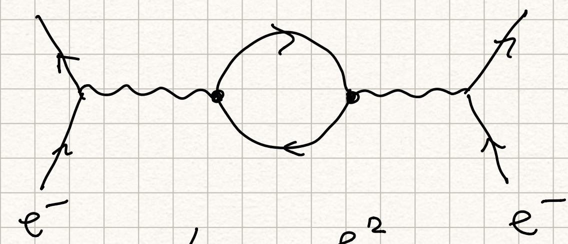
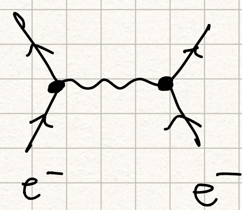
$\Rightarrow$ LECTURE 17

VI) RENORMALIZATION OF QED

1) VACUUM POLARIZATION IN QED

$\Longrightarrow$ 



$$\alpha_{em} = \frac{e^2}{4\pi}$$

SCALAR QED (γ INTERACTING WITH CHARGED SPIN-0 PART. π^+)

$$\mathcal{L}_{KG} = (\partial_\mu \phi)^\dagger (\partial^\mu \phi) - m^2 \phi^\dagger \phi$$

IMPOSE U(1) GAUGE INV

$$\phi(x) \xrightarrow{U(1)} e^{i\chi(x)} \phi(x)$$

$$\partial^\mu \Rightarrow D^\mu = \partial^\mu + ieA^\mu.$$

$$A^\mu \longrightarrow A^\mu - \frac{1}{e}\partial^\mu \chi$$

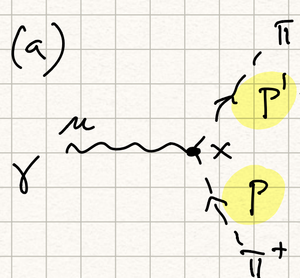
$$\mathcal{L} = \mathcal{L}_{KG} + \mathcal{L}_{INT}$$

$$= \left[(\partial_\mu - ie\underline{\underline{A_\mu}})\phi^\dagger\right]\left[(\partial^\mu + ieA^\mu)\phi\right] - m^2 \overline{\overline{\phi^\dagger}}\phi$$

$$\mathcal{L}_{INT} = -ie\left[\phi^\dagger(\partial^\mu\phi) - (\partial^\mu\phi^\dagger)\phi\right]A_\mu + e^2\,\phi^\dagger\phi\,A_\mu A^\mu$$

$$S = \mathbb{1} + \frac{1}{(\dots)}(2\pi)^4\,\delta^4(\dots)\,\mathcal{M}$$

(a)



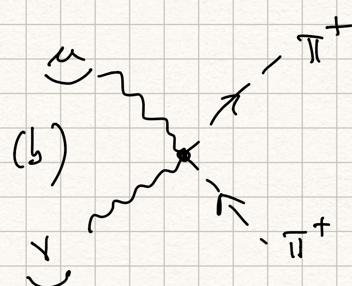
$p' \rightarrow e^{+ip'x} \Rightarrow +ip'^\mu$

$p \rightarrow e^{-ipx} \Rightarrow -ip^\mu$

$$\mathcal{M}^{(1)}_{fi} = \underbrace{(i)\,(-ie)\,\left(-ip^\mu - ip'^\mu\right)}_{\text{VERTEX } \Gamma^\mu} \cdot \varepsilon_\mu$$

$$\boxed{\Gamma^{\mu}_a = -ie\,(p+p')^{\mu}} \longleftrightarrow -i\underbrace{q}_{e}\,\gamma^\mu$$

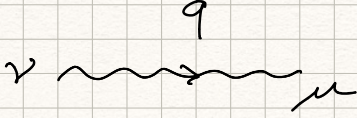
(b)



$$\boxed{\Gamma^{\mu\nu}_b = 2ie^2\,g^{\mu\nu}}$$

$$\mathcal{M}^{(1)}_{fi} = \Gamma^{\mu\nu}_b \cdot \varepsilon_\mu\,\varepsilon^*_\nu = 2ie^2\,\varepsilon_\mu\,\varepsilon^{\mu *}$$

