⇒ LECTURE 14

SCALAR FIELD

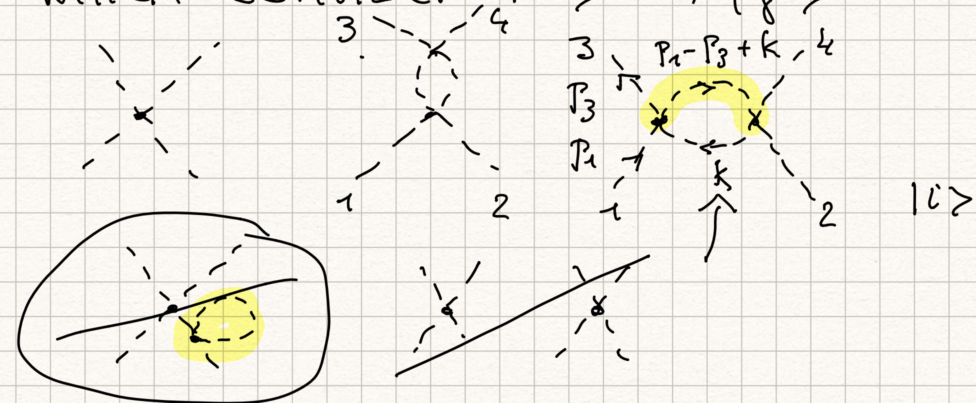
$\langle f | S | i \rangle$ IN MOMENTUM SPACE

$$\mathcal{L}_1 = - \frac{\lambda}{4!} \phi^4$$

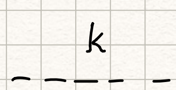
$S^{(1)}, \; S^{(2)}, \; \ldots$

FEYNMAN RULES FOR SCALAR FIELD

⇝ CONSIDERS ALL CONNECTED GRAPHS
TOPOLOGICALLY DIFFERENT
WHICH CONNECT $|i\rangle \to |f\rangle$

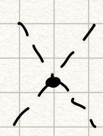


1) $\forall$ PROPAGATOR (INTERNAL LINE)



$$\frac{i}{k^2 - m^2 + i\varepsilon}$$

2) $\forall$ VERTEX



$(-i\lambda)$

3) $\forall$ EXTERNAL LINES $\Longrightarrow$ FACTOR $\underline{1}$

NO LOOPS ATTACHED TO EXT. LINES

4) $\forall$ VERTEX $\Rightarrow$ IMPOSE ENERGY-MOMENTUM CONSERVATION

5) $\int$ OVER INDEPENDENT LOOP MOMENTA

$$\int \frac{d^4 k}{(2\pi)^4}$$

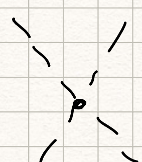
6) DETERMINE SYMMETRY FACTOR $\Rightarrow$ DIVIDE BY IT

7) GLOBAL ENERGY-MOMENTUM CONSERVATION

$$(2\pi)^4 \, \delta^4\left(\sum_i p_i - \sum_f p_f \right)$$

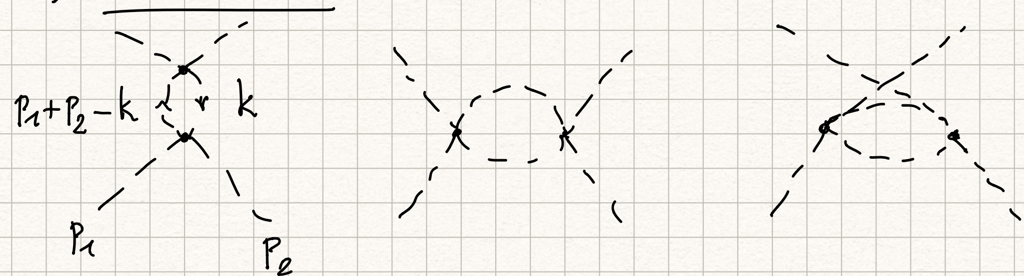
$\hookrightarrow$ <u>1° ORDER</u>

$$\langle f \,|\, S^{(1)} \,|\, i \rangle$$



$$= (-i\lambda) \quad (2\pi)^4 \, \delta^4(p_1 + p_2 - p_3 - p_4)$$

$\hookrightarrow$ <u>2° ORDER</u>



$$\langle f | S^{(2)} | i \rangle \sim \underline{\underline{\int \frac{d^4k}{(2\pi)^4}}} \underbrace{\frac{i}{k^2 - m^2 + i\varepsilon} \, \frac{i}{(p_1+p_2-k)^2 - m^2 + i\varepsilon}}$$

