⇒ LECTURE 15

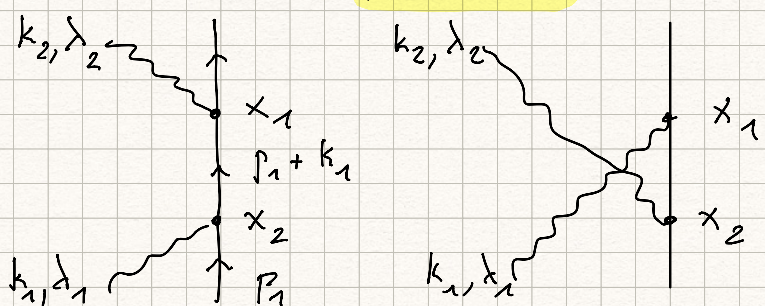
4) FEYNMAN DIAGRAMS & RULES IN QED

$$\mathcal{H}_1 = -e \, N\left(\bar{\Psi} \gamma^\mu A_\mu \Psi\right)$$

$$(e > 0)$$

$$S^{(2)} = \frac{(ie)^2}{2!} \int d^4x_1 \int d^4x_2 \, T\Big((\bar{\Psi}\gamma^\mu A_\mu \Psi)_{x_1} \cdot (\bar{\Psi}\gamma^\nu A_\nu \Psi)_{x_2} \Big)$$

$$\hookrightarrow \quad N\Big((\bar{\Psi}\gamma^\mu A_\mu \Psi)_{x_1} (\bar{\Psi}\gamma^\nu A_\nu \Psi)_{x_2} \Big)$$



COMPTON SCATTERING

(a) (b)

$$|i\rangle = |\gamma(k_1, \lambda_1)\, e^-(p_1, s_1)\rangle$$

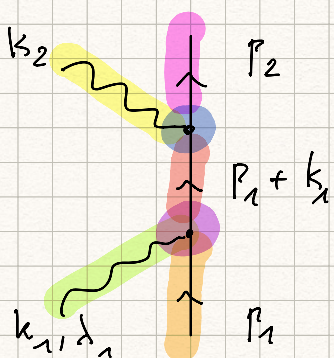
$$|f\rangle = |\gamma(k_2, \lambda_2)\, e^-(p_2, s_2)\rangle$$

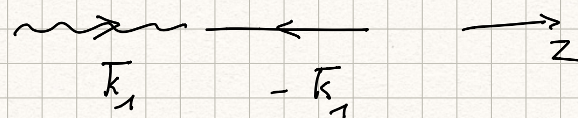
$$\langle f | S_a^{(2)} | i \rangle$$

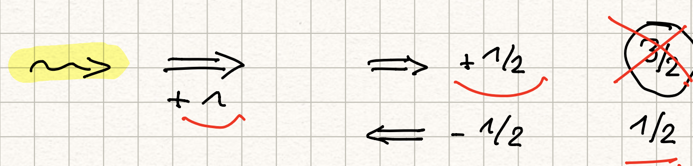
$$= (2\pi)^4 \, \delta^4(k_1 + p_1 - k_2 - p_2)$$

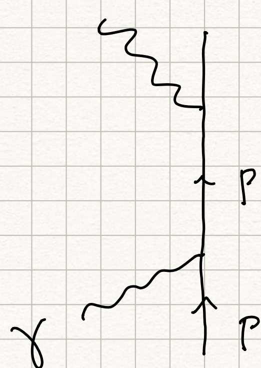
$$\cdot \; \varepsilon_\nu(k_1, \lambda_1) \; \varepsilon^*_\mu(k_2, \lambda_2)$$

$$\cdot \; \bar{u}(p_2, s_2) \, (ie\gamma^\mu) \, \frac{i(\not{p}_1 + \not{k}_1 + m)}{(p_1 + k_1)^2 - m^2 + i\varepsilon} \, (ie\gamma^\nu) \, u(p_1, s_1)$$





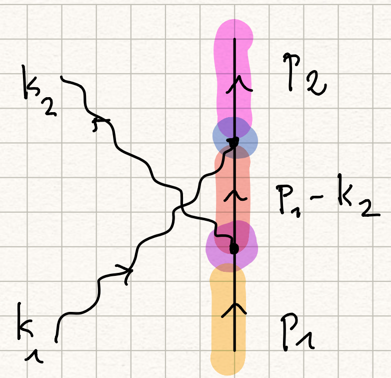




$1/2$

SP(N) $3/2$

$1/2, \; 3/2$



$$\langle f \,|\, S_b^{(2)} \,|\, i \rangle$$

$$= (2\pi)^4 \, \delta^4 (k_1 + p_1 - k_2 - p_2)$$

$$\cdot \, \varepsilon_\nu (k_1, \lambda_1) \; \varepsilon^*_\mu (k_2, \lambda_2)$$

$$\cdot \, \bar{u}(p_2, s_2) \, (ie\gamma^\nu) \, \frac{i(\not{p}_1 - \not{k}_2 + m)}{(p_1 - k_2)^2 - m^2 + i\varepsilon} \, (ie\gamma^\mu)$$

