Theoretical Physics 6a (QFT): SS 2020
Exercise sheet 8

08.06.2020

Exercise 1. (40 points) : Wick's Theorem

In the proof of Wick's theorem, we need the evaluation of the following commutator:

$$\left[\phi_1^+, N(\phi_2\phi_3)\right]_- = N(\underbrace{\phi_1}\phi_2\phi_3) + N(\phi_2\underbrace{\phi_1}\phi_3). \tag{1}$$

where we have defined $\phi_i \equiv \phi(x_i)$, and where ϕ_i^+ stands for the positive frequency part (annihilation part) in the field ϕ_i. Prove the above identity.
Hint: Start by explicitly splitting the fields ϕ_2 and ϕ_3 into positive and negative frequency parts.

Exercise 2. (60 points) : Scalar QED

The Lagrangian of scalar QED is given by:

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + (D_\mu\phi)^*(D^\mu\phi) - m^2\phi^*\phi, \tag{2}$$

where $D_\mu = \partial_\mu + ieA_\mu$.

(a)(30 points) Derive the Feynman rules for scalar QED using the Lagrangian of Eq. (2) by identifying the interaction term and using Wicks theorem.

(b)(30 point) Use the Feynman rules to calculate the matrix elements for the two processes:

- $\gamma\gamma \to \phi^+\phi^-$
- $\gamma\phi^- \to \gamma\phi^-$.