$$\int_{x_1}^{x_2} \frac{dx}{x} = \ln \frac{x_2}{x_1}$$

$k \to \infty$

$$\frac{i}{k^2 + i\varepsilon} \, \frac{i}{k^2 + i\varepsilon}$$

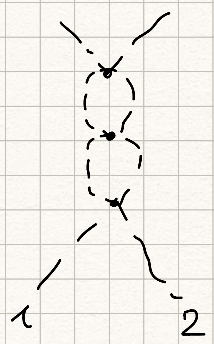
$x_2 >>$

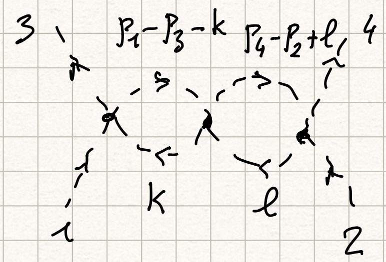
$$\underset{k\to\infty}{\sim} \frac{1}{k^4}$$

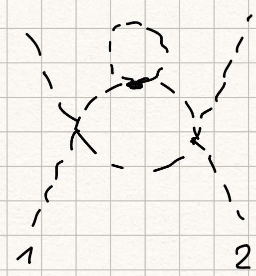
LOG DIVERGENCE

$\rightsquigarrow$ RENORMALIZATION

$\hookrightarrow$







$$\Rightarrow (2\pi)^4 \delta^4(p_1+p_2-p_3-p_4) \frac{(-i\lambda)^3}{S} \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4l}{(2\pi)^4} \frac{i}{k^2-m^2+i\varepsilon} \, \frac{i}{(p_1-p_3-k)^2-m^2+i\varepsilon}$$

$$\frac{i}{l^2-m^2+i\varepsilon} \, \frac{i}{(p_4-p_2+l)^2-m^2+i\varepsilon}$$

4) FEYNMAN RULES FOR QED (SPIN 1/2)

$\Rightarrow$ S - MATRIX AT 1° ORDER

$$\mathcal{L}_1 = -q\,\bar{\psi}\gamma^\mu\psi A_\mu$$

e^- $\quad q = \underbrace{-e}\qquad e>0 \qquad \frac{e^2}{4\pi} = \alpha \approx \frac{1}{137}$

$$\mathcal{L}_1 = e\,\bar{\psi}\gamma^\mu\psi A_\mu$$

$$\mathcal{H}_1 = -e\,N(\bar{\psi}\gamma^\mu\psi A_\mu)$$

$$\langle 0|\mathcal{H}|0\rangle = 0$$

$$S^{(1)} = -i\int d^4x\ \mathcal{H}_1(x)$$

$$= ie\int d^4x\ N\big(\bar{\psi}(x)\,\gamma^\mu\psi(x)\,A_\mu(x)\big)$$

$$\psi(x) = \psi^+(x) + \psi^-(x)$$

$\Uparrow$ ABSORBS e^- $\qquad$ $\Uparrow$ CREATES e^+

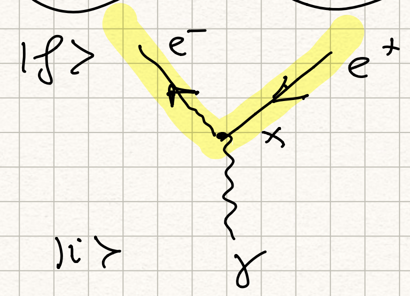
$$\bar{\psi}(x) = \bar{\psi}^-(x) + \bar{\psi}^+(x)$$

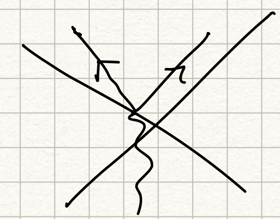
$\Uparrow$ CREATES e^- $\qquad$ $\Uparrow$ ABSORBS e^+