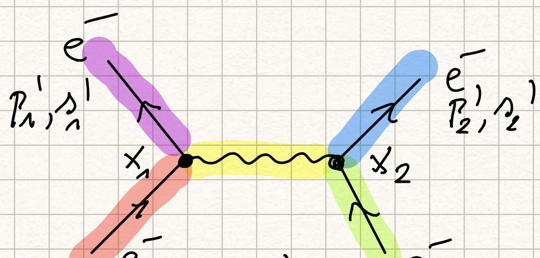
$$\cdot \, u(p_1, s_1)$$

$$\hookrightarrow \quad N\left((\bar{\psi} \gamma^\mu A_\mu \psi)_{x_1} \, (\bar{\psi} \gamma^\nu A_\nu \psi)_{x_2} \right)$$

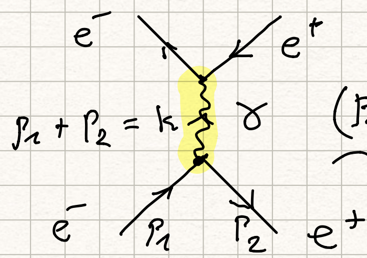
$e^- + e^- \longrightarrow e^- + e^-$ (MØLLER SCATT.)

$e^+ + e^+ \longrightarrow e^+ + e^+$ (BHABHA SCATT.)

$\longrightarrow$ $e^- + e^+ \longrightarrow e^- + e^+$ (PAIR CREATION e^-e^+ ANNIHILATION)

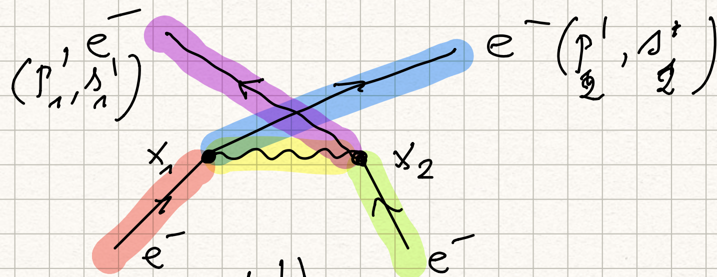


(c)



$(p_1 + p_2)^2 > 0$

$$N\left((\bar{\psi}\gamma^\mu A_\mu \psi)_{x_1} \; (\bar{\psi}\gamma^\nu A_\nu \psi)_{x_2} \right)$$



(d)

MØLLER SCATTERING

$$e^-(p_1, s_1) + e^-(p_2, s_2) \longrightarrow e^-(p_1', s_1') + e^-(p_2', s_2')$$

$$|i\rangle = \sqrt{2E_{p_2}}\sqrt{2E_{p_1}}\, a^\dagger(p_2, s_2)\, a^\dagger(p_1, s_1)\,|0\rangle$$

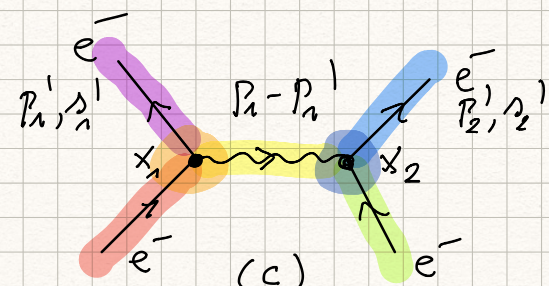
$$|f\rangle = \sqrt{2E_{p_2'}}\sqrt{2E_{p_1'}}\, a^\dagger(p_2', s_2')\, a^\dagger(p_1', s_1')\,|0\rangle$$

$\frac{1}{n!}$ in $S^{(n)}$ CAN BE OMITTED

BY CONSIDERING ONLY

TOPOLOGICALLY DIFFERENT DIAGRAMS

$$\langle f \,|\, S_c^{(2)} \,|\, i \rangle$$



(c)

$$= \int d^4x_1 \int d^4x_2 \quad i\, D_{F\mu\nu}(x_2 - x_1)$$

$$\langle 0 |\, a(p_1', s_1')\, a(p_2', s_2') \quad \sqrt{2E_{p_1}'}\, \sqrt{2E_{p_2}'}$$

$$\cdot\, N\left(\bar{\psi}^-_{x_1} (ie\gamma^\mu)\, \psi^+_{x_1}\, \bar{\psi}^-_{x_2} (ie\gamma^\nu)\, \psi^+_{x_2} \right)$$

$$a^\dagger(p_2, s_2)\, a^\dagger(p_1, s_1)\, |0\rangle \quad \sqrt{2E_{p_1}}\, \sqrt{2E_{p_2}}$$