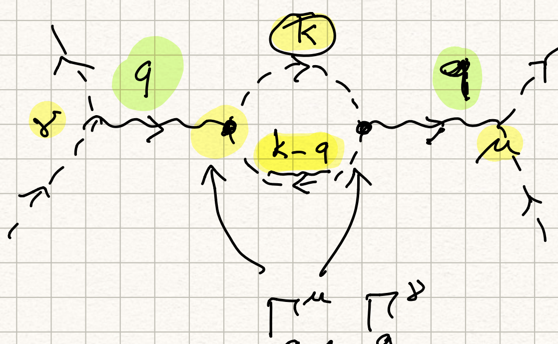
↳ 1-LOOP CORRECTION TO PHOTON PROPAGATOR

- 

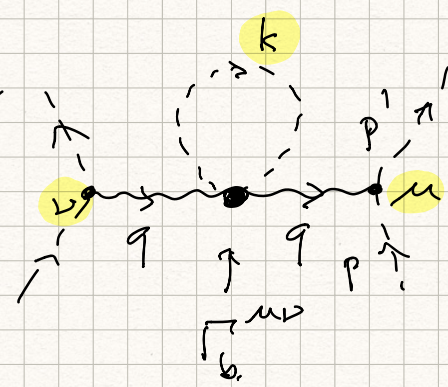
$$iD_0^{\mu\nu}(q) = \frac{i(-g^{\mu\nu})}{q^2 + i\varepsilon}$$

↑ LOWEST ORDER IN (e^2)

- 1° ORDER IN e^2



(a)



(b)

$$\mathcal{M}_a^{\mu\nu} = \int \frac{d^4k}{(2\pi)^4} \frac{i^2(-ie)(2k-q)^\nu(-ie)(2k-q)^\mu}{[k^2 - m^2 + i\varepsilon][(k-q)^2 - m^2 + i\varepsilon]}$$

$$= e^2 \int \frac{d^4k}{(2\pi)^4} \frac{(2k-q)^\nu (2k-q)^\mu}{[k^2 - m^2 + i\varepsilon][(k-q)^2 - m^2 + i\varepsilon]}$$

$$\mathcal{M}_b^{\mu\nu} = -\int \frac{d^4k}{(2\pi)^4} \frac{(2e^2 g^{\mu\nu})}{[k^2 - m^2 + i\varepsilon]}$$

$$\mathcal{M}^{\mu\nu} = \mathcal{M}_a^{\mu\nu} + \mathcal{M}_b^{\mu\nu}$$

$$\mathcal{M}^{\mu\nu} = -e^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{\left[k^2 - m^2 + i\varepsilon\right]\left[(k-q)^2 - m^2 + i\varepsilon\right]}$$

$$\cdot \Big\{ -4k^\mu k^\nu + 2k^\mu q^\nu + 2k^\nu q^\mu - q^\mu q^\nu + 2g^{\mu\nu}\left[(k-q)^2 - m^2\right] \Big\}$$

$k \to \infty$ $\qquad \displaystyle\int d^4k \, \frac{1}{k^4} \, k^\mu k^\nu$

QUADRATIC DIV.

GAUGE INV.

$$q_\mu \mathcal{M}^{\mu\nu} = 0 \qquad q_\nu \mathcal{M}^{\mu\nu} = 0$$

- FEYNMAN PARAMETRIZATION

BRING TO COMMON DENOMINATOR

$$\frac{1}{AB} \qquad \begin{cases} A = k^2 - m^2 + i\varepsilon \\ B = (k-q)^2 - m^2 + i\varepsilon \end{cases}$$

$$\frac{1}{AB} = \int_0^1 dx \, \frac{1}{\left[A + (B-A)x\right]^2}$$

FEYNMAN PARAMETER

$$= -\frac{1}{(B-A)} \left.\frac{1}{\left[A + (B-A)x\right]}\right|_0^1$$

$$= -\frac{1}{(B-A)}\left\{\frac{1}{B}-\frac{1}{A}\right\}$$

$$= \frac{1}{A-B}\left\{\frac{A-B}{A.B}\right\} \overset{!}{=} \frac{1}{A.B}$$

$$\frac{1}{A_1 \cdots A_n} = \underbrace{\int_0^1 dx_1 \cdots \int_0^1 dx_{n-1}}_{n-1} \frac{1}{[\,\cdots\cdots\,]^{n}}$$

