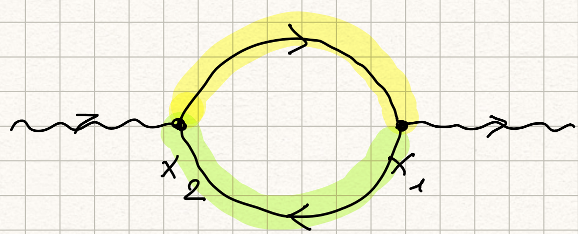
⇒ LECTURE 16

$$S^{(2)}_{QED} = \frac{(ie)^2}{2!}\int d^4x_1 \int d^4x_2$$

$$N\Big(\big(\bar\psi_\alpha \gamma^\mu_{\alpha\beta} A_\mu \psi_\beta\big)_{x_1} \big(\bar\psi_\gamma \gamma^\nu_{\gamma\delta} A_\nu \psi_\delta\big)_{x_2}\Big)$$



$$\psi_\alpha \bar\psi_\beta = (iS_F)_{\alpha\beta}$$

$$= (ie)^2 \int d^4x_1 \int d^4x_2 \quad A^-_\mu(x_1)\, A^+_\nu(x_2)$$

$$(-1) \quad \underbrace{\gamma^\mu_{\alpha\beta}\big(iS_F(x_1-x_2)\big)_{\beta\gamma}\big(\gamma^\nu\big)_{\gamma\delta}\big(iS_F(x_2-x_1)\big)_{\delta\alpha}}_{}$$

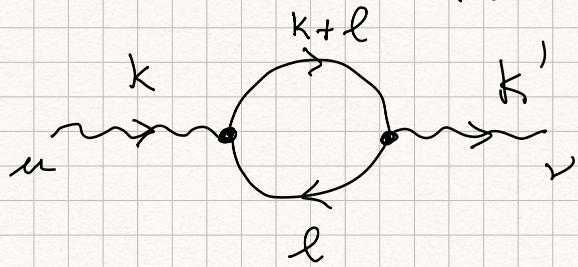
$$\mathrm{Tr}\Big(\gamma^\mu \, iS_F(x_1-x_2)\, \gamma^\nu \, iS_F(x_2-x_1)\Big)$$

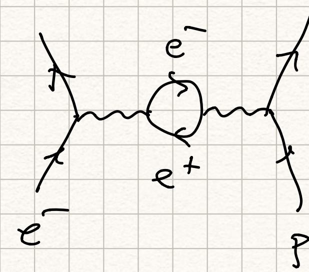
⇑

TRACE IN DIRAC SPACE

WHEN THERE IS A CLOSED FERMION LOOP

$\hookrightarrow$ MOMENTUM SPACE





$$|i\rangle = |\gamma(k,\lambda)\rangle$$
$$|f\rangle = |\gamma(k',\lambda')\rangle$$

$$\langle f | S^{(2)} | i \rangle = (2\pi)^4 \delta^4(k-k') \cdot \varepsilon_\mu(k,\lambda)\, \varepsilon_\nu^*(k',\lambda') (-1) \cdot \int \frac{d^4 l}{(2\pi)^4} \mathrm{Tr}\left\{ (ie\gamma^\mu) \frac{i(\not{k}+\not{l}+m)}{(k+l)^2 - m^2 + i\varepsilon} (ie\gamma^\nu) \cdot \frac{i(\not{l}+m)}{l^2 - m^2 + i\varepsilon} \right\}$$

PHOTON POLARIZATION

$$l \gg$$

$$\rightsquigarrow \int \underline{d^4 l}\; \frac{\not{l}}{l^2} \frac{\not{l}}{l^2} \sim \int_0^\infty \underline{\underline{d|l|}}\; |l|^3 \frac{\overline{|l|}^2}{|l|^3}$$

$$\int_{x_1}^{\overline{x_2 \to \infty}} \underline{\underline{dx}}\; \underline{x} = \frac{1}{2}\left(x_2^2 - x_1^2\right)$$

QUADRATIC DIV.

$$\hookrightarrow \quad N\Big(\left(\bar{\psi}\gamma^{\mu}A_{\mu}\psi\right)_{x_1} \left(\bar{\psi}\gamma^{\nu}A_{\nu}\psi\right)_{x_2} \Big)$$