SIGN $+1$

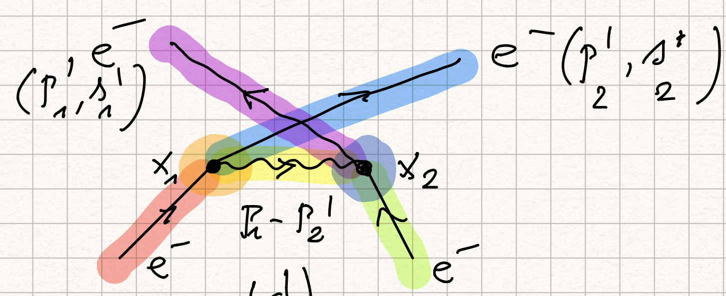
$$= (2\pi)^4\, \delta^4(p_1 + p_2 - p_1' - p_2')$$

$$\cdot\, \frac{(-i g_{\mu\nu})}{(p_1 - p_1')^2 + i\varepsilon}$$

$$\cdot\, \bar{u}(p_1', s_1')\, (ie\gamma^\mu)\, u(p_1, s_1)$$

$$\cdot\, \bar{u}(p_2', s_2')\, (ie\gamma^\nu)\, u(p_2, s_2)$$

$\rightarrow$



(d)

$$= \int d^4x_1 \int d^4x_2 \quad i D_{F\mu\nu}(x_2 - x_1)$$

$$\langle 0 | \, a(p_1', s_1') \, a(p_2', s_2') \quad \sqrt{2E_{p_1}'} \sqrt{2E_{p_2}'}$$

$$\cdot \, N\left(\bar{\psi}^-_{x_1} (ie\gamma^\mu) \, \psi^+_{x_1} \; \bar{\psi}^-_{x_2} (ie\gamma^\nu) \, \psi^+_{x_2} \right)$$

$$a^+(p_2, s_2) \, a^+(p_1, s_1) \, | 0 \rangle \quad \sqrt{2E_{p_1}} \sqrt{2E_{p_2}}$$

SIGN : (-1) !

$$\langle f | S_d^{(2)} | i \rangle$$

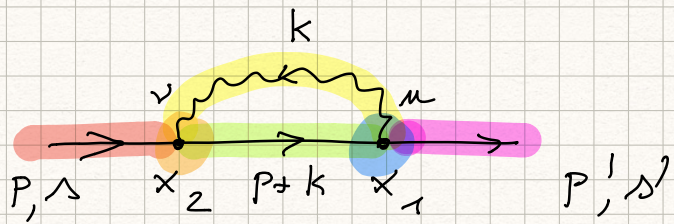
$$= (2\pi)^4 \, \delta^4(p_1 + p_2 - p_1' - p_2')$$

$$\cdot \frac{(-i g_{\mu\nu})}{(p_1 - p_2')^2 + i\varepsilon}$$

$$\cdot \, (-1) \cdot \bar{u}(p_2', s_2') (ie\gamma^\mu) \, u(p_1, s_1)$$

$$\cdot \, \bar{u}(p_1', s_1') (ie\gamma^\nu) \, u(p_2, s_2)$$

$$\mathcal{L} \to N\left((\bar\psi \gamma^\mu A_\mu \psi)_{x_1} \; (\bar\psi \gamma^\nu A_\nu \psi)_{x_2} \right)$$



$$\langle f |\, S^{(2)} \,| i \rangle$$

$$= (2\pi)^4 \, \delta^4(p - p')$$

$$\int \frac{d^4k}{(2\pi)^4} \frac{(-i g_{\mu\nu})}{k^2 + i\varepsilon}$$

$$\cdot\; \bar u(p', s') \, (ie\gamma^\mu) \, \frac{i(\not p + \not k + m)}{(p+k)^2 - m^2 + i\varepsilon} \, (ie\gamma^\nu) \, u(p, s)$$

$$\underbrace{k \to \infty}$$

$$\int d^4k \; \frac{1}{k^2} \quad \bar u \, \gamma^\mu \frac{\not k}{k^2 + i\varepsilon} \gamma^\nu \, u$$

$$\sim \int \frac{d^4k}{k^3} \qquad\qquad \int_{x_1}^{x_2} dx = x_2 - x_1$$

LINEAR DIVERGENCE

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