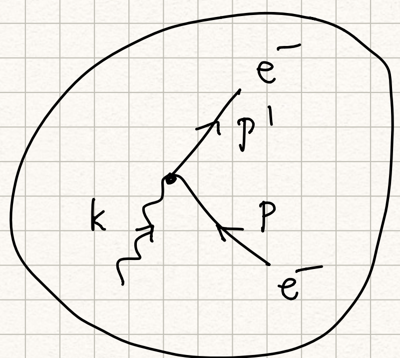
$$A^\mu(x) = A^+(x) + A^-(x)$$

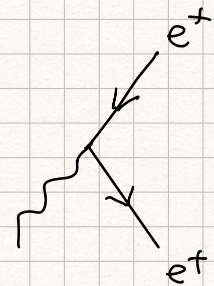
$\Uparrow$ ABSORBS γ $\qquad$ $\Uparrow$ CREATES γ

$$N\left(\bar{\psi}^-(x)\,\gamma^\mu\,\psi^-(x)\,A^+(x)\right)$$









$$\langle f \,|\, S^{(1)} \,|\, i \rangle$$

$$\begin{cases} p^2 = m^2 = p'^2 \\ k^2 = 0 \end{cases}$$

$$p' = p + k \implies p'^2 = (p+k)^2$$
$$= p^2 + k^2 + 2p.k$$

$$m^2 = m^2 + 0 + 2p.k$$

$$\Downarrow$$

$$\underline{\underline{p.k = 0}} \quad \text{ONLY WHEN} \quad \underline{\underline{k^0 = 0}}$$

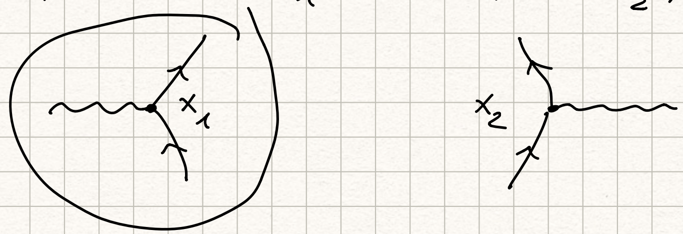
NO PHYSICAL PROCESSES AT 1° ORDER

$\Rightarrow$ S-MATRIX AT 2° ORDER

$$S^{(2)} = \frac{(-i)^2}{2!}\int d^4x_1 \int d^4x_2 \; T\left(\mathcal{H}_1(x_1)\,\mathcal{H}_1(x_2)\right)$$

$$= \frac{(ie)^2}{2!}\int d^4x_1 \int d^4x_2 \; T\left((\bar\psi\gamma^\mu A_\mu\psi)_{x_1}\,(\bar\psi\gamma^\nu A_\nu\psi)_{x_2}\right)$$

$$= N\left((\bar\psi\gamma^\mu A_\mu\psi)_{x_1}\,(\bar\psi\gamma^\mu A_\mu\psi)_{x_2}\right)$$



DISCONNECTED GRAPH

$\downarrow$

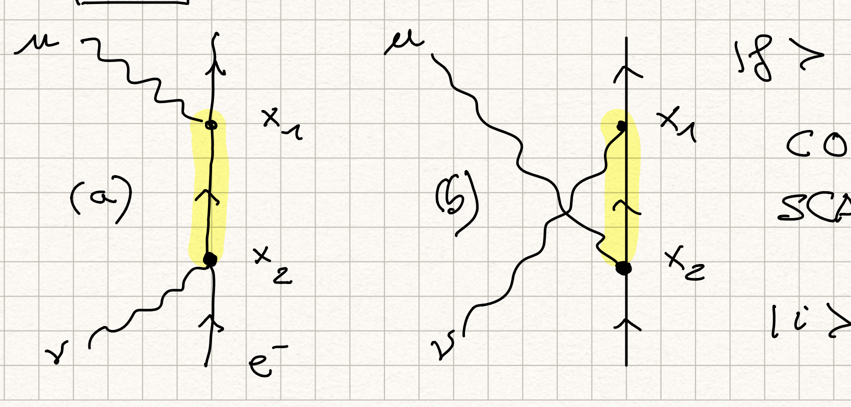
NO CONTRIBUTION

$$+\Big\{N\left((\bar\psi\gamma^\mu A_\mu\psi)_{x_1}\,(\bar\psi\gamma^\nu A_\nu\psi)_{x_2}\right)$$