VACUUM DIAGRAM

$\Downarrow$

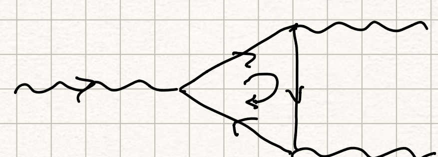
NO CONTRIBUTION

$\Rightarrow$ FEYNMAN RULES FOR QED

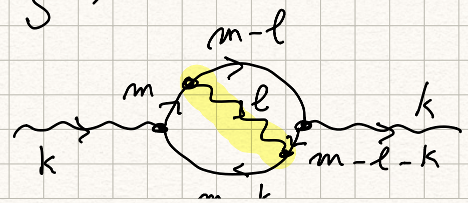
$\langle f \,|\, S^{(n)} \,|\, i \rangle$ IN MOMENTUM SPACE

DRAW ALL CONNECTED, TOPOLOGICALLY DIFFERENT DIAGRAMS CONNECTING $|i\rangle$ TO $|f\rangle$

e.g. $S^{(3)}$



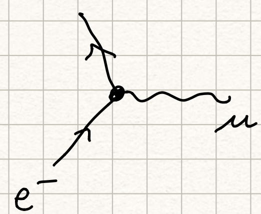
$S^{(4)}$



$$\int \frac{d^4\ell}{(2\pi)^4} \int \frac{d^4 m}{(2\pi)^4} \cdots$$

FEYNMAN RULES IN QED

1) ∀ VERTEX



$-iq\gamma^\mu$

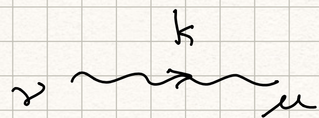
$\downarrow$

$(ie\gamma^\mu)$

$e^-: \ q = -e$

$p: \ q = +e$

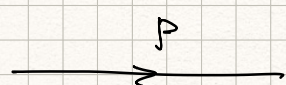
2) ∀ INTERNAL γ LINE



$$iD_F^{\mu\nu}(k) = \frac{i(-g^{\mu\nu})}{k^2 + i\varepsilon}$$

LORENTZ GAUGE

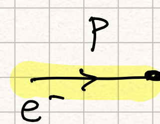
3) ∀ INTERNAL FERMION LINE



$$iS_F(p) = \frac{i(\not{p} + m)}{p^2 - m^2 + i\varepsilon}$$

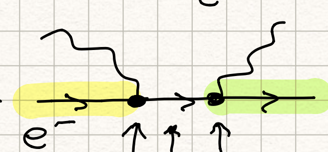
4) EXTERNAL LINES

↳ INITIAL e^-



$u(\bar{p}, s)$

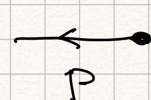
e.g.



↳ FINAL e^-

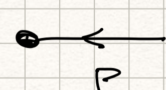
$\bar{u}(\bar{p}, s)$

↳ INITIAL e^+



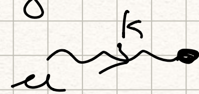
$\bar{v}(\bar{p}, s)$

↳ FINAL e^+



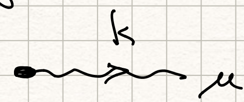
$v(\bar{p}, s)$

↳ INITIAL γ



$\varepsilon^{\mu}(k,\lambda)$

↳ FINAL γ



$\varepsilon^{\mu *}(k,\lambda)$

5) FERMION ARROW FOR SPINOR FACTORS

READ EXPRESSION LEFT TO RIGHT
YOU GO AGAINST ARROW

6) $\forall$ CLOSED FERMION LOOP

$\Rightarrow$ FACTOR (-1)