FEYNMAN PAR INT.

$$\mathcal{M}^{\mu\nu} = -e^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{[k^2-m^2+i\varepsilon]\,[(k-q)^2-m^2+i\varepsilon]}$$

$$\cdot\left\{-4k^\mu k^\nu + 2k^\mu q^\nu + 2k^\nu q^\mu - q^\mu q^\nu + 2g^{\mu\nu}[(k-q)^2-m^2]\right\}$$

$$B - A = q^2 - 2k.q$$

$$A = k^2 - m^2 + i\varepsilon$$

$$\downarrow$$

$$= -e^2 \int_0^1 dx \int \frac{d^4k}{(2\pi)^4} \frac{1}{[k^2-m^2+(q^2-2k.q)x+i\varepsilon]^2}$$

$$\cdot\left\{\cdots\right\}$$

- CHANGE INT. VARIABLE

$$k^2 - 2k\cdot q x = (\underbrace{k - qx}_{k'})^2 - q^2x^2$$

$$k \to k' = k - qx \implies k = k' + qx$$

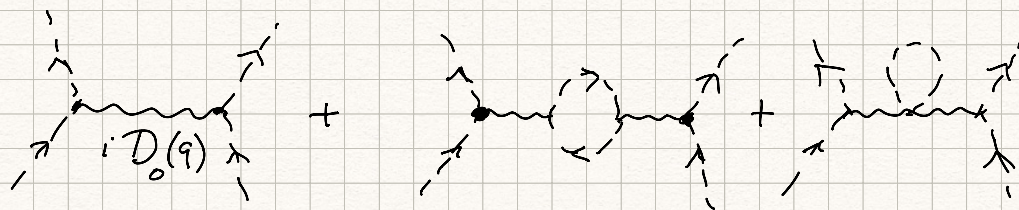
$$= -e^2 \int_0^1 dx \int \frac{d^4k'}{(2\pi)^4} \frac{1}{[k'^2 + q^2x(1-x) - m^2 + i\varepsilon]^2}$$

$$\cdot \Big\{ -4\,(k'+qx)^\mu (k'+qx)^\nu$$
$$+ 2q^\mu(\cancel{k'}+qx)^\nu$$
$$+ 2q^\nu(\cancel{k'}+qx)^\mu - q^\mu q^\nu$$
$$+ 2g^{\mu\nu}\big[(k'-q(1-x))^2 - m^2\big] \Big\}$$

$$\Big\downarrow \quad \int d^4k' \; k'^\mu = 0$$

$$= -e_B^2 \int_0^1 dx \int \frac{d^4k}{(2\pi)^4} \frac{1}{[k^2 + q^2x(1-x) - m^2 + i\varepsilon]^2}$$

$$\cdot \Big\{ -4k^\mu k^\nu$$
$$+ q^\mu q^\nu\big[\underbrace{-4x^2 + 4x}_{4x(1-x)} - 1\big]$$
$$= \mathcal{M}^{\mu\nu} \qquad + 2g^{\mu\nu}\big[k^2 + q^2(1-x)^2 - m^2\big] \Big\}$$



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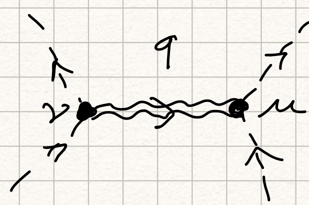
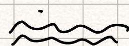
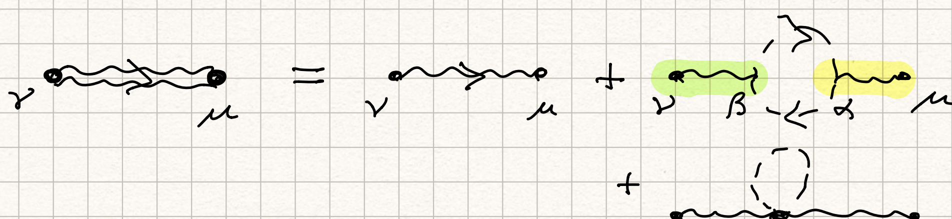
ABSENT FOR SPIN 1/2