$$+N\left((\bar\psi\gamma^\mu A_\mu\psi)_{x_1}\,(\bar\psi\gamma^\nu A_\nu\psi)_{x_2}\right)\Big\}$$

$$\psi_\alpha(x_1)\bar\psi_\beta(x_2) = \langle 0|\,T(\psi_\alpha(x_1)\bar\psi_\beta(x_2))\,|0\rangle$$

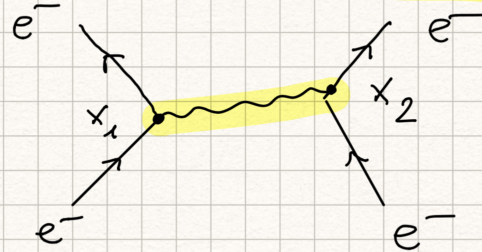
$\Rightarrow$



COMPTON SCATTERING

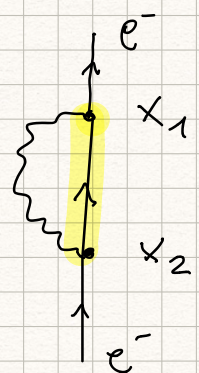
2° TERM IDENTICAL $\Rightarrow$ FACTOR <u>2</u>

$$+ \; N\Big((\bar{\Psi}\gamma^{\mu} A_{\mu} \Psi)_{x_1} \, (\bar{\Psi}\gamma^{\nu} A_{\nu} \Psi)_{x_2} \Big)$$



$$+ \; \Big\{ N\Big((\bar{\Psi}\gamma^{\mu} A_{\mu} \Psi)_{x_1} \, (\bar{\Psi}\gamma^{\nu} A_{\nu} \Psi)_{x_2} \Big)$$

$$+ \; N\Big((\bar{\Psi}\gamma^{\mu} A_{\mu} \Psi)_{x_1} \, (\bar{\Psi}\gamma^{\nu} A_{\nu} \Psi)_{x_2} \Big) \Big\}$$

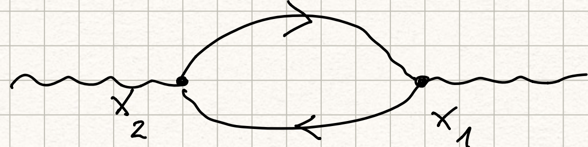


SELF-ENERGY OF e^-

↳ e.g. <u>LAMB SHIFT IN H-SPECTRUM</u>

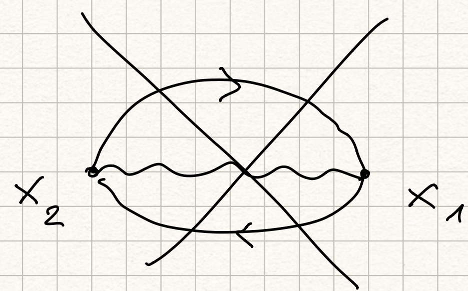
$$E(2s_{1/2}) - E(2p_{1/2}) \neq 0$$

$$+ \; N\Big((\bar{\Psi}\gamma^{\mu} A_{\mu} \Psi)_{x_1} \, (\bar{\Psi}\gamma^{\nu} A_{\nu} \Psi)_{x_2} \Big)$$



PHOTON/VACUUM POLARIZATION

$$+ \; N\Big((\bar\psi \gamma^\mu A_\mu \psi)_{x_1} \; (\bar\psi \gamma^\nu A_\nu \psi)_{x_2} \Big)$$