$\underbrace{Tr}\left(\qquad \right)$

7) AT EACH VERTEX

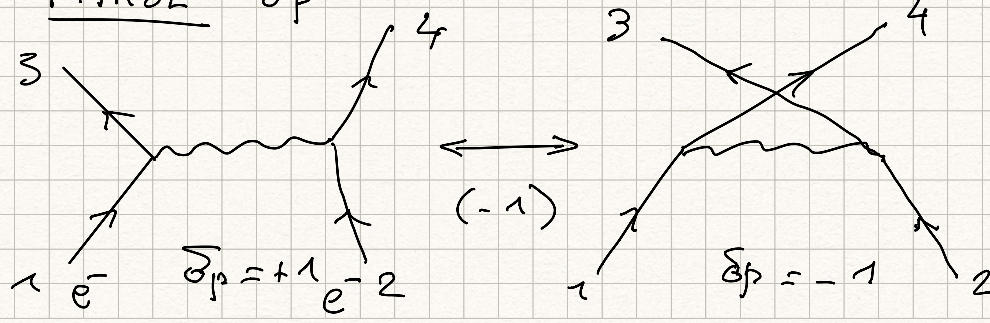
ENERGY - MOMENTUM CONSERVATION

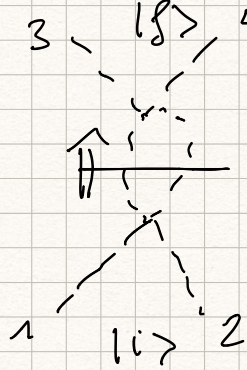
$\forall$ LOOP MOMENTUM $\displaystyle\int \frac{d^4 \ell}{(2\pi)^4} \ldots$

8) GLOBAL ENERGY - MOM. CONS.

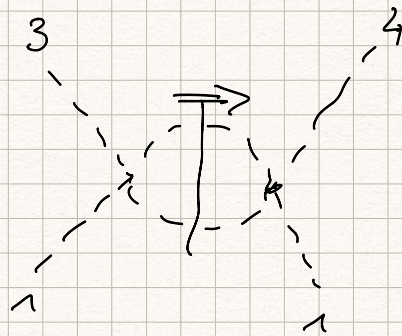
$$(2\pi)^4 \, \delta^4\left(\sum_i p_i - \sum_f p_f \right)$$

9) PHASE δ_P

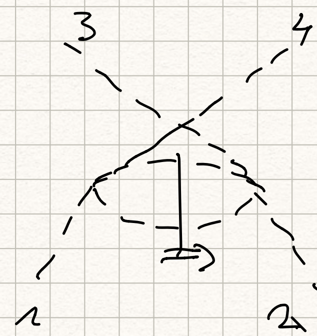




s- CHANNEL



t- CHANNEL



u- CHANNEL

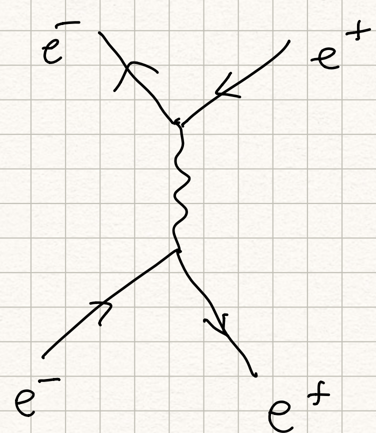
MANDELSTAM INV.

$1 + 2 \to 3 + 4$

$$s = (p_1 + p_2)^2$$
$$t = (p_1 - p_3)^2$$
$$u = (p_1 - p_4)^2$$

$$p_i^2 = m_i^2$$

$$s + t + u = \sum_{i=1}^{4} m_i^2$$



V QED PROCESSES AT LOWEST ORDER

$$\underline{\underline{\sigma}}$$

$$\underline{\underline{S_{fi}}} = \langle f | S | i \rangle$$

$$\langle i | i' \rangle = \delta_{ii'}$$

$$\sum_f |S_{fi}|^2 = 1$$

$$\underline{\underline{|\tilde{i}\rangle}} = |e^-(p,s)\rangle$$

$$\langle e^-(p',s') | e^-(p,s) \rangle = \delta_{ss'} (2E_{\bar{p}}) (2\pi)^3 \delta^3(\bar{p} - \bar{p}')$$

$$\boxed{|i\rangle = N |\tilde{i}\rangle}$$

$$\langle i | i' \rangle = N^2 \langle \tilde{i} | \tilde{i}' \rangle$$

$$\delta_{ii'} = N^2 \delta_{ss'} (2E) \underbrace{(2\pi)^3 \delta^3(\bar{p}_i - \bar{p}_i')}_{\delta_{\bar{p}_i \bar{p}_i'}}$$

<u>UNIT VOLUME</u>

$$\boxed{N = \frac{1}{(2E)^{1/2}}}$$

$$1 + 2 \to 1' + \dots + N'$$

$$S_{fi} \equiv \underbrace{\delta_{fi}}_{\text{NO INTERACTION}} + \frac{1}{(2E_1)^{1/2}} \frac{1}{(2E_2)^{1/2}} \prod_f \left(\frac{1}{2E_f}\right)^{1/2} \cdot (2\pi)^4 \delta^4\Big(p_1 + p_2 - \sum_f p_f\Big) \cdot \underbrace{\mathcal{M}_{fi}}_{\text{INVARIANT AMPLITUDE}}$$

