\left(1-\frac{\varepsilon}{2}\right)$$

$$\int \frac{d^Dk}{(2\pi)^D} \frac{1}{[k^2-\Delta]^2} = \frac{i}{(4\pi)^2}\left(\frac{4\pi\mu^2}{\Delta}\right)^{\varepsilon} \underbrace{\Gamma(\varepsilon)}_{\frac{1}{\varepsilon}-\gamma_E}$$

$$\Sigma(\not p) = \frac{e^2}{(4\pi)^2}\, 2\, (4\pi\mu^2)^{\varepsilon} \int_0^1 dx \left[-(1-\varepsilon)\not p x + 2m\left(1-\frac{\varepsilon}{2}\right)\right] \left[\frac{1}{\varepsilon}-\gamma_E\right]\left[1-\varepsilon \ln \Delta\right]$$

$$= \frac{e^2}{8\pi^2} \int_0^1 dx \left\{ \left[\frac{1}{\varepsilon} - \gamma_E + \ln \frac{4\pi\mu^2}{\Delta}\right]\left[-\not p x + 2m\right] + \not p x - m + O(\varepsilon) \right\}$$

$$
\begin{aligned}
\Sigma(\not p) = \not p \Big\{ \Big(-\frac{e^2}{16\pi^2} \Big) \Big[\frac{1}{\varepsilon} - \gamma_E + \ln 4\pi - 1 \\
+ 2 \int_0^1 dx \; x \; \ln \frac{\mu^2}{\Delta} \Big] \Big\} \\
+ m \Big\{ \Big(\frac{e^2}{4\pi^2} \Big) \Big[\frac{1}{\varepsilon} - \gamma_E + \ln 4\pi - \frac{1}{2} \\
+ \int_0^1 dx \; \ln \frac{\mu^2}{\Delta} \Big] \Big\}
\end{aligned}
$$

$$
= \not p \; f_2(p^2) + m \; f_m(p^2)
$$

$$
e^2 = Z_3^{-1} \, e_R^2
$$

$$
= e_R^2 \left(1 + O(e_R^2) \right)
$$

↳ NEED 2 RENORMALIZATION CONSTANTS

$$
m = Z_m \, m_R
$$

↑ PHYSICAL

MASS RENORM.

$$
Z_m = 1 + \delta_m
$$

↳ $O(e^2)$

$$
i S_F(x-y) = \langle 0 | T\{ \psi(x) \bar\psi(y) \} | 0 \rangle
$$

$$
\psi = Z_2^{1/2} \psi_R
$$

$$
S = Z_2 \, S^R(p)
$$

$$S^R(\not p) = \frac{1}{Z_2}\,\frac{1}{\not p - m - \Sigma(\not p)}$$

$$Z_2 \equiv 1 + \delta_2 \qquad\qquad \delta_2 = O(e^2)$$

↳ COUNTER TERMS

$$S_R(\not p) = \frac{1}{(1+\delta_2)\not p - \underbrace{(1+\delta_2)(1+\delta_m)}_{1+\delta_2+\delta_m+O(e^4)} m_R - \Sigma(\not p)}$$

RENORM. SCHEME $\overline{MS}$ (MODIFIED MINIMAL SUBTRACTION)

$$\delta_2^{\overline{MS}} = -\frac{\alpha_{em}}{4\pi}\left[\frac{1}{\varepsilon} - \gamma_E + \ln 4\pi\right] + O(\alpha^2)$$

$$\delta_2^{\overline{MS}} + \delta_m^{\overline{MS}} = -\frac{\alpha_{em}}{4\pi}\,4\left[\frac{1}{\varepsilon} - \gamma_E + \ln 4\pi\right] + O(\alpha^2)$$

$$\delta_m^{\overline{MS}} = -\frac{\alpha_{em}}{4\pi}\,3\left[\frac{1}{\varepsilon} - \gamma_E + \ln 4\pi\right] + O(\alpha^2)$$

$$S_R(\not{p}) = \frac{1}{\not{p} - m_R - \Sigma_R(\not{p})}$$

$$\boxed{\begin{aligned}\Sigma_R^{\overline{MS}}(\not{p}) = {} & \not{p}\left(-\frac{\alpha_{em}}{4\pi}\right)\left\{-1 + 2\int_0^1 dx\, x \ln\frac{\mu^2}{\Delta}\right\} \\ & + m_R\left(\frac{\alpha_{em}}{\pi}\right)\left\{-\frac{1}{2} + \int_0^1 dx \ln\frac{\mu^2}{\Delta}\right\} \\ & + O(\alpha^2)\end{aligned}}$$

$$\Delta = \left(m_R^2 - p^2 x\right)(1-x)$$

$$\rightarrow m_R^2\,(1-x)^2$$

$\hookrightarrow$ POLE MASS

$S_R(\not{p})$ HAS POLE AT $\not{p} = m_p$

POLE MASS

$$\not{p} - m_R - \Sigma_R^{\overline{MS}}(\not{p})\Big|_{\not{p}=m_p} = 0$$

$$\Longrightarrow \quad m_p - m_R = \Sigma_R^{\overline{MS}}(\not{p} = m_p)$$

$$m_p = m_R\left(1 + O(\alpha_{em})\right)$$

$$\Longrightarrow \quad m_R = m_p - \Sigma_R^{\overline{MS}}(m_p)$$

$$= m_p\left[1 - \frac{\alpha_{em}}{4\pi}\left(+1 - \ln\frac{\mu^2}{m_p^2} - 2 + 4\ln\frac{\mu^2}{m_p^2} \underbrace{- 2\int_0^1 dx\,(2-x)\ln(1-x)^2}_{5}\right)\right]$$

$$\boxed{m_R^{\overline{MS}}(\mu) = m_p\left[1 - \frac{\alpha_{em}}{4\pi}\left(4 + 3\ln\frac{\mu^2}{m_p^2}\right) + O(\alpha_{em}^2)\right]}$$

RUNNING MASS (μ)

$\mu \uparrow \qquad m_R \searrow$