$\equiv$ 

 FULL γ PROPAGATOR

$$iD^{\mu\nu}(q)$$



$$+ \; O(e^4)$$

$$iD^{\mu\nu}(q) = iD_0^{\mu\nu}(q) + iD_0^{\mu\alpha}(q)\,\mathcal{M}_{\alpha\beta}\,iD_0^{\beta\nu}(q)$$

$$\longrightarrow \; D_0^{\mu\nu} = -\frac{g^{\mu\nu}}{q^2+i\varepsilon} \quad \text{(LORENZ GAUGE)}$$

$$\mathcal{M}^{\mu\nu} \equiv i\,\underbrace{\Pi^{\mu\nu}}$$

VACUUM POLARIZATION

$$D^{\mu\nu}(q) = D_0^{\mu\nu}(q) - D_0^{\mu\alpha}(q)\,\Pi_{\alpha\beta}(q)\,D_0^{\beta\nu}(q)$$

$$D^{\mu\nu}(q) = \frac{1}{q^2+i\varepsilon}\left\{-g^{\mu\nu} - \frac{\Pi^{\mu\nu}(q)}{q^2+i\varepsilon}\right\}$$

GAUGE INV.

$$q_\mu \Pi^{\mu\nu}(q) = 0 \qquad q_\nu \Pi^{\mu\nu}(q) = 0$$

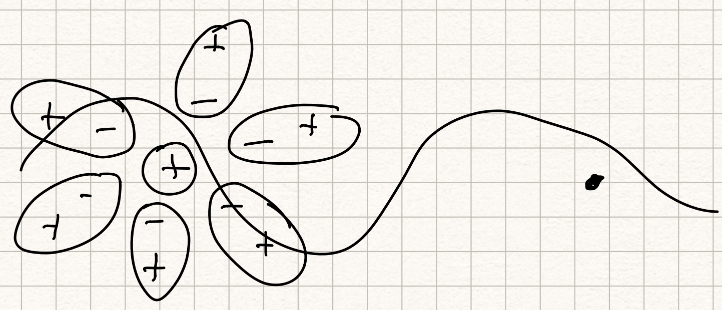
$$\Pi^{\mu\nu}(q) \equiv g^{\mu\nu} q^2 \Pi_1(q^2) + q^\mu q^\nu \Pi_2(q^2)$$

$$\to \quad q^\nu q^2 \Pi_1 + q^2 q^\nu \Pi_2 = 0$$

$$\Pi_2 = -\Pi_1 \qquad \Pi(q^2) = \Pi_1(q^2)$$

$$\Pi^{\mu\nu}(q) = \left(q^2 g^{\mu\nu} - q^\mu q^\nu\right) \Pi(q^2)$$

LOGARITHMIC DIV.



CHARGE (+) IS SCREENED BY VACUUM POL.

PHYSICALLY: WE DON'T SEE "BARE" CHARGE e_B

↳ BUT CHARGE SCREENED BY VACUUM FLUCTUATIONS

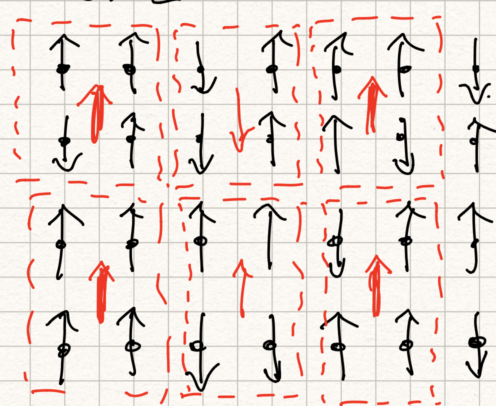
||

PHYSICAL CHARGE

$$\alpha_{em} = \frac{e^2}{4\pi} = \frac{1}{137}$$

<u>ANALOGY</u>:

ISING MODEL



COARSE GRAINING TF

$$H_1 = -J_1 \, S_i \, S_j \qquad (J > 0)$$

$$H_2 = -J_2 \, S_i \, S_j$$

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