↳ CROSS SECTION $\underbrace{1}_{\text{BEAM}} + \underbrace{2}_{\text{TARGET}} \to 1' + \dots + N'$

TARGET PARTICLE

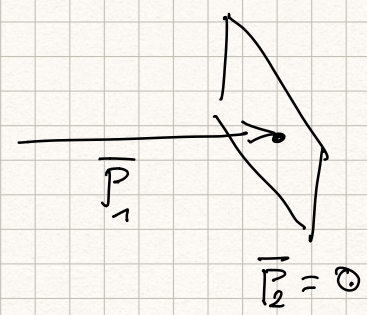
$$\sigma = \frac{w}{\phi} = \frac{\text{TRANSITION PROB PER UNIT TIME}}{\text{INCOMING FLUX}}$$

$$w = \underbrace{\frac{1}{(2E_1)(2E_2)}}_{\text{INITIAL PARTICLES}} \underbrace{\prod_f \frac{d^3 \bar{p}_f}{(2\pi)^3 (2E_f)}}_{\text{PHASE SPACE}} (2\pi)^4 \delta^4\Big(p_1 + p_2 - \sum_f p'_f\Big) |\mathcal{M}_{fi}|^2$$

ϕ # BEAM PARTICLES HITTING TARGET PARTICLE PER UNIT TIME & PER UNIT SURFACE

- <u>LAB</u> SYSTEM

BEAM HITS A FIXED TARGET



$$|\overline{J}| = \underbrace{\rho}_{=1} |\overline{v}| \Longrightarrow \Phi = v_{rel} = \frac{|\overline{p}_1|}{E_1} \approx 1$$

- COLLIDER SYSTEM

$e^- e^+$

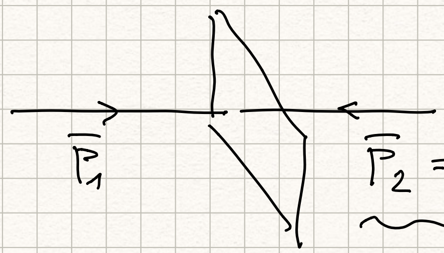
$e^- \longrightarrow \bullet \longleftarrow e^+$ NOVOSIBIRSK, KEK, BEPC-II, DESY, LEP, SLAC

$p\,p$

$p \longrightarrow \bullet \longleftarrow p$ LHC

$p\,\overline{p}$

$p \longrightarrow \bullet \longleftarrow \overline{p}$ TEVATRON



$$\bar{p}_2 = -\bar{p}_1 \longrightarrow E_1 = E_2$$

$$v_{rel} = \emptyset = \underbrace{\frac{|\bar{p}_1|}{E_1}} + \underbrace{\frac{|\bar{p}_2|}{E_2}}$$

$$= |\bar{p}_1| \left(\frac{E_1 + E_2}{E_1 E_2} \right)$$

$$= |\bar{p}_1| \; \frac{2}{E_1}$$

$$= \underbrace{2}_{|\bar{p}_1| \approx E_1}$$

↳ $d\sigma$

$$d\sigma = \frac{1}{v_{rel} (2E_1)(2E_2)} \prod_f \frac{d^3 \bar{p}_f}{(2\pi)^3 (2E_f)} (2\pi)^4 \delta^4 \left(p_1 + p_2 - \sum_f p'_f\right) |\mathcal{M}_{fi}|^2$$

$\underbrace{\text{GeV}}$

$$[d\sigma] = .. \text{GeV}^{-2}$$

SURFACE

$$d\sigma = (d\sigma \text{ in GeV}^{-2}) \cdot \underbrace{(\hbar c)^2}_{(0.197\ \text{GeV fm})^2}$$

in ... fm^2

$$1 \text{ barn} = 10^{-28}\ m^2$$
$$= 100\ fm^2$$

$1\ fm = 10^{-15}$

$$d\sigma = (d\sigma \text{ in GeV}^{-2}) \cdot (\hbar c)^2 \cdot 10^{-2}\ b$$
$$\cdot 10^{6}\ \mu b$$
$$\cdot 10^{9}\ nb$$