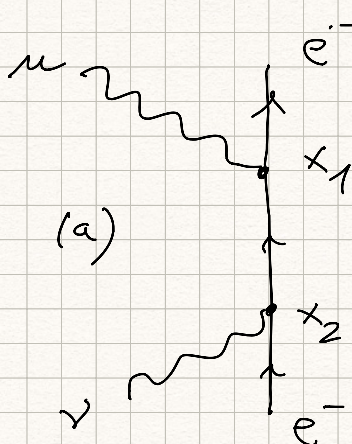
VACUUM DIAGRAM

↓

ABSORBED IN ~~DEF.~~ DEF. OF VACUUM

↳ COMPTON PROCESS TO $S^{(2)}$

$$\longrightarrow \; N\Big((\bar\psi^- \gamma^\mu A^-_\mu \psi)_{x_1} \; (\bar\psi \gamma^\nu A^+_\nu \psi^+)_{x_2} \Big)$$



(a)

$$\psi(x_2) = \psi^+(x_2) + \psi^-(x_2)$$

$$A^\nu(x_2) = A^{\nu +}(x_2) + A^{\nu -}(x_2)$$

$$\langle f \,|\, S_a^{(2)} \,|\, i \rangle$$

$$|i\rangle = |\, \gamma(k_1, \lambda_1)\; e^-(p_1, s_1) \rangle$$

$$= \sqrt{2|\vec{k}_1|}\sqrt{2E_{p_1}}\; a^+(k_1, \lambda_1)\, a^+(p_1, s_1)\, |0\rangle$$

$$|f\rangle = \sqrt{2|\vec{k}_2|}\,\sqrt{2E_{\vec{p}_2}}\;a^{+}(k_2,\lambda_2)\,a^{+}(p_2,s_2)|0\rangle$$

$$\psi^{+}(x_2)\,|e^{-}(p_1,s_1)\rangle = e^{-ip_1 x_2}\,u(p_1,s_1)|0\rangle$$

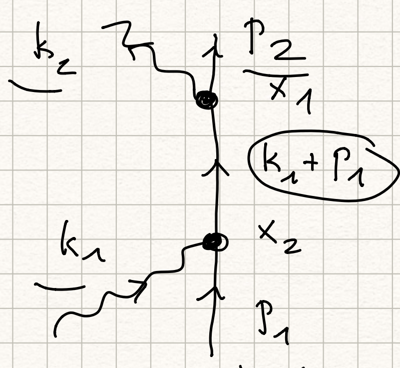
$$\langle e^{-}(p_2,s_2)|\,\bar{\psi}^{-}(x_1) = \langle 0|\,e^{+ip_2\cdot x_1}\,\bar{u}(p_2,s_2)$$

$$A^{\nu +}(x_2)\,|\gamma(k_1,\lambda_1)\rangle = e^{-ik_1\cdot x_2}\,\varepsilon^{\nu}(k_1,\lambda_1)|0\rangle$$

$$\langle\gamma(k_2,\lambda_2)|\,A^{\mu -}(x_1) = \langle 0|\,e^{+ik_2\cdot x_1}\,\varepsilon^{\mu *}(k_2,\lambda_2) \qquad \lambda_i = \pm 1$$

$$\langle f|\,S_a^{(2)}\,|i\rangle$$

$$= -e^2\int d^4x_1\int d^4x_2\,\langle f|\,\bar{\psi}^{-}(x_1)\,\gamma^{\mu}A^{-}_{\mu}(x_1)\; iS_F(x_1-x_2)\,\gamma^{\nu}A^{+}_{\nu}(x_2)\,\psi^{+}(x_2)\,|i\rangle$$



$$\rightsquigarrow \int\frac{d^4k}{(2\pi)^4}\,e^{-ik\cdot(x_1-x_2)}\,\frac{i(\not{k}+m)}{k^2-m^2+i\varepsilon}$$

$$k = k_1 + p_1$$

$$= (2\pi)^4 \, \delta^4(k_1 + p_1 - k_2 - p_2)$$

$$\varepsilon^*_\mu(k_2, \lambda_2) \, \varepsilon_\nu(k_1, \lambda_1)$$

$$\bar{u}(p_2, s_2)(ie\gamma^\mu) \, \frac{i(\not{k}_1 + \not{p}_1 + m)}{(k_1 + p_1)^2 - m^2 + i\varepsilon}$$

$$\cdot (ie\gamma^\nu) \, u(p_1, s_1)$$